A Fuzzy Time Series Forecasting Model for Multi-Variate Forecasting Analysis with Fuzzy C-Means Clustering

Emrah Bulut, Okan Duru, and Shigeru Yoshida

Abstract—In this study, a fuzzy integrated logical forecasting method (FILF) is extended for multi-variate systems by using a vector autoregressive model. Fuzzy time series forecasting (FTSF) method was recently introduced by Song and Chissom [1]-[2] after that Chen improved the FTSF method. Rather than the existing literature, the proposed model is not only compared with the previous FTS models, but also with the conventional time series methods such as the classical vector autoregressive model. The cluster optimization is based on the C-means clustering method. An empirical study is performed for the prediction of the chartering rates of a group of dry bulk cargo ships. The root mean squared error (RMSE) metric is used for the comparing of results of methods and the proposed method has superiority than both traditional FTS methods and also the classical time series methods.

Keywords—C-means clustering, Fuzzy time series, Multi-variate design

I. INTRODUCTION

FORECASTING science reached to a maturing period by contributions of many scholars and its long history in economic and econometric literature. Although, the conventional forecasting science has superior particulars on many examples, it still has limitations and gaps exist in the literature. Among these limitations, uncertainty is the most cited problem in the forecasting research. In the engineering field, uncertainty exists in many automated systems and fuzzy set theory is proposed to deal with such problems. Since it is first introduced by Zadeh, the pioneering impact of fuzzy logic is unavoidable. In the last three decades, fuzzy logic is applied to many problems and fuzzy time series is one of the unique contributions of the literature.

[1]-[2] first presented the fuzzy time series (FTS) by introducing time-invariant FTSF model. Chen [3] improved the existing approach and the accuracy of Chen’s method is found superior than Song and Chissom’s approach. Huang [4] presented a method for FTSF by using heuristic modelling. Duru [5] developed fuzzy integrated logical forecasting (FILF) model which is improved the classical FTS by an integrated approach in univariate time series and it is applied for dry bulk freight index (BDI). In addition, there are many studies have developed and implemented the fuzzy time series forecasting on different study fields [6]-[13].

This paper investigates the FILF method and improves it for multi-variate modeling. For that purpose, an empirical study is designed for forecasting of the time-based shipping freight rates and the proposed model is compared with Chen’s algorithm and its bivariate version. Also a number of traditional methods such as autoregressive integrated moving average (ARIMA) is compared with results.

The clustering procedure is one of the major contributions of the FTS method. By applying clustering algorithm, the unusual fluctuations and outliers are eliminated and dataset is bundled in representative interval sets. Song and Chissom choose 1000 as the length of intervals and many studies have applied this length of intervals for the FTSF without specifying any reason [9]. However, the way of the choosing effective length of intervals affects the forecasting result and the accuracy of forecasting. Therefore, this study proposes the fuzzy C-means clustering method which is widely used as a clustering method and applies it for the length of intervals of the FTSF. Additionally, the numbers of fuzzy clusters are defined by using half of standard deviation.

In the literature, [14] first analyzed the relationship between tonnage and freight rate. [15] reported that ship prices adjust to freight and activity rates, and proposed equations to forecast it. [16] described a theoretical model in which freight markets and ship markets are interdependent because a ship is a capital asset of considerable longevity and [17]-[18] applied this model to the dry bulk cargo market and the tanker market. [19] proposed the supply and demand analysis for modeling ship prices by using the theoretical Error Correction model. System dynamics is also applied as a forecasting method in maritime economics [20]-[21]. To overcome forecasting problems and fluctuating of freight rates in the shipping market, time series models have recently been developed in the shipping literature [22]-[25]. In the study of [5], the FTS is used for forecasting the levels of Baltic Dry Index (BDI) and its superiority over the previous FTS method is noted.

In this paper, first order fuzzy logical relationships are used for pattern recognition and crisp predictions are generated by using time charter series of two ship sizes. Data is collected from several periodicals for monthly averages. Fuzzy clusters are based on the C-means algorithm which optimizes the shape and mid-point of the cluster. Each tonnage is assumed to be linked with the pricing of other tonnages.

