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Vibration Suppression of Timoshenko Beams with Embedded Piezoelectrics Using POF

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Abstract**—**This paper deals with the design of a periodic output feedback controller for a flexible beam structure modeled with Timoshenko beam theory, Finite Element Method, State space methods and embedded piezoelectrics concept. The first 3 modes are considered in modeling the beam. The main objective of this work is to control the vibrations of the beam when subjected to an external force. Shear piezoelectric sensors and actuators are embedded into the top and bottom layers of a flexible aluminum beam structure, thus making it intelligent and self-adaptive. The composite beam is divided into 5 finite elements and the control actuator is placed at finite element position 1, whereas the sensor is varied from position 2 to 5, i.e., from the nearby fixed end to the free end. 4 state space SISO models are thus developed. Periodic Output Feedback (POF) Controllers are designed for the 4 SISO models of the same plant to control the flexural vibrations. The effect of placing the sensor at different locations on the beam is observed and the performance of the controller is evaluated for vibration control. Conclusions are finally drawn.

Keywords**—**Smart structure, Timoshenko beam theory, Periodic output feedback control, Finite Element Method, State space model, SISO, Embedded sensors and actuators, Vibration control.

I. INTRODUCTION

ACTIVE control of vibrations relieves a designer from strengthening the structure from dynamic forces and the structure itself from extra weight and cost. The need for intelligent structures such as smart structures arises from the high performance requirements of such structural members in numerous applications. Intelligent structures are those which incorporate actuators and sensors that are highly integrated into the structure and have structural functionality, as well as highly integrated control logic, signal conditioning and power amplification electronics [2].

Many researchers have proposed and demonstrated AVC schemes for vibration suppression in recent years. The early studies were mostly focused on surface glued piezoceramics and PZT's to the structure. But, these have some disadvantages such as difficulties to protect the ceramics and the connection wires, protection from external environment, leads coming out while vibrating, bad coupling with the surface, low-signal-to-noise ratio, e.t.c.,. These problems can be solved with the embedded piezoceramics in between the master structures. This paper deals with the active vibration control of a Timoshenko beam for a SISO case using embedded piezoelectrics as both actuators and sensors using POF method.

The work presented in this paper is organized in the following sequence. A review of related literature about the types of beam models and embedded shear actuation is given in section 2. Section 3 gives a introduction to the modeling technique (finite element model, sensor and actuator model, state space model) of the smart flexible cantilever beam using Timoshenko beam theory. A brief review of the controlling technique, viz., the periodic output feedback control technique is presented in section 4. The design of the proposed controller to control the first three dominant modes of vibration of the system for different embedded sensor locations along the length of the beam for the various SISO models of the same plant is discussed in Section 5. The control simulation results and discussions are presented in Section 6. Section 7 concludes the paper with conclusions followed by the acronyms and the references.

II. REVIEW OF VARIOUS BEAM THEORIES

A precise mathematical model is required for the controller design for vibration control to predict the structure's response. Two beam models in common use in structural mechanics are the Euler-Bernoulli beam model and the Timoshenko beam model, which were discussed in brief in [38], [39].

In the Euler-Bernoulli model [2], [3], [5], [10] bending effects, stresses, moments and deformations are considered. The effect of shear, axial displacement is neglected as a result of which accurate model of the system is not available for sophisticated control. The assumption that we make while developing this model is that the cross sections of the beams remain plane and normal to the deformed longitudinal axis before and after bending. Extensive research was carried out using this theory by many researchers. The concept of smart structures was presented in the survey paper by Culshaw [22] and Rao [6]. Smart structures and its numerous applications was presented by Baily and Hubbard [5]. Manjunath and

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Bandyopadhyay [23] presented the active vibration control scheme using the POF method. The effect of failure of one of the actuators in a multivariable smart system and its control using the periodic output feedback control law was discussed in [24].

In the Timoshenko model [8], [9], [12] - [15], [37] - [39] the axial displacement and the shear is taken into consideration while developing the model for the structure. Thus, the theory corrects the drawbacks and the assumptions made in Euler-Bernoulli model Theory. The cross sections remain plane and rotate about the same neutral axis as the Euler-Bernoulli model, but do not remain normal to the deformed longitudinal axis. The deviation from normality is produced by a transverse shear that is assumed to be constant over the cross section. Because of the above-mentioned reasons, the Timoshenko Beam model is far more superior to the Euler-Bernoulli model in precisely predicting the beam response.

In this context, a brief review of the research work done on Timoshenko beams with surface mounted sensors / actuators and using embedded sensors / actuators is presented. Aldraihem *et al.* [13] have developed a laminated beam model using two theories; namely, Euler-Bernoulli beam theory and Timoshenko Beam theory. Abramovich [14] has presented analytical formulation and closed form solutions of composite beams with piezoelectric actuators, which was based on Timoshenko beam theory.

Using a higher-order shear deformation theory, Chandrashekhara and Varadarajan [12] presented a finite element model of a composite beam to produce a desired deflection in beams with clamped-free (C-F), clamped-clamped (C-C) and simply supported ends. Shear embedded piezoelectrics are used nowadays to suppress the vibrations. Sun and Zhang [9], [15] suggested the idea of exploiting the shear mode to create transverse deflection in sandwich structures. Here, he proved that embedded shear actuators offer many advantages over surface mounted extension actuators.

Aldraihem and Khdeir [18] proposed analytical models and exact solutions for beams with shear and extension piezoelectric actuators and the models were based on Timoshenko beam theory and higher-order beam theory (HOBT). Exact solutions were obtained by using the state-space approach. In a more recent work, Zhang and Sun [15] formulated an analytical model of a sandwich beam with shear piezoelectric actuator that occupies the entire core. The model derivation was simplified by assuming that the face layers follow Euler-Bernoulli beam theory, whereas the core layer obeys Timoshenko beam theory. Furthermore, a closed form solution of the static deflection was presented for a cantilever beam. Abramovich [14] studied the effects of actuator location and number of patches on the actuator's performance for various configurations of the piezo patches and boundary conditions under mechanical and / or electric loads.

