Analytical and Experimental Methods of Design for Supersonic Two-Stage Ejectors

S. Daneshmand, C. Aghanajafi, and A. Bahrami

Abstract—In this paper the supersonic ejectors are experimentally and analytically studied. Ejector is a device that uses the energy of a fluid to move another fluid. This device works like a vacuum pump without usage of piston, rotor or any other moving component. An ejector contains an active nozzle, a passive nozzle, a mixing chamber and a diffuser. Since the fluid viscosity is large, and the flow is turbulent and three dimensional in the mixing chamber, the numerical methods consume long time and high cost to analyze the flow in ejectors. Therefore this paper presents a simple analytical method that is based on the precise governing equations in fluid mechanics. According to achieved analytical relations, a computer code has been prepared to analyze the flow in different components of the ejector. An experiment has been performed in supersonic regime 1.5<M<2 and pressure and velocity have been measured in different parts of ejectors. Finally the analytical and experimental results have been compared. It is seen that the results have a reasonable accordance.

Keywords—Ejector, Wind Tunnel, Supersonic, Diffuser, Mach number, Mixing Chamber

I. INTRODUCTION

The ejectors are vacuum pumps which have no moving component. In these instruments the energy of a fluid is used to transfer another fluid. They are classified, according to the fluid velocity, into subsonic, sonic and supersonic ejectors and, according to the type of fluid, into water high velocity ejectors, steam ejectors and gas ejectors [1]. The liquid ejectors are used to produce low vacuum or to mix the liquids and the vapor and gas ejectors are used to produce high vacuum or to maintain an existing vacuum in a system [2]. Having a very simple structure, the gas ejectors can be used in a wide range of gaseous parameter variations. In fact the ejectors are the nozzles which are placed downstream the diffusers and drive the flow into the outlet of diffuser. Accordingly they can produce various pressure fractions. To reduce the pressure at the wind tunnel outlet, these nozzles easily pump fluids like air and steam [3]. The ejectors have a lot of industrial applications. Especially they are used in steam power plants, refineries, and chemical production companies. They can supply the required vacuum in power plant condensers, or transfer gas and vapor specially the corrosive ones in refineries. These devices are also used in food industries for thickening the solutions [4]. They work steadily in their operating range and their installation cost is low in comparison with the vacuum pumps. In accordance with the environment in which they would be used they are made of different materials. Because of large energy loss and formation of normal shock wave, in subsonic compressible and permanent supersonic wind tunnels a large power unit is needed to produce required vacuum and pressure fraction [5]. Preparation and using such power units usually are not economic. Therefore such wind tunnels are designed with a small test chamber and their performance time is very low. Generating large compression factor, the experiment setup is turned on by a jet motor suction and an ejector system. Therefore the setup works steadily and with no limitation [6].

II. EJECTOR FUNCTION

An ejector is shown in Figure 1. The fluid is blown with a high kinetic energy from section 1 in central part into section 2 and drives the considered fluid into the diffuser. In fact the power unit is initially placed in the entrance of section 1 and removes fluid from section 2 by blowing a high energy air flow from section 1 into section 2 [7]. Figure 2 shows a permanent suction ejector which is used in wind tunnels. In spite of the ejector shown in Figure 1, in this ejector power unit is placed at the end of the circuit. The power unit suction makes the air enter the nozzle 1. The air velocity increases in the nozzle throat. It leads pressure in nozzle 2 to reduce. So that the entering air into the wind tunnel is sucked to its end. As it is seen in Figures 1 and 2, according to the design type and its requirements, the active and passive nozzles can be placed in section 1 or 2. As it is shown in Figure 2 ejectors include 4 basic components [8]. Firstly an active nozzle in which the fluid flows with high energy and velocity (section 1). Secondly a passive nozzle in which the fluid flows with low energy and velocity, wherein vacuum must be generated and from which the fluid is removed (section 2). Thirdly a mixing chamber that is usually cylindrical with constant cross section in which the two fluid flows, from active nozzle and from passive nozzle, are mixed by formation of a shear layer so that a uniform profile is produced at the end of the chamber (section 3). And finally a diffuser which is placed at the end of ejector and by which fluid velocity is reduced and its pressure is increased and the fluid is transferred to the outlet or to power unit. In a circuit, required vacuum can be generated by using several serried ejectors. As the first ejector output would be input of the latter. In the active nozzle of subsonic ejectors which is a convergent nozzle the fluid velocity reaches at least the speed of the sound. Easy utilization and effective performance in any regime are the