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II. METHODOLOGY

A. Fuzzy C-Means Clustering

Cluster analysis is an unsupervised learning method for statistical data analysis and is applied in many fields. It is the process of dividing all data elements into classes or clusters so that objects in the same class are as similar as possible. There are many clustering methods used such as K-means clustering, fuzzy C-means clustering, hierarchical clustering and so on [26]-[27]. One of the most widely used clustering methods is the fuzzy C-Means (FCM) algorithm [27]-[29] in which one item of data can belong to more than one cluster and related to each element in a set of membership levels.

FCM that is based on minimization of the objective function is defined as follows:

\[ J_u = \sum_{i=1}^{N} \sum_{j=1}^{C} u_{ij}^m ||x_i - c_j||^2, \quad 1 \leq m \leq \infty \]  

(1)

Where any real number is greater than 1, \( u_{ij} \) is the degree of membership of \( x_i \) in the cluster \( j \), \( x_i \) is the \( i \)th of \( d \)-dimensional measured data, \( c_j \) is the \( d \)-dimensional center of cluster, and \( ||x_i - c_j|| \) is any norm expressing the similarity between any measured data and the center.

In the second step, the membership function \( U \) is appointed randomly and the center of cluster \( (c_j) \) is computed by:

\[ U = [u_{ij}] \text{matrix}, \quad U^0 \]

\[ c_j = \frac{\sum_{i=1}^{N} u_{ij}^m x_i}{\sum_{i=1}^{N} u_{ij}^m} \]  

(2)

According to the center of clusters, \( U^k \) is calculated again by using Eq. 3 and this iteration will stop \( \max_{ij} \left( \|u_{ij}^{(k+1)} - u_{ij}^k\| \right) < \varepsilon \), where \( \varepsilon \) is a termination criterion between 0 and 1, whereas \( k \) are the iteration steps.

\[ u_{ij} = \frac{1}{\sum_{k=1}^{C} \left( \|x_i - c_j\| / \|x_i - c_k\| \right)^{\frac{2}{m-1}}} \]  

(3)

In this study, the fuzzy C-means clustering method is used to determine the lengths of intervals for the VAR-FILF method and the termination criterion, \( \varepsilon \), is defined as 0.001. However, the determination of the initial center of the cluster is still uncertain in the fuzzy C-means clustering method and the number of clusters has an important impact on the performance of the FTSF. Song and Chissom applied 1000 as the length of the intervals and this scale is used in almost all FTS forecasting (FTSF) studies [3]-[4]. Huarng [9] proposed two different methods which are the average of the first differences of data and the distribution-based length method for choosing the first length of intervals for FTSF. Yolcu [30] improved Huarg’s method with a proposed method based on a single variable constrained optimization.

The definition of the number of clusters is an important question in FTS clustering since the results may change due to the number of clusters. In this study, the clusters are structured in the half of standard deviation. An increment can be applied to the value of the half of standard deviation for smoothing by either a reduction or an increase.

B. Fuzzy Time Series

[1]-[2] first introduced the FTSF method. In this method, all historical data transformed to fuzzy numbers. FTSF has superiority than the traditional forecasting method such as not involving non-stationary, limited number of observations and non-linearity. Chen [3] improved the FTSF method by performing simple calculations and his study gave superior result than the one [2] suggested.

The definitions of Chen’s FTSF method are as follows:

**Definition 1**  
Y(t) (t=…, 1, 2, 3,...) is a subset of real numbers (R). Let \( Y(t) \) be the universe of discourse defined by the fuzzy set \( \mu_i(t) \). If \( F(t) \) includes of \( \mu_i(t) (t=1,2,...) \), \( F(t) \) is called a fuzzy time series on \( Y(t) \).

**Definition 2**  
If there exists a fuzzy relationship \( R(t-1, t) \) such that \( F(t) = F(t-1)R(t-1, t) \), where \( \alpha \) is an operator, then \( F(t) \) is said to be caused by \( F(t-1) \). The relationship between \( F(t) \) and \( F(t-1) \) can be denoted by \( F(t-1) \rightarrow F(t) \).

**Definition 3** Suppose \( F(t) \) is computed by \( F(t-1) \) only, and \( F(t) = F(t-1)R(t-1, t) \). For any \( t \), if \( R(t-1, t) \) is dependent of \( t \), then \( F(t) \) is considered a time–invariant fuzzy time series. Otherwise, \( F(t) \) is time variant.

**Definition 4** Suppose \( F(t-1) = \hat{A}i \) and \( F(t) = \hat{A}j \), a fuzzy logical relationship can be defined as \( \hat{A}i \rightarrow \hat{A}j \), where \( \hat{A}i \) and \( \hat{A}j \) are called the left-hand side (LHS) and right-hand side (RHS) of the fuzzy logical relationship (FLR), respectively.