A FEM approach was used by Benjeddou *et. al.* [17] to model a sandwich beam with shear and extension piezoelectric elements which is used in our work. The FE model employed the displacement field of Zhang and Sun [9] [15]. It was shown that the finite element results agree quite well with the analytical results. Raja *et. al.* [16] extended the finite element model of Benjeddou's research team to include a vibration control scheme. In [39], Manjunath and Bandyopadhyay discussed the modeling and vibration control technique for Timoshenko beam with embedded sensors and actuators for SISO systems using FOS law. Here, in this paper, we control the same models using POF law when the beam is subjected to an external disturbance. An improved 2-node Timoshenko beam model was presented by Friedman and Kosmataka [26] that is used in our work. Azulay *et al.* [27] have presented analytical formulation and closed form solutions of composite beams with piezoelectric actuators, which is used in our work. Work on cross-ply beams was done by Abramovich and Lishvits [33] which is also used in our work.

III. Mathematical Modeling of Smart Sandwiched Beam with Embedded Shear Sensors and Actuators

The mathematical modeling of the sandwiched beam is presented in [26], [27], [33], [38], [39]. Accurate model of the system is obtained when the shear effects and the axial displacement of the beam is considered in modeling of the smart structure.

A. Finite Element Modeling [F E M] of the Sandwiched Beam Element [35] [28]

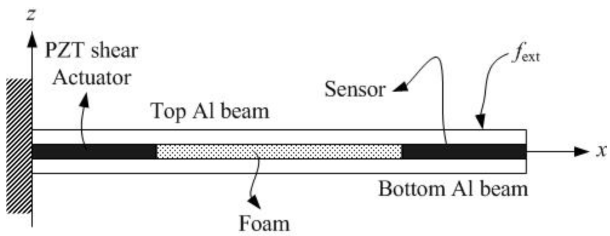


Fig. 1 A flexible sandwiched Timoshenko beam with embedded shear sensor and actuator with disturbing force applied at free end

A sandwiched beam (piezo-laminated composite beam) as shown in Fig. 1 is considered. This beam consists of 3 layers, viz., the piezo-*patch* with the rigid foam is sandwiched in between two aluminum beam layers. For shear actuation, rigid foam is introduced as a core along with PZT to obtain an equivalent sandwiched model. The assumption made is that the middle layer is perfectly glued to the carrying structure. The thickness of the adhesive is neglected (thus, neglecting the effect of shear-lag, no slippage or delamination between the core layers during vibrations), as a result of which strong coupling exists between the master structure and the piezo-patches. The beam is stacked properly and then used as a composite structure for AVC [9], [15], [26], [27], [33]. The beam properties and the assumptions made are presented in [39].

Equations of motion of a general non-symmetric piezo-laminated composite beam with shear deformation and rotary inertia is obtained as [26], [27], [33], [34], [37], [39]

$$\frac{\partial}{\partial x}\left(A_{11}\frac{\partial u}{\partial x}+B_{11}\frac{\partial \theta}{\partial x}+E_{11}\right)=\frac{\partial}{\partial t}\left[\left(I_1\dot{u}\right)+\left(I_2\dot{\theta}\right)\right], \quad (1)$$

$$\frac{\partial}{\partial x}\left(A_{55}\left(\theta+\frac{\partial w}{\partial x}\right)+G_{55}\right)=\frac{\partial}{\partial t}\left[\left(I_1\dot{w}\right)+q_0\right], \quad (2)$$

$$\frac{\partial}{\partial x}\left(B_{11}\frac{\partial u}{\partial x}+D_{11}\frac{\partial \theta}{\partial x}+F_{11}\right)-A_{55}\left(\theta+\frac{\partial w}{\partial x}\right)-G_{55} =\frac{\partial}{\partial t}\left[\left(I_3\dot{\theta}\right)+\left(I_2\dot{u}\right)\right]. \quad (3)$$

Let

$$w=a_1+a_2\,x+a_3\,x^2+a_4\,x^3, \quad (4)$$

$$\theta=b_1+b_2\,x+b_3\,x^2, \quad (5)$$

$$u=c_1+c_2\,x+c_3\,x^2 \quad (6)$$

be the solutions of the Eqns. (1) to (3) where a_i , b_j and c_j 's are the unknown coefficients $(i=1,.....,4)$ and $(j=1,.....,3)$ subject to the boundary conditions

$$\begin{array}{lllll} \text{at} & x=0 & w=w_1 & \theta=\theta_1 & u=u_1, \\ & x=L & w=w_2 & \theta=\theta_2 & u=u_2, \end{array} \quad (7)$$

where x is the local axial coordinate of the element. The unknown coefficients in the Eqns. (4)-(6) are obtained as [39]

$$[u]=[N_u]\begin{Bmatrix} u_1 \\ w_1 \\ \theta_1 \\ u_2 \\ w_2 \\ \theta_2 \end{Bmatrix},\ [w]=[N_w]\begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix},\ [\theta]=[N_\theta]\begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix}, \quad (8)$$

where N_u, N_w N_θ are the mode shape functions due to the axial displacement, transverse displacement and due to the rotation or the slope, which are defined as

$$[N_u]=[N_1 \quad N_2 \quad N_3 \quad N_4 \quad N_5 \quad N_6], \quad (9)$$

$$[N_w]=[N_7 \quad N_8 \quad N_9 \quad N_{10}], \quad (10)$$

$$[N_\theta]=[N_{11} \quad N_{12} \quad N_{13} \quad N_{14}] \quad (11)$$

with the elements of the shape function given in [39].