S. Daneshmand is with the Mechanical Engineering Department, Islamic Azad University, Majlesi Branch.
C. Aghanajafi is with the Mechanical Engineering Department, University of K.N.T.
A. Bahrami is with the Mechanical Engineering Department, Islamic Azad University, Majlesi Branch.
advantages of these ejectors. Figure 2 shows an ejector of this type [9]. In the supersonic ejectors active nozzle is of convergent-divergent type. The fluid velocity may be more than the speed of sound. The efficiency of these ejectors is very high in comparison with the subsonic ejectors one but working with these ejectors is difficult. If power unit does not supply the appropriate pressure ratio according to area ratio, the throat area to the outlet area, the performance of the ejector will be extremely reduced by formation of normal shock wave. A sample of supersonic ejectors is shown in Figure 3. Let’s mention that there is no difference between the passive nozzle and diffuser in these ejectors and the subsonic ones [10].

III. AN ANALYTICAL METHOD TO STUDY THE EJECTOR FUNCTION

Generally, mixing of two active and passive masses in mixing chamber generates a shear layer. It is obvious that because of high viscosity and turbulence and three dimensional nature of the flow, numerical methods are not able to accurately analyze the flow in ejectors without spending high costs and time. Therefore a simple analytical method which is based on precise scientific laws is needed. This method should analyze ejector operation and calculate flow characteristics all over it. To obtain such a method, governing equations in fluid mechanics has been used. A FORTRAN code has been prepared to study the flow in different parts of ejectors. The data determined by the code have been compared with practical data which was accumulated in a wind tunnel experiment in supersonic regime 1.5<M<2. Pressure and velocity in different parts of

IV. WIND TUNNEL

Figure 4 shows the used wind tunnel and ejectors. This wind tunnel is of the type of suction continues performing wind tunnel. This tunnel contains a reticulated entrance, calming chamber, converging-diverging nozzle, test section, diffusers, ejectors, jet motor and etc. Test section dimensions are 600*600*1400 mm3. The upper and lower walls have been made by porous material to eliminate boundary layer and normal shock wave. The ratio of porosity surface to total surface is variable according to flow regime. On two lateral walls of the test section removable transient doors have been installed for model installation and flow observation around it. The angle of attack changing system which is controlled by computer can set the model’s angle of attack in the range of -6<\alpha<18°. In this tunnel all types of pressure distribution, temperature distribution tests can be performed and loads are imposed to the model statically and dynamically. In Figure 5 the ejectors and the points in which pressure was measured is shown.

V. COMPUTER CODE FOR TWO STAGE EJECTORS

A FORTRAN code is prepared to design the supersonic ejectors. This code is contained of:
1. Input data of ejectors
2. Output data file of ejectors

In the first part of the code temperature, pressure, surface ratio, flow rate and pressure loss in mixing chamber, and the angle of diffusers are inputted. The output of the code is a file contains all required data.

VI. GOVERNING EQUATIONS AND THEIR SOLUTION STEPS

1. The determination of inlet flow rates are according to following equations:
\[ \dot{M}_{21} = \dot{m}_1 + \dot{m}_0, \quad m_{11} = \frac{0.4P_{011} \times 10^4}{\sqrt{T_{011}}} A_{11} \]

\[ \dot{M}_{21} = \frac{0.4 \times P_{012} \times 10^4 \times A_{12}}{\sqrt{T_{012}}}, \quad \dot{m}_{22} = m_{11} + \dot{m}_{12} \]

In these equations \( \dot{m}_{12}, \dot{m}_{11} \) are inlet flow rates through sections 1 and 2 in the first ejector.

2. Ejector factors determination:

\[ m_1 = \frac{\text{passive}}{\text{active}} = \frac{m_{21}}{m_{11}}, \quad n_2 = \frac{\dot{m}_{22}}{\dot{m}_{12}} \]

3. Determination of diffuser angle:

The first diffuser angle can be determined as:

\[ \theta_1 = \frac{T_{011}}{T_{011}} \]

The values of \( T_{011} \) and \( T_{011} \) are clarified in the code input data. The second diffuser angle is:

\[ \theta_2 = \frac{T_{022}}{T_{011}} \]

In this equation \( T_{022} \) is defined as:

\[ T_{022} = T_{011} \times (1 + n_1 \theta_1) \times \frac{1}{1 + n_1} \]

In all of the above mentioned equations \( T \) denotes temperature.

