Chaotic Properties of Hemodynamic Response in Functional Near Infrared Spectroscopic Measurement of Brain Activity

Ni Ni Soe, Masahiro Nakagawa

Abstract—Functional near infrared spectroscopy (fNIRS) is a practical non-invasive optical technique to detect characteristic of hemoglobin density dynamics response during functional activation of the cerebral cortex. In this paper, fNIRS measurements were made in the area of motor cortex from C4 position according to international 10-20 system. Three subjects, aged 23 - 30 years, were participated in the experiment.

The aim of this paper was to evaluate the effects of different motor activation tasks of the hemoglobin density dynamics of fNIRS signal. The chaotic concept based on deterministic dynamics is an important feature in biological signal analysis. This paper employs the chaotic properties which is a novel method of nonlinear analysis, to analyze and to quantify the chaotic property in the time series of the hemoglobin dynamics of the various motor imagery tasks of fNIRS signal. Usually, hemoglobin density in the human brain cortex is found to change slowly in time. An inevitable noise caused by various factors is to be included in a signal. So, principle component analysis method (PCA) is utilized to remove high frequency component. The phase pace is reconstructed and evaluated the Lyapunov spectrum, and Lyapunov dimensions. From the experimental results, it can be conclude that the signals measured by fNIRS are chaotic.

Keywords—Chaos, hemoglobin, Lyapunov spectrum, motor imagery, near infrared spectroscopy (NIRS), principal component analysis (PCA).

I. INTRODUCTION

RECENTLY, a variety of non-invasive techniques have been adopted for measuring brain activity by means of not only electrical signal but also changes in cerebral hemodynamic and oxygenation. For electrical signal, electroencephalography (EEG) is very popular technique and positron emission tomography (PET), magnetoencephalography (MEG), or functional magnetic resonance imaging (fMRI) has been used to measure changes in regional cerebral blood flow (CBF). These methods for CBF are expensive, not widely available, associated with injection of radioactive compounds, and have a low temporal resolution.

Near infrared spectroscopy (NIRS) and functional magnetic resonance imaging (fMRI) are rapidly developing technologies that both allow non-invasive monitoring of regional changes in cortical tissue oxygenation, particularly changes in deoxymoglobin concentration ([Hb]), in response to various stimuli including visual, auditory, and motor stimuli [9]-[10].

NIRS is a simple, inexpensive and a new noninvasive optical technique, measures changes in concentrations of oxygenated and deoxygenated hemoglobin (oxy-Hb and deoxy-Hb, respectively) mainly in cerebral venous blood. Summation of the changes in oxy-Hb and deoxy-Hb indicates changes in the total hemoglobin concentration (total-Hb). The NIRS method is based on near-infrared light absorption changes that depend on concentration changes of the chromophores [HbO2] and [Hb] in the tissue under investigation. Changes in [total-Hb] can be used as a measure of blood volume changes. NIRS promises flexibility of use, portability, metabolic specificity, good spatial resolution, localized information, high sensitivity in detecting small substance concentrations and affordability. NIRS has no doubt certain disadvantages. It is slow to operate because of the inherent latency of the hemodynamic response. The signal strength is affected by hair on the head. Furthermore, relative motion of the optodes on the hair may introduce motion artifacts and drifts in the hemodynamic signal. Nevertheless, NIRS’ ability to record localized brain activity with a spatial resolution in the order of centimeter (depending on the probe geometry) provides with an excellent opportunity to control a variety of motor and cognitive activities. Actually, there have been many research areas on NIRS not only in brain but also in pharmaceutical, Agricultural application and so on [11].