C. Vector Autoregressive Fuzzy Integrated Logical Forecasting (FILF)

The classical FILF model is characterized with a differencing operation and the last value contribution. For that purpose, additional definitions are given as follows:

**Definition 5** The lag, or a backward linear function for raw data that defines the first order differences of the original series, is as follows:

\[ \Delta Y(t) = Y(t) - Y(t-1) \]  

(5)

**Definition 6** \( \beta \) is an adjustment coefficient that defines the combination function of the last actual value of the fuzzified data set and the forecasted value for \( t + 1 \). The fuzzified data can be the raw time series data, the first differenced data or the second differenced set as well.

\[ F_s(t+1) = Y(t) + \beta \times F(t+1)(1-\beta) \]  

(6)

**Property** The adjustment coefficient \( \beta \) can be defined by experimental studies, and can also be calculated by a simulation of the function to minimize errors in the estimation period of the data.

**Definition 7** A FILF algorithm is described by its order:

FILF \((i, d, \beta)\)

\( i \): number of fuzzy sets.
\( d \): order of differencing operator \((\Delta^d Y(t))\).
\( \beta \): value of adjustment coefficient.

**Example 1** If the FILF algorithm is specified with 6 fuzzy numbers \((A_k, i=1, 2, ..., 6)\), the first order differenced series \((d=1)\), and the adjustment coefficient is 0.5 \((\beta=0.5)\), then the specification is FILF \((6,1,0.5)\).

**Program 1. The FILF procedure**

1. Define the universe of discourse \(U\). If the original data is differenced, the differenced data will be defined by the universe of discourse \(U\).
2. Divide \(U\) into intervals according to linguistic terms.
3. Define the fuzzy sets \( \mathcal{A}_k \) on \( U\), and fuzzify the historical data.
4. Derive the FLSs based on the historical data.
5. Classify the derived FLSs into groups.
6. Utilize three defuzzification rules to calculate the forecasted values.
7. Regulate the forecasted values by the combination function of the actual value of fuzzified data set and forecasted value.

A vector autoregressive FILF model is a particular case of bivariate FTS modeling. Rather than a single model, the VARFILF is based on two models of two variables. The bivariate FTS is explained in the following definitions.

**Definition 8** Let Panamax, \(P\), and Handymax, \(H\), be two fuzzy time series. Suppose that \( P(t-1)=A_1, H(t-1)=N_1 \), and \( P(t)=A_2 \). A bivariate FLR is defined as \( A_i, N_i \rightarrow A_j \), where \( A_0, N_0 \) are referred to as the LHS and \( A_j \) as the RHS of the bivariate FLR.

The current fuzzy time series models utilize discrete fuzzy sets to define their fuzzy time series. Their discrete fuzzy sets are defined as follows:

Assume there are \( m \) intervals, which are \( u_1 = [d_1, d_2], u_2 = [d_2, d_3], u_3 = [d_3, d_4], ..., u_m = [d_{m-1}, d_m] \), and \( u_0 = [d_0, d_{-1}] \), and \( u_0 = [d_0, d_{-1}] \), and \( u_0 = [d_0, d_{-1}] \), and \( u_0 = [d_0, d_{m+1}] \).

Let \( \mathcal{A}, \mathcal{A}_1, ..., \mathcal{A}_m \) be fuzzy sets which are linguistic values of the data set. Define fuzzy sets \( \mathcal{A}_1, \mathcal{A}_2, ..., \mathcal{A}_m \) on the universe of discourse \( U \) as follows:

\[
\begin{align*}
\mathcal{A}_1 & = a_{11}/u_1 + a_{12}/u_2 + a_{13}/u_3 + \ldots + a_{1m}/u_m, \\
\mathcal{A}_2 & = a_{21}/u_1 + a_{22}/u_2 + a_{23}/u_3 + \ldots + a_{2m}/u_m, \\
\vdots & \quad \vdots \\
\mathcal{A}_m & = a_{m1}/u_1 + a_{m2}/u_2 + a_{m3}/u_3 + \ldots + a_{mm}/u_m,
\end{align*}
\]

Where \( a_{ij} \in [0,1], 1 \leq i \leq k, \) and \( 1 \leq i \leq m \). The value of \( a_{ij} \) indicates the grade of membership of \( u_j \) in the fuzzy set \( \mathcal{A}_i \).