4. Pressure determination:

\[ P_{03} = \frac{P_{04}}{\delta_g} \]

In this equation \( P_{04} \) is known and \( \delta_g \) is pressure Coefficient. Coefficient which can be calculated as:

\[ \delta_g = 1 - (0.01 \times M - 0.0003196 \alpha - 0.01499 M \times \alpha) \]

\[ \alpha = \frac{A_1}{A_2} \]

\( \alpha \) is one of the input data and Mach number is obtained as it will be mentioned in section 1.

5. Determination of \( q(M_3^*) \)

This parameter is calculated as:

\[ q(M_3^*) = q(M_1^*) \times \sqrt{(n + 1)(1 + n \theta)} \times \frac{P_{03}}{P_{01}} \left(1 + \frac{1}{\alpha}\right) \]

In this equation \( q(M_1^*) = 1 \) and \( \theta, n \) value is related to designed ejector. \( P_{01} \) is given in program data and \( P_{03} \) is calculated before in section 4.

6. Regarding to \( q(M_3^*) \) which has been calculated in section 5 and by using gas dynamics functions table \( M_3^* \) is determined because:

In the equation above \( q(M_1^*) \) is \( \frac{A}{A'} \) in which \( A' \) is throat cross section area.

7. \( Z(M_3^*) \) value is calculated as:

\[ Z(M_3^*) = \frac{1}{M_3^*} + M_3^* \]

8. Mach number \( (M) \) is determined using following equation:

\[ M_3 = \sqrt{\frac{0.833 \times M_3'^2}{1 - 0.1666 M_3'^2}} \]

9. Once \( Z(M_3^*) \) is determined \( Z(M_3^*) \) could be calculated as:

\[ Z(M_3^*) = (1 + n) z(M_3^*) - z(M_3^*) \]

If friction is considered this equation is written as:

\[ Z(M_3^*) = \frac{(Z(M_3^*) \times \gamma M_3^* \gamma (n + 1)(1 + n \theta) - z(M_3^*))}{n \gamma \sqrt{\theta}} \]

In this equation \( \gamma, n \) and \( h_i \) are due to ejector and \( Z(M_3^*) \) = 2, in which

\[ h_i = \frac{2 \gamma}{\gamma + 1} \]

And \( L_3 \) and \( d_3 \) are the length and diameter of mixing chamber respectively.

10. \( M_2^* \) and \( q(M_2^*) \) values are calculated as follows:

\[ M_2^* = \frac{z(M_2^*)}{2} - \frac{\sqrt{(Z(M_2^*) - M_2^*)}}{2} \]

or

\[ q(M_2^*) = \frac{1}{\gamma + 1} \quad \frac{1}{M_2^*} \quad (1 + \frac{\gamma - 1}{\gamma + 1} \quad M_2^* \quad (1 - \frac{\gamma - 1}{\gamma + 1} \quad M_2^* \quad )^{\frac{1}{\gamma - 1}}} \]

11. According to \( q(M_2^*) \) determination \( \pi_0 \) is calculated as:

\[ \pi_0 = \frac{1}{n \alpha \sqrt{\theta}} \times \frac{q(M_2^*)}{q(M_1^*)} \]

12. \( \pi(M_2^*) \) is determined as:

\[ \pi(M_2^*) = (1 - \frac{\gamma - 1}{\gamma + 1} \quad M_2^* \quad (1 - \frac{\gamma - 1}{\gamma + 1} \quad M_2^* \quad )^{\frac{1}{\gamma - 1}}} \]

or

\[ \pi(M_2^*) = (1 - 0.6666) \]

Wherein \( \pi \) is pressure fraction.

13. Total flow pressure in passive ejector \( P_{02} \) is determined as:

\[ P_{02} = P_{02} \times A_{11} \]
In which $P_0$ is given in program input data and $\pi_0$ is already calculated in section 11.

14. Static pressure due to passive ejector $P_2$ is

$P_2 = \frac{P_01}{\pi_0}$

Wherein $\pi(M_2*)$ should be used.

15. Finally wind tunnel pressure $P_t$ is calculated as:

$$P_t = \frac{P_01}{\pi(M_2)} = \frac{P_01}{P_2} = \frac{\pi_0}{P_02}$$

VII. WIND TUNNEL EJECTOR TEST

Wind tunnel ejector was tested in Mach numbers of 1.5 and 2. The results are shown in table 1.

