Abstract—In syntactic pattern recognition a pattern can be represented by a graph. Given an unknown pattern represented by a graph \( g \), the problem of recognition is to determine if the graph \( g \) belongs to a language \( L(G) \) generated by a graph grammar \( G \). The so-called IE graphs have been defined in [1] for a description of patterns. The IE graphs are generated by so-called ETPL\((k)\) graph grammars defined in [1]. An efficient, parsing algorithm for ETPL\((k)\) graph grammars for syntactic recognition of patterns represented by IE graphs has been presented in [1]. In practice, structural descriptions may contain pattern distortions, so that the assignment of a graph \( g \), representing an unknown pattern, to a graph language \( L(G) \) generated by an ETPL\((k)\) graph grammar \( G \) is rejected by the ETPL\((k)\) type parsing. Therefore, there is a need for constructing effective parsing algorithms for recognition of distorted patterns. The purpose of this paper is to present a new approach to syntactic recognition of distorted patterns. To take into account all variations of a distorted pattern under study, a probabilistic description of the pattern is needed. A random IE graph approach is proposed here for such a description ([2]).

Keywords—Syntactic pattern recognition, Distorted patterns, Random graphs, Graph grammars.

I. INTRODUCTION

THE so-called IE graphs have been defined in [1] for a description of patterns in syntactic pattern recognition. Nodes in an IE graph denote pattern primitives. Edges between two nodes in an IE graph represent spatial relations between pattern primitives. We do not present a definition of an IE graph here. Instead of it, we show an example pattern and its representation by an IE graph. In order to do it, let us assume that a set of edge labels describing spatial relations between primitives of an analyzed pattern is shown in Fig. 1. Now, let us assume that we analyze an example pattern shown in Fig. 2. The analyzed pattern shown in Fig. 2 can be represented by an IE graph shown in Fig. 3.

In practice structural descriptions may contain pattern distortions. For example, because of errors in the primitive extraction process, an IE graph \( g \) representing a pattern under study may be distorted, either in primitive properties or in their spatial relations, so that the assignment of the IE graph \( g \) to a graph language \( L(G) \) generated by an ETPL\((k)\) graph grammar \( G \) is rejected by the ETPL\((k)\) type parsing. Therefore, there is a need for constructing effective parsing algorithms for recognition of distorted patterns, which is the motivation to do research.

The purpose of this paper is to present an idea of a new approach to syntactic recognition of distorted patterns. To take into account all variations of a distorted pattern under study, a probabilistic description of the pattern is needed. A random IE graph approach is proposed here for such a description ([2]). In this paper the idea of an efficient (that is the time complexity is \( O(n^3) \)) parsing algorithm presented in [1] is extended so that distorted patterns, represented by random IE graphs, can be recognized.

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II. REPRESENTATION OF DISTORTED PATTERNS

Let us assume that due to distortions possible (distorted) IE graphs associated with the example pattern shown in Fig. 2 might also be like IE graphs shown in Fig. 4.

The IE graphs shown in Fig. 4 can be considered as outcome IE graphs obtained from a random IE graph ([2]) shown in Fig. 5.

The random IE graph shown in Fig. 5 consists of random nodes \{A, B, C, D\} and random edges \{U, V, X, Y\}. The ranges of A, B, C and D are \{tree\}, \{house, tree\}, \{bus, car\} and \{bus, car\} respectively. The ranges of U, V, X and Y are \{r\}, \{u\}, \{t\} and \{r, s\} respectively. Thus, in the proposed approach, a distorted pattern can be represented by a random IE graph. An example random IE graph with given nodes and edges distributions is shown in Fig. 6.

In the proposed approach it is assumed that random nodes and edges are mutually independent. Thus, the probability of an outcome IE graph, obtained form a random IE graph, is equal to the product of the probabilities of the node labels and the edge labels. For example, Fig. 7 shows an outcome IE graph \(r_1\) obtained from the random IE graph shown in Fig. 6. Similarly, Fig. 8 shows an outcome IE graph \(r_2\) obtained from the random IE graph shown in Fig. 6.

The probabilities of the outcome IE graphs \(r_1\) (Fig. 7) and \(r_2\) (Fig. 8) are the following:

\[
p(r_1) = p(b_1) p(a_2) p(b_2) p(r_{1,3}) p(u_{2,3}) p(s_{3,4}) = (0.8) (0.9) (0.7) (0.8) (0.7) = 0.1422
\]

and

\[
p(r_2) = p(a_1) p(d_2) p(c_3) p(r_{1,3}) p(u_{2,3}) p(s_{3,4}) = (0.2) (0.1) (0.3) (0.2) (0.7) = 0.0002
\]

where indexes denote node numbers.
III. PARSING OF RANDOM GRAPHS FOR RECOGNITION OF DISTORTED PATTERNS

Given an unknown distorted pattern represented by a random IE graph $R$, the problem of recognition of a pattern under study is to determine if an outcome IE graph $r$, obtained from the random IE graph $R$, belongs to a graph language $L(G)$ generated by an ETPL($k$) graph grammar $G$.

It is generally impossible to explore all possible derivation paths during parsing because of combinatorial explosion. Therefore, we propose the following strategy of random IE graph parsing for an efficient (that is the time complexity is $O(n^2)$) analysis of distorted patterns: during a derivation step a number of simultaneously derived graphs spread through the search tree, but only the best graph, that is with maximum probability, is expanded.