Writing the 3 shape functions N_u, N_w N_θ in matrix form, we get the relation between the vector of inertial forces N and the vector of nodal displacements $\mathbf{q}$ (displacement field) as

$$\{\mathrm{N}\}=[S]\{\mathbf{q}\}. \quad (12)$$

The mass matrix of the particular regular beam element [30]-[32] is given by

$$[M]=\int_0^l [N]^T [I][N]\,dx. \quad (13)$$

The stiffness matrix of the particular regular beam element [29], [30]-[32] is given by

$$[K]=\int_0^l [B]^T [D][B]\,A\,dx. \quad (14)$$

The values of the matrix coefficients are given in the [39]. The mass and stiffness matrices of the regular beam element are obtained using foam as the core between two facing aluminum layers. The mass and stiffness matrices of the piezoelectric beam element are obtained by using a shear piezoelectric patch between the two facing aluminum layers.

B. Sensor and Actuator Equations

In this section, modeling of the sensor and actuator equation [4], [36] is presented.

The sensor equation is modeled as

$$V^s(t)=\mathbf{p}^T\,\dot{\mathbf{q}}, \quad (15)$$

where $\dot{\mathbf{q}}$ is the time derivative of the modal coordinate vector (strain rate) and $\mathbf{p}^T$ is a constant vector which depends on the type of sensor and its finite element location in the embedded structure.

The input voltage to the actuator is $V^a(t)$ and is given by

$$V^a(t)=K_c\,V^s(t), \quad (16)$$

where K_c is the gain of the controller.

This controlling voltage when applied to the actuator from the controller results in the generation of moments. The work done by this moment results in the generation of the control force that is applied by the actuator as

$$\mathbf{f}_{ctrl}=\mathbf{h}\,V^a(t), \quad (17)$$

where $\mathbf{h}^T$ is a constant vector which depends on the type of actuator and its finite element location in the embedded structure.

If any external forces described by the vector $\mathbf{f}_{ext}$ are acting then, the total force vector becomes

$$\mathbf{f}^t=\mathbf{f}_{ext}+\mathbf{f}_{ctrl}. \quad (18)$$

C. Dynamic equation of the smart structure

The governing equation of the smart structure is obtained as

$$\mathbf{M}\,\ddot{\mathbf{q}}+\mathbf{K}\mathbf{q}=\mathbf{f}_{ext}+\mathbf{f}_{ctrl}=\mathbf{f}^t, \quad (19)$$

where $\mathbf{M}$ is the assembled mass matrix of the smart structure, $\mathbf{K}$ is the assembled stiffness matrix of the smart structure [36], $\mathbf{q}$ is the nodal variable vector and $\ddot{\mathbf{q}}$ is the acceleration vector.

The generalized coordinates are introduced in Eqn. (19) by using a transformation $\mathbf{q}=\mathbf{T}\,\mathbf{g}$ in order to reduce it further such that the resultant equation represents the dynamics of the desired number of modes of vibration and the uncoupled equations are obtained [23] - [25], [29]. $\mathbf{T}$ is the modal matrix containing the eigenvectors representing the number of modes of vibration of the cantilever beam. The equation (19) after applying the transformation and further simplifying becomes

$$\mathbf{M}^* \ddot{\mathbf{g}} + \mathbf{K}^* \mathbf{g} = \mathbf{f}^*_{ext} + \mathbf{f}^*_{ctrl} \,, \tag{20}$$

where the matrices $\mathbf{M}^*, \mathbf{K}^*$, $\mathbf{f}^*_{ext}$ and $\mathbf{f}^*_{ctrl}$ are the generalized mass matrix, the generalized damping matrix, the generalized external force vector and the generalized control force vector. The structural damping matrix is introduced into Eqn. (20) by using

$$\mathbf{C}^* = \alpha\, \mathbf{M}^* + \beta\, \mathbf{K}^* \tag{21}$$

where $\mathbf{C}^*$ is the generalized damping matrix (also called as Rayleigh damping), which is of the form given in Eqn. (21), α and β being the structural constants [36]. The dynamic equation of the smart structure is finally given by

$$\mathbf{M}^* \ddot{\mathbf{g}} + \mathbf{C}^* \dot{\mathbf{g}} + \mathbf{K}^* \mathbf{q} = \mathbf{f}^*_{ext} + \mathbf{f}^*_{ctrl} \,. \tag{22}$$

D. *State Space Model of the Smart Structure*

The governing equation in (22) is written in the state space form and is obtained as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & I \\ -\mathbf{M}^{*-1}\mathbf{K}^* & -\mathbf{M}^{*-1}\mathbf{C}^* \end{bmatrix}_{(6\times 6)} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \tag{23}$$

$$\begin{bmatrix} 0 \\ \mathbf{M}^{*-1}\mathbf{T}^T\mathbf{h} \end{bmatrix}_{(6\times 1)} \mathbf{u}(t) + \begin{bmatrix} 0 \\ \mathbf{M}^{*-1}\mathbf{T}^T\mathbf{f} \end{bmatrix}_{(6\times 1)} r(t),$$

where $u(t)$ is the control input, $r(t)$ is the external input to the system (impulse disturbance), $\mathbf{f}$ is the external force coefficient vector.

The sensor voltage is taken as the output of the system and is given by

$$V^s(t) = \mathbf{p}^T \dot{\mathbf{q}} = y(t), \tag{24}$$

which can be written in the state space form as

$$y(t) = \begin{bmatrix} 0 & \mathbf{p}^T\mathbf{T} \end{bmatrix}_{(1\times 6)} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} . \tag{25}$$

Thus, the state space model of the smart system (the state space equation and the output equation) is obtained by combing the Eqns. (23) and (25) as

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t) + \mathbf{E}\,\mathbf{r}(t), \quad y(t) = \mathbf{C}^T\,\mathbf{x}(t) \tag{26}$$

where $r(t), u(t), \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}, x(t)$ and $y(t)$ represents the external force input, the control input, system matrix, input matrix, output matrix, transmission matrix, external load matrix, state vector, system output (sensor output).