<table>
<thead>
<tr>
<th>First ejector</th>
<th>Second ejector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mach=1.5</td>
<td>Mach=2</td>
</tr>
<tr>
<td>$P_{01}=0.84$</td>
<td>$P_{01}=0.84$</td>
</tr>
<tr>
<td>$P_{02}=0.57$</td>
<td>$P_{02}=0.84$</td>
</tr>
<tr>
<td>$P_{03}=0.62$</td>
<td>$P_{03}=0.62$</td>
</tr>
<tr>
<td>$P_{04}=0.64$</td>
<td>$P_{04}=0.64$</td>
</tr>
<tr>
<td>$P_{01n}/P_{02}=1.58$</td>
<td>$P_{01n}/P_{02}=1.65$</td>
</tr>
<tr>
<td>$P_{04}=0.57$</td>
<td>$P_{04}=0.57$</td>
</tr>
<tr>
<td>$P_{01n}/P_{02}=1.58$</td>
<td>$P_{01n}/P_{02}=1.58$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>First ejector</th>
<th>Second ejector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mach=2</td>
<td>Mach=1.5</td>
</tr>
<tr>
<td>$P_{01}=0.84$</td>
<td>$P_{01n}=0.49$</td>
</tr>
<tr>
<td>$P_{01n}=0.84$</td>
<td>$P_{01n}=0.84$</td>
</tr>
<tr>
<td>$P_{02}=0.56$</td>
<td>$P_{02}=0.57$</td>
</tr>
<tr>
<td>$P_{03}=0.51$</td>
<td>$P_{03}=0.51$</td>
</tr>
<tr>
<td>$P_{04}=0.67$</td>
<td>$P_{04}=0.67$</td>
</tr>
<tr>
<td>$P_{01n}/P_{02}=1.65$</td>
<td>$P_{01n}/P_{02}=1.65$</td>
</tr>
<tr>
<td>$P_{04}=0.57$</td>
<td>$P_{04}=0.57$</td>
</tr>
<tr>
<td>$P_{01n}/P_{02}=1.58$</td>
<td>$P_{01n}/P_{02}=1.58$</td>
</tr>
</tbody>
</table>

VIII. COMPUTER CODE RESULTS

Using analytical relation which has been derived a computer code was prepared. Running this code the following results are obtained:

<table>
<thead>
<tr>
<th>First ejector</th>
<th>Second ejector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mach=1.5</td>
<td>Mach=2</td>
</tr>
<tr>
<td>$P_{01n}=0.8410$</td>
<td>$T_{01}t=293.13$</td>
</tr>
<tr>
<td>$T_{01t}=293.13$</td>
<td>$q(M_1*)=0.95$</td>
</tr>
<tr>
<td>$P_{02n}=0.5268$</td>
<td>$P_{02}=0.5268$</td>
</tr>
<tr>
<td>$q_{M_1}=0.69$</td>
<td>$q_{M_2}=0.90$</td>
</tr>
<tr>
<td>$m_1=45.388$</td>
<td>$m_2=45.388$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mass flow rate of ejectors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{11}=35.2973$</td>
</tr>
<tr>
<td>$m_{22}=80.6849$</td>
</tr>
<tr>
<td>$m_{12}=52.6438$</td>
</tr>
<tr>
<td>$m_{22}=133.3287$</td>
</tr>
</tbody>
</table>

IX. CONCLUSION

Regarding to their low cost and simplicity of operation, ejectors are appropriate devices to produce vacuum through various sections. The cost of their installation, service and maintenance is low and they can be made of different materials. Since comparing theoretical results and experimental ones it is seen that theoretical results can analyze flow manner with a very good approximation and present acceptable results consuming low cost and time. Through performing further tests and flow property equalization in more points with more accuracy, existing problems such as no effective performance of ejector, no appropriate area of throat cross section, no suitable mixing chamber length, and no effective performance of jet motor can be solved. Better results, therefore are achieved. It is also observed that the more energy of passing fluid through active nozzle which enters mixing chamber, the more generated vacuum and the more nozzle efficiency. As it is difficult to make use of supersonic ejectors, it is better to utilize subsonic ejectors and to avoid fluid velocity to reach the speed of sound by choosing suitable power and appropriate nozzle throat area. Accordingly subsonic ejectors have better efficiency. Designing ejector as it is shown in Figure 1 the oil of low-pressure wells can be sucked out by using high-pressure wells. In wind tunnels in which jet motors is located at the end, the proposal is to use multi-step ejector systems, unless motor parts will be broken down. The principal of multi-step ejector system is that the motor exit is located perpendicular to basic circuit in active section. It provides required vacuum in basic circuit by exerting air and motor gas mixture to the high energy and velocity fluid. In fact without being located in basic circuit, required vacuum is provided at the end of test chamber. Accordingly subsonic and supersonic sections are achieved.

REFERENCES