According to the brain activity research, many researchers have been analyzed fNIRS signals on cognition tasks [12], infant brain development [13], and spectral analysis of event related of fNIRS [14]. fNIRS was used also in the study of human cortical taste cognitive processing [15]. Sitaram et al. [16] and Coyle et al. [17] reported optical response using NIRS signals during overt and covert hand movements. For BCI application, motor tasks such as left hand, right hand imagery are common tasks [16]-[18]. D. Friedman et al. reported a direct EEG-based BCI using motor imagery [19]. It is important that, for a BCI to be user-friendly, the mental task should be easy to learn. With this view and based on the above findings, motor imagery tasks were chosen as the basis for the experiments. Information about peoples’ current activity and their mental tasks can be used for multiple applications not only in healthy person but also in patients. This paper demonstrates the new application of chaos analysis on NIRS signal.
Chaos analysis, such as the embedding theorem, the mutual information method, false nearest neighboring method, the largest Lyapunov exponent and so on, is a powerful non-linear method [20] which enables the extraction of characteristic quantities. It offers tools that bridge the gap between experimentally observed irregular behavior and deterministic chaos theory [21]-[28]. The interest in chaotic research has risen rapidly and several attempts have been made in many related fields such as mathematics, physics, chemistry and biomedical engineering as well as applications to the human brain and heart [3], [5], [6].

The aim of this paper is to investigate the chaotic property of hemoglobin changes of the blood within motor cortex by Lyapunov analysis. Lyapunov exponents of a given time series are of great importance. Motor imagery of left-hand, and relax condition were chosen as the tasks as it has been shown to work well in previous research on EEG based fractal analysis [3]-[4]. To verify the chaotic property of NIRS signal, the real movement of motor cortex was also analyzed. This is the first time that Lyapunov exponent analysis has been used to NIRS signal to the best of our knowledge.

In this paper, it was described that the experimental paradigm of motor imagery; method of signal acquisition; preliminary signal analysis for artifact removal technique by PCA in the hemodynamic response to motor imagery; chaotic analysis of the NIRS signal using reconstruction of the attractor and Lyapunov spectrum; and finally the results of signal processing and analysis. Lyapunov spectrum was analyzed not only for forward time series of NIRS signal but also for reverse time series of NIRS signal to validate the Lyapunov exponents. This paper was concluded with a discussion of the application of these techniques to the classification problem towards developing a NIRS–BCI system by time dependent Lyapunov and fractal dimension.

II. MATERIALS AND METHODS

A. Subjects and motor imagery tasks

Three subjects with the age ranging from 23 years and 30 years voluntarily participated in this study. None of the recruited subjects had neurological or psychiatric history or was on medication.

Motor imagery is described as the mental rehearsal of a motor act without any overt motor output [1]. Mental imagination of movements involves similar brain regions that are used in programming and preparing movements. According to this view, the main difference between motor performance and motor imagery is that in the latter case execution would be blocked at some corticospinal level. Motor imagery, defined as mental simulation of a movement, has been shown to represent an efficient mental strategy [2]-[5]. The imagination of different types of movements, for example, right hand, left hand, or foot movement, results in a characteristic change of the hemoglobin density in the blood of the brain by fNIRS over the sensorimotor cortex of a participant. So, in this work, the motor imagery tasks were chosen for the experiments.

B. Experimental setup

During the experiment, the subjects were seated in a position that allowed them to perform the task comfortably. The subjects were asked to maintain a stable body position and avoid eye blinks, teeth bites and head movements during

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<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
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<tr>
<td>0-10s</td>
<td>Baseline</td>
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<tr>
<td>10s</td>
<td>Imagery/Real of left hand movement</td>
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<tr>
<td>600s</td>
<td>Baseline</td>
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(a) Experimental protocol

![Experimental protocol](image)

The placement of NIRS optodes position on the scalp according to international 10-20 system.

Fig. 1 (a) Experimental paradigm for imagery movement and (b) NIRS optodes position on the scalp. A pair of illuminators and detectors formed one channel.