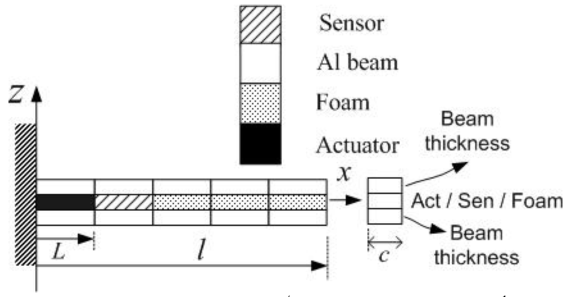


Fig. 2 Smart beam (actuator at 1st position, sensor at 2nd position)

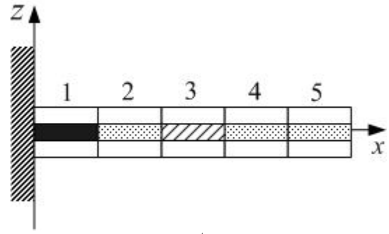


Fig. 3 Smart beam (actuator at 1st position, sensor at 3rd position)

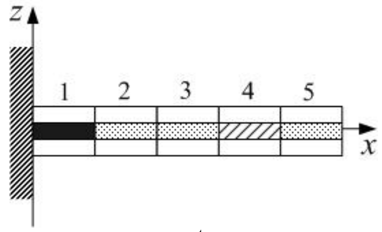


Fig. 4 Smart beam (actuator at 1st position, sensor at 4th position)

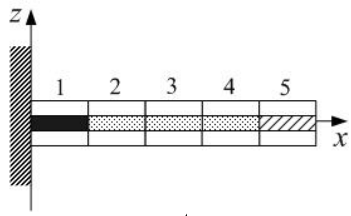


Fig. 5 Smart beam (actuator at 1st position, sensor at 5th position)

The values of the **A, B, C, D, E** matrices for the 4 SISO models are given shown in the Figs. 2-5 is given in [39] along with their characteristics.

IV. Design of Periodic Output Feedback Controller

In the following section, we develop the control strategy for the SISO representation of the developed smart structure model using the periodic output feedback control law [20], [21], [23], [24], [38], [40]-[42] with 1 actuator input u and 1 sensor output y for the various SISO models of the same plant as shown in Figs. 2 - 5. The problem of pole assignment by piecewise constant output feedback was studied by Chammas and Leondes [40]-[42] for LTI systems with infrequent observations. They have shown that by the use of a periodically time-varying piecewise constant output feedback gain, the poles of the discrete time control system could be assigned arbitrarily (within the natural restriction that they should be located symmetrically with respect to the real axis) using the POF technique. Since the feedback gains are piecewise constants, their method could easily be implemented, guarantees the closed loop stability and indicated a new possibility. Such a control law can stabilize a much larger class of systems.

Consider a LTI CT system [21], [23] - [25], [38], [40]-[42] given by

$$\dot{x} = Ax + Bu,\ \ y = Cx, \tag{27}$$

which is sampled with a sampling interval of τ secs given by the discrete Linear Time Invariant (LTI) system (called as the tau system) as

$$x(k+1) = \Phi_\tau x(k) + \Gamma_\tau u(k),\ \ y(k) = C\,x(k), \tag{28}$$

where $x \in \Re^n$, $u \in \Re^m$, $y \in \Re^p$ and Φ_τ, Γ_τ and C are constant matrices of appropriate dimensions. The following control law is applied to this system. The output y is measured at the time instant $t = k\tau$, $k = 0,1,2,.....$ We consider constant hold function because they are more suitable for implementation. An output-sampling interval is divided into N sub-intervals of length $\Delta = \tau / N$ and the hold function is assumed to be constant on these sub-intervals as shown in the Fig. 6. Thus, the control law becomes

$$\begin{aligned} u(t) &= K_l\ y(k\tau), \\ \left(k\tau + l\Delta\right) &\le k\tau \le \left(l+1\right)\Delta,\quad K_{l+N} = K_l \end{aligned} \tag{29}$$

for $l = 0,1,2,.....,(N-1)$. Note that a sequence of N gain matrices $\{K_0, K_1, ..., K_{N-1}\}$, when substituted in (29), generates a time-varying piecewise constant output feedback gain $\mathbf{K}(t)$ for $0 \le t \le \tau$.

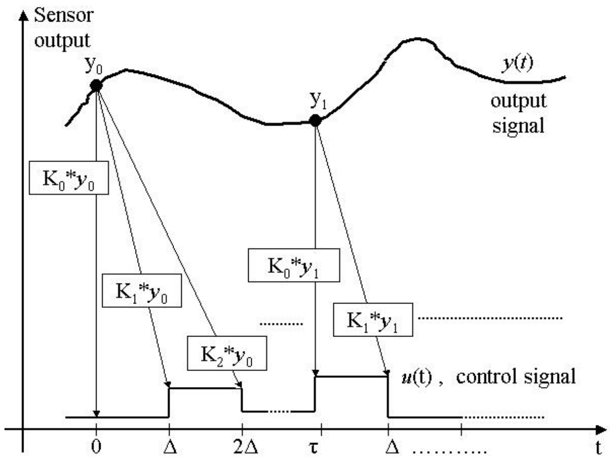


Fig. 6 Graphical illustration of the POF control law

Consider the following system, which is obtained by sampling the system in (27) at sampling interval of $\Delta = \tau / N$ and denoted by $\left(\Phi, \Gamma, C\right)$ called as the delta system :

$$x(k+1) = \Phi\,x(k) + \Gamma u(k),\ \ y(k) = C\,x(k), \tag{30}$$

Assume $\left(\Phi_\tau, C\right)$ is observable and $\left(\Phi, \Gamma\right)$ is controllable with controllability index ν such that $N \ge \nu$, then it is possible to choose a gain sequence K_l, such that the closed-loop system, sampled over τ, takes the desired self-conjugate set of eigen values. Here, we define