The degree of each data is found out according to their membership grade fuzzy sets. When the maximum membership grade is existed in \( \mathcal{A}_i \), the fuzzified data is treated as \( \mathcal{A}_i \). The fuzzy sets \( \mathcal{A}_1, \mathcal{A}_2, ..., \mathcal{A}_m \) are defined by

\[
\begin{align*}
\mathcal{A}_1 & = 1/u_1 + 0.5/u_1 + 0.5/u_1 + \ldots + 0.5/u_m, \\
\mathcal{A}_2 & = 0.5/u_1 + 1/u_1 + 0.5/u_1 + \ldots + 0.5/u_m, \\
\mathcal{A}_3 & = 0.5/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + \ldots + 0.5/u_m,
\end{align*}
\]


defines the universe of discourse \( U \). The fuzzified data is treated

\[
\begin{align*}
\mathcal{A}_{k-1} & = 0/u_1 + 0/u_2 + \ldots + 0/u_m + 0.5/u_{m+2} + 1/u_{m+1} + 0.5/u_m, \\
\mathcal{A}_k & = 0/u_1 + 0/u_2 + \ldots + 0/u_m + 0.5/u_{m+1} + 1/u_m.
\end{align*}
\]

The detailed application steps can be described as follows:

1. Collect and arrange the historical data. Define the universe of discourse \( U \).
2. Define \( \mathcal{A}_k \) on \( U \). The data mean and the standard deviation \( \sigma \).
3. Calculate fuzzy sets which are in the half of the standard deviations. Mean of data is located in the middle of fuzzy set and upper bound and lower bound is in distance of \( \sigma/4 \).
4. The transformation of data from crisp to fuzzy sets by the C-means clustering simulation.
5. Organize the bivariate FLRs into groups of same LHS fuzzy sets named the FLR Group (FLRG). LHSs of groups indicate input value of one period previous data. RHS is variety of outputs that experienced in estimation period.
6. Calculate the prediction outputs. The forecasted value at time \( t \), \( FV_t \), is determined by the following three IF-THEN rules. Assume the bivariate inputs at time \( t-1 \) is \( A_i, N_i \). 

**Rule 1** IF the FLRG of \( A_0, N_0 \) does not exist; \( A_i, N_i \rightarrow \varphi \), THEN the value of \( FV_t \) is \( \bar{A}_i \) (Naïve result), and calculate centroid of the fuzzy set \( \bar{A}_i \), which is located on midpoint, for inference point forecast.

**Rule 2** IF the FLRG of \( A_i, N_i \) does exist; \( A_i, N_i \rightarrow \bar{A}_i \), THEN the value of \( FV_t \) is \( \bar{A}_i \) and calculates centroid of the fuzzy set \( \bar{A}_i \), which is located on midpoint, for inference point forecast.

**Rule 3** IF the FLRG of \( A_i, N_i \) does exist; \( A_i, N_i \rightarrow \bar{A}_i \), THEN the value of \( FV_t \) is calculated as follows:

\[
FV_t = \bar{A}_i + \bar{A}_2 + \ldots + \bar{A}_m
\]

and calculate centroid of the resulting fuzzy set, which is the arithmetic average of \( m_{1j}, m_{2j}, ..., m_{pj} \), the midpoints of \( u_{i1}, u_{i2}, ..., u_{ip} \), respectively.

**Step 7** The adjustment of the forecasted value is performed by minimizing the error metrics (See the next section). The adjustment coefficient, \( \beta \), is calculated.

**D. Error Metrics**

The performance verification of the proposed method is evaluated by using the root mean squared error (RMSE) which frequently used in forecasting science. The RMSE metric gives an average deviation interval, and increases effects of larger errors by squares of them. Eq. (8) indicates the RMSE function.

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{n} (Y(t) - FV_t)^2}{n}} \quad (i=1, 2 \ldots n)
\]
III. EMPIRICAL STUDY AND APPLICATION

The VAR-FILF model is used to forecast the time charter rates of Panamax and Handymax bulk carriers. The proposed method is applied to the first order differencing of raw data because it ensures stationarity of data [5]. The application of the VAR-FILF model is applied step by step as follows.