The recording. Possible sources of distraction or noise, such as sound or light, were minimized. NIRS signals were recorded from each subject performing relax, real and imagery movements of left hand movement sequentially. Fig. 1(a) shows the schematic diagram of the protocol. At each task, the subjects performed 3 trials and each for duration of total 620 seconds. A single trial comprised of a baseline block, a preparation block and a motor task block. Each trial started and ended with a baseline block during which the subject fixated on the cross displayed on the screen for 20s. This was followed by a beep indicating the subject to get ready for the motor task. Following this, the subject performed the motor task as indicated on the screen for a period of 600 s. The type of motor task to be performed was instructed by the operator such as “Please imagine left hand movement”.

Between any two adjacent trials, the subjects had sufficient relax so that the effect of muscle fatigue was minimized. Time gap of 20 minutes is given between any two successive tasks to ensure that a previous state stimulus doesn't affect the subsequent state of NIRS signal.

C. sNIRS signal acquisition

A multichannel fNIRS instrument (OMM-3000 from Shimadzu Corporation, Japan) for acquiring oxygenated hemoglobin and deoxygenated hemoglobin concentration...
changes during motor imagery. The system operated at three different wavelengths of 780 nm, 805 nm and 830 nm. The illuminator and detector optodes were placed on the scalp. The detector optodes were fixed at a distance of 3 cm from the illuminator optodes. The optodes were placed on the motor cortex at the position of C4 according to the 10 to 20 system for standard electrode positions (American Electroencephalographic Society, 1994). A pair of illuminator and detector optodes formed one channel on sensorimotor cortex was shown in the Fig. 1(b). Near-infrared rays leave illuminator, pass through the skull and the brain tissue of the cortex and are received by the detector optodes.

The photomultiplier cycles through all the illuminator–detector pairings to acquire data at every sampling period. The optodes were attached to a tight elastic headband in order to press them against the scalp. Changes in total-Hb, oxy-Hb, and deoxy-Hb were sampled every 0.025 s and stored on a floppy disk for offline analysis by a computerized system.

It is also important to maintain a good connection with the scalp and also to push hair aside. Dark hair obstructing the photon’s path can significantly attenuate the signal, and every effort must be made to overcome this problem; combing hair out of the way, fixing with hair gel and hair clips are an imperative part of the experimental regime. A stable connection between the optodes and scalp is vital to ensure that the path length of the photons does not change and also to prevent ambient light saturating the detector. Before the beginning of the protocol, the oxygenation response was checked. If no oxygenation change could be detected, the optodes were replaced until a response was signaled. Example of oxy-Hb, deoxy-Hb, and total-Hb density changes for the dynamics of brain oscillations, of left hand motor imagery are presented in Fig. 2 (a-c).

The PCA method actually consists of three disjoint steps: Computing the principal components, projecting the data to obtain a low-dimensional representation, and reconstructing the cleaned data set. But many NIRS signals represent one dimensional time series. Clearly projective subspace techniques are not available for one dimensional time series to suppress noise contributions, hence time series analysis techniques often rely on embedding a one dimensional sensor signal in a high-dimensional space of time delayed coordinates [29]. So, NIRS signals on each channel which represent one dimensional time series need to be transformed to multidimensional signal vectors for the latter techniques to be applicable before computing the principal components. The transformation can be achieved by embedding an observed signal in its delayed coordinates. The time embedding of the signals transforms one dimensional time series into multidimensional signal vectors. This is a necessary step to have subspace projection techniques available. However, this step unavoidably introduces nonlinearity into the signal analysis process. For the observed one dimensional time series, \( x = (x_1, x_2, ... x_d) \) can transform to a multidimensional sequence of \( M = N - (m-1)\tau_d \) by time lag \( \tau_d \) and embedding dimension \( m \) [§. E]. The legged vectors then constitute the column of the trajectory matrix \( X_i = [x_i, x_i\tau, ... x_i(\tau D)\tau] \), \( i = 1, 2, ..., M \). Further processing of the data matrix is performed by PCA. It encompasses the following steps: After embedding, the data
points $X_{1}, \ldots, X_{M} \in \mathbb{R}^{m}$, of the trajectory matrix are computed the mean value:
\[ \bar{X} = \frac{1}{M} \sum_{i=1}^{M} X_{i}. \]
(1)