$$\mathbf{K} = \begin{bmatrix} K_0 \\ K_1 \\ K_2 \\ \vdots \\ K_{N-1} \end{bmatrix}, \tag{31}$$

$$\mathbf{u}(k\tau) = \mathbf{K}\ y(k\tau) = \begin{bmatrix} u(k\tau) \\ u(k\tau + \Delta) \\ \vdots \\ u(k\tau + \tau - \Delta) \end{bmatrix}, \tag{32}$$

then, a state space representation for the system sampled over τ is

$$\begin{aligned} x(k\tau + \tau) &= \mathbf{\Phi}^N\ x(k\tau) + \mathbf{\Gamma}\,u\,(k\tau), \\ y(k\tau) &= C\,x(k\tau), \end{aligned} \tag{33}$$

where $\mathbf{\Gamma} = \left[\Phi^{N-1}\Gamma, ..., \Gamma\right]$.

Applying POF in Eqn. (29), i.e., $\mathbf{K}\ y(k\tau)$ is substituted for $\mathbf{u}(k\tau)$, the closed loop system becomes

$$x\left(k\tau + \tau\right) = \left(\Phi^N + \mathbf{\Gamma K}C\right) x(k\tau). \tag{34}$$

The problem has now taken the form of static output feedback [19], [20], [25], [40]. Equation (34) suggests that an output injection matrix G be found such that

$$\rho\left(\Phi^N + GC\right) < 1, \tag{35}$$

where $\rho(\ )$ denotes the spectral radius. By observability, one can choose an output injection gain G to achieve any desired self-conjugate set of eigen values for the closed-loop matrix $\left(\Phi^N + GC\right)$ and from $N \ge \nu$, it follows that one can find a POF gain which realizes the output injection gain G by solving

$$\mathbf{\Gamma K} = G \tag{36}$$

for $\mathbf{K}$. The controller obtained from this equation will give the desired behaviour, but might require excessive control action. To reduce this effect, we relax the condition that $\mathbf{K}$ exactly satisfy the linear equation and include a constraint on it. Thus, we arrive at the following in the inequality equation

$$\left\|\mathbf{K}\right\| < \rho_1,\ \left\|\mathbf{\Gamma}\ \mathbf{K} - G\ \right\| < \rho_2. \tag{37}$$

Using the schur complement, it is straight forward to bring these conditions in the form of linear matrix inequalities [1], [7], [11] as

$$\begin{bmatrix} -\rho_1{}^2 I & \mathbf{K} \\ \mathbf{K}^T & -I \end{bmatrix} < 0,\ \begin{bmatrix} -\rho_2{}^2 I & \left(\mathbf{\Gamma}\ \mathbf{K} - G\right) \\ \left(\mathbf{\Gamma}\ \mathbf{K} - G\right)^T & -I \end{bmatrix} < 0 \tag{38}$$

In this form, the LMI toolbox of MATLAB can be used for the synthesis of $\mathbf{K}$ [1], [7], [11]. The POF controller obtained by this method requires only constant gains and is hence easier to implement.

Werner and Furuta [20], [21] proposed a performance index so that $\mathbf{\Gamma K} = G$ need not be forced exactly. This constraint is replaced by a penalty function, which makes it possible to enhance the closed loop performance by allowing slight deviations from the original design and at the same time improving the behaviour. The performance index $J(k)$ is given by

$$J(k) = \sum_{i=0}^{\infty} \begin{bmatrix} x_l^T & u_l^T \end{bmatrix} \begin{bmatrix} \overline{Q} & 0 \\ 0 & R \end{bmatrix} \begin{bmatrix} x_l \\ u_l \end{bmatrix} + \sum_{k=1}^{\infty} \left(x_{kN} - x_{kN}^*\right)^T \overline{P} \left(x_{kN} - x_{kN}^*\right), \tag{39}$$

where $R \in \Re^{m \times m}$, $\overline{Q}$, $\overline{P} \in \Re^{n \times n}$ are positive definite and symmetric weight matrices, x_l and u_l denote the states and the inputs of the delta system respectively and x_{kN}^* denotes the state that would be reached at the instant kN, given $x_{(k-1)N}$, if K is solved to satisfy (36) exactly, i.e., $x_{kN}^* = \left(\Phi^N + GC\right) x_{(k-1)N}$.

The first term represents the 'averaged' state and control energy whereas the second term penalizes the deviation of G. A trade-off between the closed loop performance and closeness to the chosen design is expressed by the above cost function.

V. POF Controller Design for the SISO models

The FEM model of the smart cantilever beam based on Laminate Beam Theory is developed using MATLAB. Different state space models of the smart cantilever beam are obtained by keeping the actuator location fixed (i.e., at fixed end) and varying the position of the sensor from the nearby fixed end to the free end as shown in the Figs. 2 - 5. A periodic output feedback controller discussed in the previous section is designed to suppress the first three modes of vibration of the smart cantilever beam. All simulations are done using MATLAB. The performance of the beam is evaluated for vibration control with the proposed control technique.

The first task in designing the periodic output feedback controller is the selection of the sampling interval τ. The maximum bandwidth for the sensor / actuator locations on the beam are calculated (here, the third vibratory mode of the plant). Then, by using the existing empirical rules for selecting the sampling interval based on bandwidth, approximately 10 times of the maximum third vibration mode frequency of the system is selected. The sampling interval τ used is 0.004 secs.

Four different configurations of the beam in Fig. 1 are considered and shown in Figs. 2 - 5 respectively. In all the four cases, the length of the beam is 30 cm and its cross section is 1 mm by 2 cm. The length of the peizo patch is 6 cm and its cross section is 1 mm by 2 cm. The only change in all the 4 models is in the location of the sensor. The material properties used for the generation of the FEM model are given in [39]. A sixth order state space model of the system is obtained on retaining the first three modes of vibration of the system as shown in Section 3. The first three natural frequencies obtained are 52.03 Hz, 97.21 Hz and 145.81 Hz.