Now, we show a derivation step of an ETPL($k$) graph grammar $G$ by means of the following example. Let us consider a graph shown in Fig. 9 (a) and a production shown in Fig. 9 (b). Suppose that the embedding transformation for the production shown in Fig. 9 (b) is the following:

$$C(r, \text{input}) = \{(d, b, r, \text{input})\}$$
$$C(u, \text{output}) = \{(e, B, r, \text{input})\}$$

The detailed discussion on the embedding transformation is presented in [1]. After the application of the production shown in Fig. 9 (b) to the node indexed by 2 of the graph shown in Fig. 9 (a) we obtain a graph shown in Fig. 9 (c).

Further, let us assume that we are given an ETPL(2) graph grammar $G$ with a starting graph $Z$ shown in Fig. 11 and a set of productions shown in Fig. 12.

After the application of the production 1 (shown in Fig. 12) to the node indexed by 2 of the starting graph $Z$ (shown in Fig. 11) we obtain a graph $q_1$ shown in Fig. 13. Similarly, after the application of the production 2 to the node indexed by 2 of the starting graph $Z$ we obtain a graph $q_2$ shown in Fig. 14. Now we calculate the following probability of a subgraph of the graph $q_1$:

$$p(q_1) = p(a_1) \cdot p(b_2) \cdot p(c_3) \cdot p(r_1, 2) \cdot p(s_2, 3) = (1.0) (0.8) (0.9) (0.8) = 0.4608$$

Similarly, we calculate the following probability of a subgraph of the graph $q_2$:

$$p(q_2) = p(a_1) \cdot p(b_2) \cdot p(a_3) \cdot p(r_1, 2) \cdot p(s_2, 3) = (1.0) (0.8) (0.1) (0.8) (0.8) = 0.0512$$

A production 1:
Fig. 12 A set of productions of an ETPL(2) graph grammar G

A production 2:

$$B \rightarrow b_1 \overset{s}{\rightarrow} c_2$$

$$C(r, input) = \{(b, a, r, input)\}$$
$$C(t, output) = \{(b, A, t, output), (c, A, r, input)\}$$

A production 3:

$$A \rightarrow d_1 \overset{s}{\rightarrow} e_2$$

$$C(s, input) = \{(d, a, s, input)\}$$
$$C(t, input) = \{(d, b, t, input)\}$$
$$C(r, output) = \{(d, c, r, output), (d, a, r, output)\}$$
$$C(v, output) = \{(d, D, v, output)\}$$

A production 4:

$$D \rightarrow g_1 \overset{t}{\rightarrow} h_2$$

$$C(t, input) = \{(g, a, t, input)\}$$
$$C(v, input) = \{(h, d, u, input), (h, b, u, input)\}$$

A production 5:

$$D \rightarrow a_1 \overset{s}{\rightarrow} b_2$$

$$C(t, input) = \{(a, a, t, input)\}$$
$$C(v, input) = \{(b, d, u, input), (b, b, u, input)\}$$

Because $p(q_1) > p(q_2)$ we choose the graph $q_1$ (Fig. 13) for further derivation. Now, after the application of the production 3 (Fig. 12) to the node indexed by 3 of the derived graph $q_1$ (Fig. 13) we obtain a graph shown in Fig. 15.

Then, after the application of the production 4 (Fig. 12) to the node indexed by 4 of the derived graph shown in Fig. 15 we obtain a graph shown in Fig. 16.
The derived graph shown in Fig. 16 belongs to a language $L(G)$ generated by the example $ETPL(2)$ graph grammar $G$ (Fig. 11 and Fig. 12). On the other hand the derived graph shown in Fig. 16 is simultaneously an outcome $IE$ graph obtained from the parsed random $IE$ graph shown in Fig. 10. Thus the parsed random $IE$ graph shown in Fig. 10 is accepted. The path of search to the solution is shown in Fig. 17.

Similarly, after the application of the production 5 (Fig. 12) to the node indexed by 4 of the derived graph shown in Fig. 15 we obtain a graph shown in Fig. 18. The derived graph shown in Fig. 18 is simultaneously an outcome $IE$ graph obtained from the parsed random $IE$ graph shown in Fig. 10.

**IV. CONCLUSION**

In this paper we have proposed a new approach to recognition of distorted patterns. To take into account all variations of a distorted pattern under study, a probabilistic description of the analyzed pattern is needed. The random $IE$ graph ([2]) approach has been proposed here for such a description. In this paper the idea of an efficient (that is the time complexity is $O(n^2)$) parsing algorithm presented in [1] is extended, so that distorted patterns, represented by random $IE$ graphs, can be recognized. The proposed approach involves $ETPL(k)$ type parsing ([1]) of random $IE$ graphs.

A system for analysis of distorted patterns, represented by random $IE$ graphs, based on the proposed here approach, has been implemented at the Institute of Computer Science, Jagiellonian University, Kraków, Poland ([3]). The time complexity $O(n^2)$ of the proposed $ETPL(k)$ type parsing of random $IE$ graphs have been checked by several experiments. The results of a one of such experiments connected with measuring parsing time with a number of nodes of an analyzed random $IE$ graph are shown in Fig. 19 ([3]).

**REFERENCES**