A covariance matrix is computed in each column using zero mean data;
\[ X_{c} = \frac{1}{M} \sum_{i=1}^{M} (X_{i} - \bar{X})(X_{i} - \bar{X})^{T}. \]
(2)

Compute the eigenvectors, $u_{1}, u_{2}, \ldots, u_{m} \in \mathbb{R}^{m}$ and eigenvalue, $\lambda_{1}, \lambda_{2}, \ldots, \lambda_{m} \in \mathbb{R}$ of the covariance matrix is computed;
\[ C = \{X_{c}X_{c}^{T}\} = UDU^{T}. \]
(3)

The most popular scheme for simultaneously approximating all the eigenvalues of a matrix $A$ is the remarkable QR algorithm. Afterwards denoising can be achieved by projecting the multidimensional signal into the subspace spanned by the eigenvectors corresponding to the largest eigenvalues. The eigenvectors with the largest eigenvalues correspond to the dimensions that have the strongest correlation in the data set.

Project the multidimensional signal to low-dimensional subspace, then the transformed data points is:
\[ X'_{i} = \sum_{j=1}^{m} u_{j}X_{i}. \]
(4)

To decide the principal component in reduction of the signal, the Scree method was utilized.

Finally, the denoised data points can be reconstructed from the transformed data points:
\[ X'_{i} = \sum_{j=1}^{m} u_{j}[X'_{j}], \]
(5)

and the noise-reduced one dimensional signal, $x$, is then obtained by reverting the embedding.

Eigenvalue is calculated for the signal shown in Fig 2. These are plotted in descending order are shown in Fig 3. As shown in Fig 3, by using the largest eigenvalue, the reconstructed signal by PCA were shown in Fig 4 (a)-(c). As shown in Fig 4, it can be verified that the high frequency component of the signal is excluded in comparison with the observation signal in Fig 2 (a)-(c).

### E. Reconstruction of attractor

The first step is to reconstruct attractors from NIRS signals to apply chaos theory in NIRS analysis in a multidimensional phase space from a single time series. For this purpose, Takens’ embedding theorem is used in this study and the trajectory of the attractor is reconstructed using the specified time delay from the time series [29]. Taking the signal on the given channel in a time series representation, NIRS data was recorded as a discrete time series $(x_{1}, x_{2}, \ldots, x_{N})$, where $x$ is the signal amplitude at the discrete time moment $N$. The $m$-dimensional phase space is reconstructed from a single observable $x_{N}$ by means of time delay method. The reconstructed trajectory, $X$, can be expressed as a matrix where each row is a phase-space vector. That is,
\[ X = [X_{1} X_{2} \ldots X_{M}]^{T}. \]
(6)

Where $X_{i}$ is the state vector of the system at discrete time $i$. For an $N$-point time series, $(x_{1}, x_{2}, x_{3})$, each $X_{i}$ is given by:
\[ X_{i} = [x_{i}, x_{i+\tau}, \ldots, x_{i+(m-1)\tau_{d}}]. \]
(7)

Where $\tau_{d}$ is the lag or reconstruction delay, and $m$ is the embedding dimension. Thus, $X$ is an $M \times m$ matrix, and the constants $m, M, \tau_{d}$ and $N$ are related as:
\[ M = N - (m-1)\tau_{d}. \]
(8)

For an infinite amount of noise-free data, the quality of the reconstructed attractor, which will affect significantly the

![Fig. 3 The Scree plot of the eigenvalue in descending order of oxy-Hb density of Fig. 2(a)](image)

![Fig. 4 The reconstructed signal of Fig 2 (a)-(c) by PCA](image)
measurement of the dynamical invariants, depends on the value of chosen the time delay \( \tau \).