A external force $\mathbf{f}_{ext}$ (impulse disturbance) of 1 Newton is applied for a duration of 50 ms at the free end of the beam for the systems shown in Figs. 2 to 5 and the open loop impulse responses (without control) of the system are observed. Controllers based on the periodic output feedback control algorithm are designed to control the first three modes of vibration of the smart cantilever beam with embedded shear sensor and actuator. The sampling interval used is 0.004 secs and is divided into 10 subintervals $(N = 10)$.

Let $\left(\Phi_{\tau i}, \Gamma_{\tau i}, C_i\right)$ with i = 1 to 4 be the discrete time systems (tau system) of the systems (Figs. 2 - 5) in Eqn. (26) sampled at a rate of $1/\tau$ seconds respectively. It is found that the tau systems are controllable and observable and the tau systems are given in [39]. The stabilizing output injection gains are obtained for the tau system such that the eigenvalues of $\left(\Phi_i^N + G_i C_i\right)$ lie inside the unit circle and the response of the system has a good settling time. The output injection gain obtained is as

$$G_1 = \begin{bmatrix} -4.05 & 5.32 & 2.24 & 1.81 & -8.62 & 3.63 \end{bmatrix} \tag{40}$$

for the model 1. Similarly, the output injection gain for the other three models is obtained. The closed loop impulse response of the 4 models of the system with the output injection gain G is also observed.

Let $\left(\Phi_i, \Gamma_i, C_i\right)$ be the discrete time systems (delta system) of the 4 models of the Fig. 1 in Eqn. (26) sampled at the rate $1/\Delta$ secs respectively, where $\Delta = \tau / N$. The number of sub-intervals, N is chosen to be 10. The delta system for the SISO models are given in [39].

The periodic output feedback gain matrix $\mathbf{K}$ for the smart system given is obtained by solving $\Gamma \mathbf{K} = C$ using the performance index method [20], [21] which reduces the amplitude of the control signal u. With the designed controller put in the loop, the closed loop impulse response (sensor output y) with the periodic output feedback gain $\mathbf{K}$ of the system are observed. The POF gain matrix $\mathbf{K}$ for the SISO models of the smart Timoshenko beam is given by

$$\begin{aligned} \mathbf{K}_1^T = [&32.10 \;\; -45.78 \;\; 45.32 \;\; -32.78 \;\; 12.92 .... \\ &....12.49 \;\; -36.69 \;\; 49.68 \;\; -49.34 \;\; 37.73 \;] \end{aligned} \tag{41}$$

for the model 1. Similarly, the POF gains are obtained for the other 3 models of the smart structure plant. The closed loop impulse responses (sensor outputs y) of all the models with periodic output feedback gain $\mathbf{K}$ of the system is observed. Also, the variation of the control signal u with time for the systems is observed and the conclusions are drawn.

VI. Simulation Results

In this section, we present the simulation results of the 4 SISO models. The following figures (Figs. 7 - 10) shows the open loop response, closed loop response with the output injection gain G, the closed loop response with the POF gain $\mathbf{K}$ and the magnitude of the control input u with time t for the smart cantilever beam with actuator at the first position and sensor location varied from second finite element position to the fifth finite element position. The simulation result shows the effectiveness of the proposed control.