Step 1 Data of time charter rates between January-2004 and December-2010 is collected and the first differences (d=1) of the sample period are computed (Table I). State the universe of discourse of a variable. Let \( D_{\text{max}} \) and \( D_{\text{min}} \) be the minimum time-charter rate and maximum time-charter rate of known first differences of data. The universe of discourse \( U \) is defined by \( U = [D_{\text{max}} - D_1, D_{\text{max}} + D_2] \), where \( D_1 \) and \( D_2 \) are two appropriate small numbers. The \( D_1 \) and \( D_2 \) are based on the round-down and round-up process respectively in three decimal on left.\[ U_{\text{Panamax}} = [-20813-187, 8895+105] = [-21000, 9000] \]
\[ U_{\text{Handymax}} = [-14363-637, 13250+750] = [-15000, 15000] \]

<table>
<thead>
<tr>
<th>Table I</th>
<th>RAW DATA OF TIME CHARTER RATES AND THEIR FIRST DIFFERENCES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date</td>
<td>PANAMAX Time charter</td>
</tr>
<tr>
<td>Jan-04</td>
<td>45719</td>
</tr>
<tr>
<td>Feb-04</td>
<td>46875</td>
</tr>
<tr>
<td>Mar-04</td>
<td>43219</td>
</tr>
<tr>
<td>..</td>
<td>..</td>
</tr>
<tr>
<td>Oct-10</td>
<td>22313</td>
</tr>
<tr>
<td>Nov-10</td>
<td>21603</td>
</tr>
<tr>
<td>Dec-10</td>
<td>19000</td>
</tr>
</tbody>
</table>

Step 2 In Table II, the descriptive statistics of time-charter data of the first order differenced PANAMAX and HANDYMAX series are indicated. Standard deviation is calculated and the half of the standard deviation which is approximately 2000 is used for finding the first length of the interval of the Panamax and the Handymax (Table II).

<table>
<thead>
<tr>
<th>Table II</th>
<th>RAW DATA OF TIME CHARTER RATES AND THEIR FIRST DIFFERENCES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st diff. of PANAMAX</td>
</tr>
<tr>
<td>Minimum value</td>
<td>-20813</td>
</tr>
<tr>
<td>Maximum value</td>
<td>8895</td>
</tr>
<tr>
<td>No of data</td>
<td>84</td>
</tr>
<tr>
<td>Standard dev.</td>
<td>4751</td>
</tr>
</tbody>
</table>

Step 3 Determine the first length of interval for the fuzzy C-means clustering. In this case, there are fifteen intervals for the time charter rates for Panamax and Handymax bulk carriers. The initial intervals of the Panamax are defined as \( u_i = [-21000, -19000], u_2 = [-19000, -17000], \ldots, u_{15} = [7000, 9000] \). The initial intervals of the Handymax are calculated as \( u_i = [-15000, -13000], u_2 = [-13000, -11000], \ldots, u_{15} = [9000,11000] \). The termination criterion is defined as the \( \epsilon \leq 0.001 \) in the C-means optimization; the midpoints of each cluster for the Panamax and Handymax are as follows:

For the Panamax; \( m_i = 19131, m_2 = 13225, m_3 = 8493, m_4 = 5074, \ldots, m_{15} = 12748 \).

For the Handymax; \( m_i = 12879, m_2 = 11309, m_3 = 6786, m_4 = 4509, m_5 = 2969, \ldots, m_{15} = 12748 \).

Step 4 & 5 Classify the FLRs into groups. The LHSs of the groups indicate the input value, which is the first order differencing of one period of previous data. The RHSs is the variety of the outputs that were exposed in the forecasting period. Table III shows FLRGs for Panamax and Handymax respectively.

Step 6 Calculate the forecasting outputs of the first difference series based on the rules in the Step 6 of the Section 2.3. The forecasted raw data is calculated by using the forecasted value of the first difference dataset as follows:

\[ F_{V_i} (\text{raw}) = Y(t) + F_{V_i} (\text{differenced data}) \]

<table>
<thead>
<tr>
<th>Table III</th>
<th>FUZZY LOGICAL RELATIONSHIP GROUPS BASED ON THE FIRST ORDER DIFFERENCING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panamax, ( \tilde{A} )</td>
<td>( \tilde{A}_1 ), ( \tilde{A}_2 ) → ( \tilde{A}_3 )</td>
</tr>
<tr>
<td>Handymax, ( \tilde{N} )</td>
<td>( \tilde{N}_1 ), ( \tilde{N}_2 ) → ( \tilde{N}_3 )</td>
</tr>
</tbody>
</table>

Step 7 Estimate the adjustment process. The \( \beta \) coefficient is calculated by minimizing the error metrics. For both Panamax and Handymax series, the \( \beta \) coefficient is estimated as 0.05 (in-sample) which indicates the series are broadly independent from the current fluctuations.