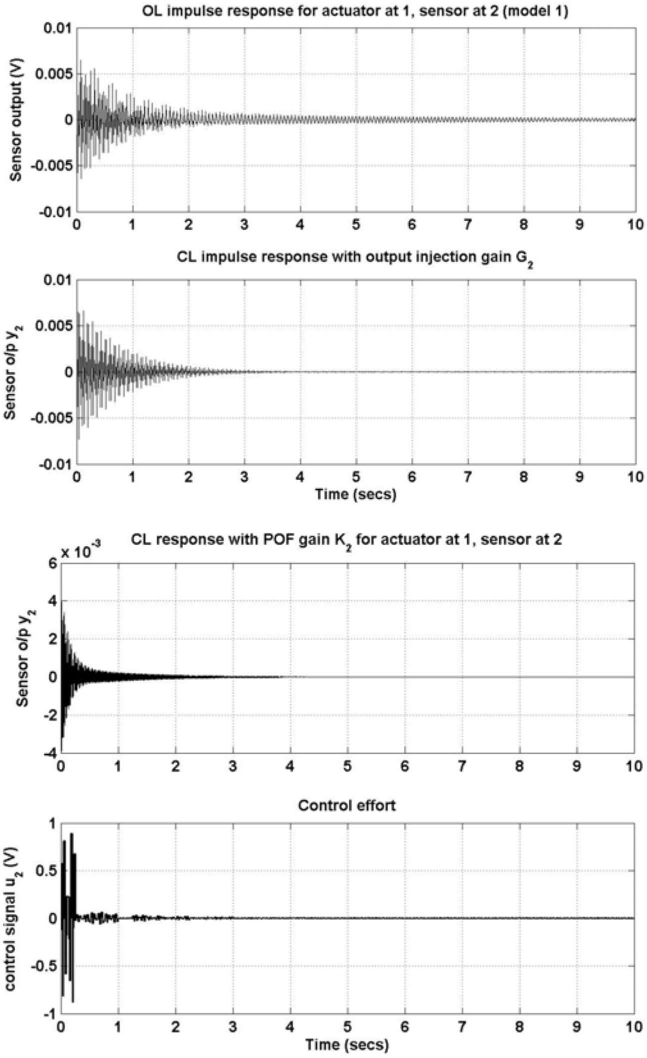


Fig. 7 OL / CL response with G and $\mathbf{K}$ / control u for model 1

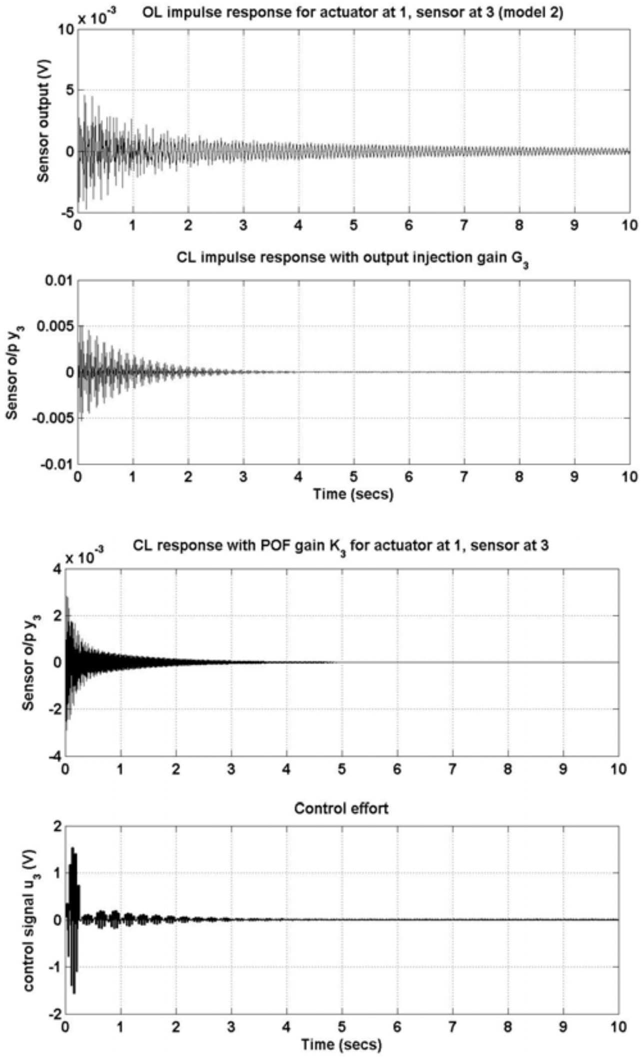
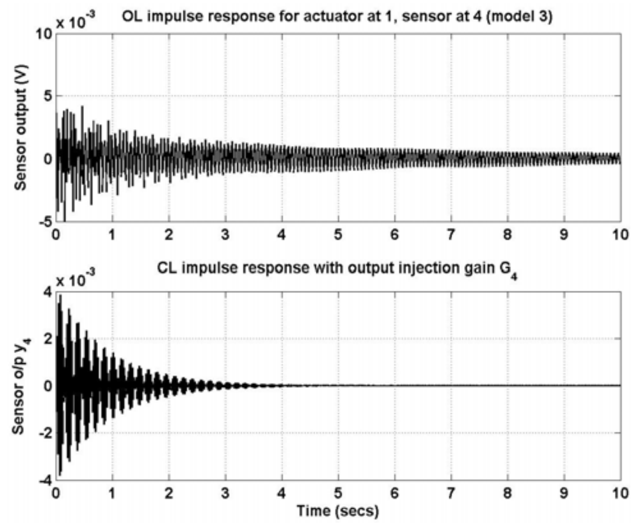