A. The Application of Benchmark Methods

For the comparative analysis, a group of benchmark methods are selected from the conventional time series analysis. The Box-Jenkins type autoregressive integrated moving average (ARIMA) and vector autoregressive models...
(VAR) are performed to the intended data and the accuracy of these approaches is also presented. As a base method, Naïve I results are also introduced.

Table IV indicates descriptive statistics for the Panamax and Handymax time charter rates. The standard deviation, kurtosis and the coefficient of variation indicates that datasets are relatively stabilized and a volatility model is not required.

For the stationarity testing, the conventional Augmented Dickey-Fuller [31] process is applied and both series are I(1) and further analysis is performed with the first order differenced series.

For the lag order selection in VAR model, cumulative test statistics are calculated by E Views 6.0 software. Table V introduces the results of VAR lag order test and a group of statistics are calculated by E Views 6.0 software. Table V indicates descriptive statistics for the Panamax and Handymax time charter rates. The standard deviation, kurtosis and the coefficient of variation indicates that datasets are relatively stabilized and a volatility model is not required.

Table VI presents the model estimations for ARIMA (2,1,0) and VAR(2) functions. The order of AR and MA terms are differenced series.

Table VI presents the model estimations for ARIMA (2,1,0) and VAR(2) functions. The order of AR and MA terms are differenced series.

Granger causality-Block exogeneity Wald test is performed for the VAR models. Panamax series is found a strong Granger-cause of Handymax series while Handymax series is a weak Granger-cause of Panamax series. From these indications, causality is found stronger from upper tonnage to lower tonnages (Table VII).

The traditional FTS methods are applied according to the related literature and the results are presented for the univariate FTS (cFTS) and bivariate FTS (Bi-cFTS) structure.

IV. RESULTS

Table VIII shows the RMSE results of the final models. The results explicitly indicate that the VAR-FILF model is superior in both series. The classical VAR (2) model is relatively better than other benchmark methods for in-sample accuracy. The classical FTS method of Chen does not perform better than the classical time series methods. Most of the FTS studies do not compare its accuracy among the classical econometrical methods and just focus on accuracy in the FTS literature. However, the presented results strongly indicate that the FTS method must be checked with the classical time series approach; otherwise FTS methods are unnecessary processes. The VAR-FILF model can be used in automatic clustering, reasoning and extrapolation mode. The number of VAR lags can also be defined by optimization of the sum of squared errors or RMSE metric itself. Pre-condition tests such as the
automated lag order selection by AIC and SBIC are also time-saving and robust alternatives.

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Handymax TC</td>
<td>Panamax TC</td>
</tr>
<tr>
<td>VAR-FILF</td>
<td>1458.58*3</td>
</tr>
<tr>
<td>Bi-cFTS</td>
<td>3805.29</td>
</tr>
<tr>
<td>eFTS</td>
<td>4577.67</td>
</tr>
<tr>
<td>ARIMA (2,1,0)</td>
<td>3207.76</td>
</tr>
<tr>
<td>NAIVE</td>
<td>4205.86</td>
</tr>
<tr>
<td>VAR (2)</td>
<td>2708.74*2</td>
</tr>
</tbody>
</table>

*1 Minimum of the column. *2 The second minimum of the column.

The possible reason of this outcome is depending on the consistency of causality which is discussed in the previous section. While the Panamax series is a strong cause of the Handymax series, the opposite direction is weakly consistent. Therefore, the Bi-cFTS model obtains an additional accuracy by reducing the squared errors.

V. CONCLUSION

In this study, the classical fuzzy time series forecasting method is extended by used VAR-FILF methods to improve the accuracy of forecasting. In addition, the C-means clustering method is proposed to optimize the distributions of the cluster sets and the half of the standard deviation is implemented for the initial intervals of the C-means clustering. The forecasting results of the VAR-FILF approach are compared with mostly used FTS methods and traditional time series analysis and the VAR-FILF method has found superior than benchmark methods.

The empirical study is related to Handymax and Panamax time charter rates as they play significant roles in the shipping economy [40].

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