Fig. 8 OL / CL response with G and $\mathbf{K}$ / control u for model 2



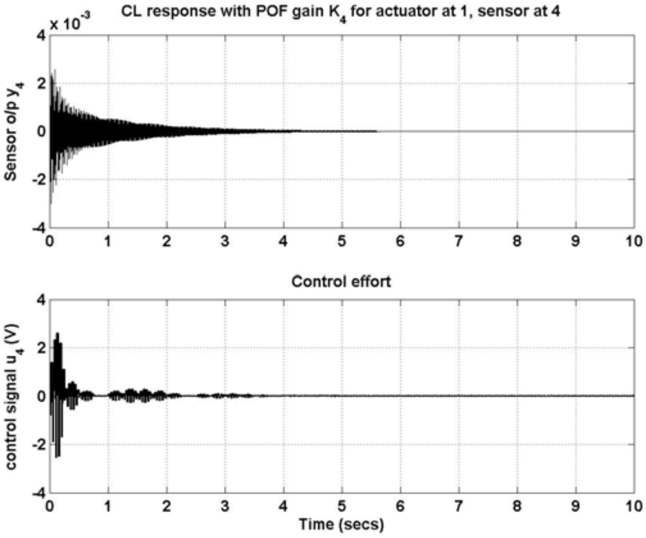


Fig. 9 OL / CL response with G and $\mathbf{K}$ / control u for model 3

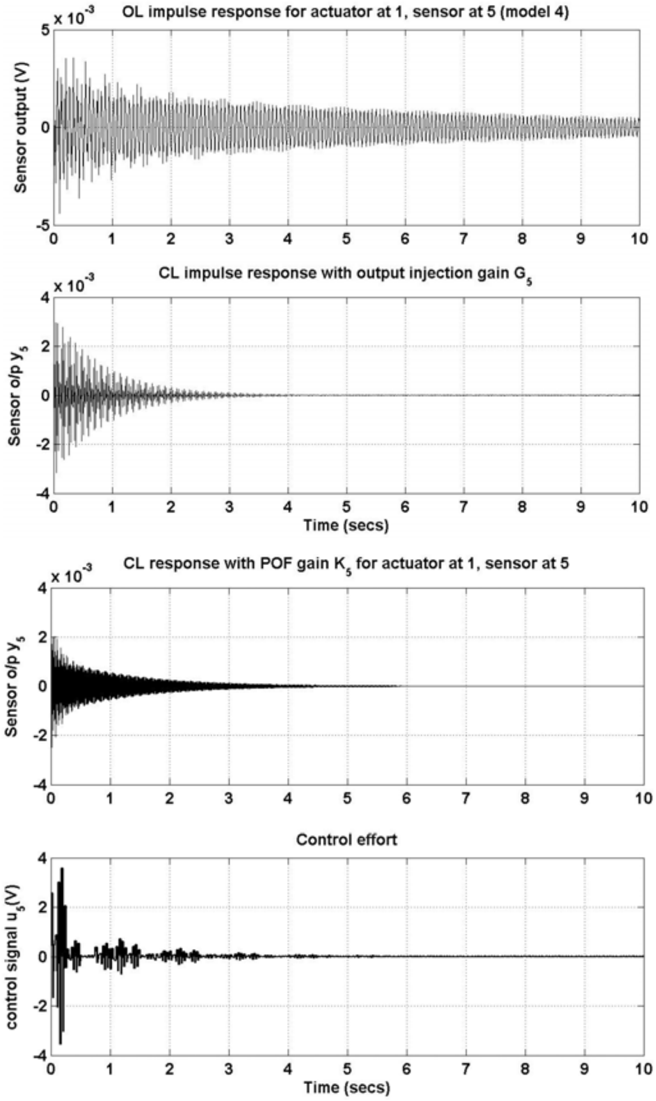


Fig. 10 OL / CL response with G and $\mathbf{K}$ / control u for model 4

The comparisons of the quantitative results of the OL and CL responses (with output injection gain G , POF gain $\mathbf{K}$) and with the magnitude of the control efforts, their settling times required is shown in Table I.

TABLE I
QUANTITATIVE COMPARITIVE RESULTS OF THE FOS SIMULATIONS
[TERMS INSIDE THE BRACKETS INDICATE SETTLING TIMES]
ONLY THE POSITIVE MAGNITUDE VALUES SHOWN

Model	**Open loop**	**Closed loop with *F***	**Closed loop with L**	**Control input *u***
1.	8 mV (12 secs)	7 mV (4 secs)	6 mV (4 secs)	1 V
2.	7 mV (15 secs)	6 mV (4.5 mV)	4 mV (5 secs)	1.8 V
3.	6 mV (18 sec)	4 mV (5 secs)	3.6 mV (5.5 secs)	2.9 V
4.	5 mV (20 secs)	(3.2 mV) (5.5 secs)	3 mV (6 secs)	3.8 V

VII. CONCLUSIONS

A Finite Element model of a smart cantilever beam based on Timoshenko Beam Theory with embedded piezoelectric shear sensors and actuators is presented for the SISO representation of the smart structure in this research paper. Some of the limitations of Euler-Bernoulli beam theory, such as the axial displacement and the shear are being considered in this work.

Different smart cantilever beam models with embedded shear sensors / actuators are developed using the Timoshenko beam theory for different sensor locations keeping the actuator location fixed. A Periodic Output Feedback (POF) controller is designed to control the first three modes of vibration of the embedded shear piezoelectric system. In the SISO case, four different models have been considered. The performance of the controller is evaluated for different sensor locations while the position of the actuator is kept constant.

It can be inferred from the response characteristics and the simulation results that the magnitude of the control signal u increases as the position of the sensor is changed from the nearby fixed end and moved towards the free end of the smart cantilever beam. The closed loop responses take more time to settle, i.e., for the vibrations to get damped out which is because of the lesser strain rate. The impulse responses with G and $\mathbf{K}$ show better performance when the sensor is at the nearby fixed end rather than at the free end. Thus, it can be inferred from the simulation results, that when the plant is placed with this controller, the system performs well and stability is guaranteed.

It is also observed that the maximum amplitude of the control voltage required to dampen out the vibrations is less when the sensor is placed at FE position 2 than at the free end and also the response settles quicker and the vibrations are damped out quickly. From the Fig. 7 - 10, it can be inferred that without control the transient response is predominant and with control, the vibrations are suppressed. It is also observed from the simulation results that modeling a smart structure by including the sensor / actuator mass and stiffness and by varying the sensor location at different positions introduces a considerable change in the structural vibration characteristics.

The output injection gain G for the SISO plant is obtained so that its poles are not placed at the origin and has a good settling time of less than 10 seconds. The designed POF controller requires constant gains and hence is easier to implement in real time. The simulation results show that a periodic output feedback controller based on Timoshenko Beam Theory is able to satisfactorily control the first three modes of vibration of the smart cantilever beam.

Surface mounted piezoelectric collocated sensors and actuators (piezo-patches bonded to the master structure at top and bottom of the single flexible beam) are usually placed at the root of the structure (near by the fixed end) to achieve most effective sensing and actuation. This subjects the sensors / actuators to high longitudinal stresses that might damage the brittle piezo-electric material.

Furthermore, surface mounted sensors / actuators are likely to be damaged by contact with surrounding objects piezopatches coming out while vibrating, connections coming out, due to thermal effects, stray magnetic fields, noise signals, etc.,. Embedded shear sensors / actuators can be used to alleviate these problems. The limitations of Euler-Bernoulli beam theory such as the neglection of the shear ϕ and axial displacements have been considered here while modeling the beam. Timoshenko beam theory corrects the simplifying assumptions made in Euler-Bernoulli beam theory and the model obtained can be a exact one.

ACRONYMS / ABBREVIATIONS

SISO	Single Input Single Output
FEM	Finite Element Method
FE	Finite Element
LMI	Linear Matrix Inequalities
MR	Magneto Rheological
ER	Electro Rheological
PVDF	Poly Vinylidene Fluoride
SMA	Shape Memory Alloys
CF	Clamped Free
CC	Clamped Clamped
CT	Continuous Time
DT	Discrete Time
OL	Open Loop
CL	Closed Loop
HOBT	Higher Order Beam Theory
LTI	Linear Time Invariant
FOS	Fast Output Sampling
AVC	Active Vibration Control
EB	Euler-Bernoulli
PZT	Lead Zirconate Titanate
DOF	Degree Of Freedom
IEEE	Institute of Electrical & Electronics Engineers

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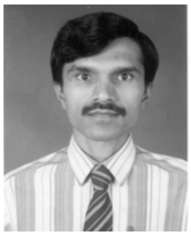

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