In this work, the time delayed mutual information which was suggested by Fraser and Swinney [30], is applied to determine a reasonable delay. Delay time (\( \tau_d \)) determined based on the first local minimum in average mutual information function of the time series data \( x_i (i = 1, 2...M) \). At first, the sequence \( x_i (i = 1, 2...M) \), of length M is considered in the time series \( x_i \). Second, by shifting the \( x_i \) by \( \alpha \), the new sequence of \( y_j (j = \alpha, \alpha+1...\alpha+M) \). Let the probability distribution in \( x_i = p_x (i) \) and the probability distribution in \( y_j = p_y (j) \). Then, the mutual information is:

\[
I(X, Y) = \sum_i \sum_j p_{xy}(i, j) \log \frac{p_{xy}(i, j)}{p_x(i)p_y(j)}
\]  

By varying the time shift \( \alpha \), let the time delay, for which \( I(X, Y) \) first takes the local minimum, be the delay \( \tau_d \).

To determine the appropriate embedding dimension \( m \) of the reconstructed attractor, false nearest neighbour (FNN) method [31] was utilized. This method can yield an acceptable minimum embedding dimension by examining the behavior of nearest neighbors as the embedding dimension is increased from \( m \) to \( m + 1 \). The result of time delay estimation of the given NIRS signal is shown in Fig. 5. Fig. 6 shows the fraction of false nearest neighbors for \( m \) on the attractors reconstructed from the NIRS signals. One may find that the fraction of NIRS signals become zero for \( m \geq 5 \), which indicates an acceptable embedding dimension. So, \( m = 5 \) was used in the following Lyapunov analysis. The phase space plot for imagery of left hand movement NIRS signals after PCA analysis were shown in Fig. 7 and \( \tau_d \) and \( m \) were selected according to the mutual information and false nearest neighboring results of Fig. 5 and Fig. 6.

Fig. 5 Approximation to the mutual information of (a) original NIRS signals of Fig. 2(a)-(c) and (b) PCA applied NIRS signal Fig. 4(a)-(c)

Fig. 6 Approximation to the embedding of (a) original NIRS signals of Fig. 2(a)-(c) and (b) PCA applied NIRS signal Fig. 4(a)-(c)
F. Lyapunov analysis

Lyapunov exponent ($\lambda$) is a quantitative measure of the sensitive dependence on the initial conditions. They characterize the average rate of divergence of the neighboring trajectories. A chaotic system has at least one positive Lyapunov exponent. Therefore, the existence of a positive $\lambda$ for almost all initial conditions in a bounded dynamical system is widely used definition of deterministic chaos. To discriminate between chaotic dynamics and periodic signals, Lyapunov exponent ($\lambda$) is often used. Due to the great interest related to the study of experimental time series in many scientific domains, a lot of methods for computing Lyapunov index for discrete time series have been developed [32]-[38]. Some methods allow computing the whole spectrum of Lyapunov index for discrete time series have been developed [32]-[38]. Some methods allow computing the whole spectrum of Lyapunov exponents [33], [35], [38] from tangent maps obtained reconstructing the dynamics on a suitable $m$ dimensional space. However, all these methods require large data sets. For shorter data sets, one can just determine the presence of chaos by computing only the largest Lyapunov index as in [32], [36]. The first successful algorithm for calculating the entire spectrum of Lyapunov exponent [33]-[34] is based on the Jacobian matrix of Eq. (9) in the tangent space is estimated from the experimental data according to the method by Sano and Sawada [26]. The current approach is principally based on the work of Sano and Sawada [35]. Let us consider a general $m$-dimensional discrete time dynamical system,

$$X_{n+1} = F(X_n), \quad X_n \in \mathbb{R}^m$$  \hspace{1cm} (10)

Where $X_i$ is a state of the system and $F : \mathbb{R}^m \rightarrow \mathbb{R}^m$ is a $m$-dimensional mapping. Considering the infinitesimal small displacement at $X_i$, $\delta X_i$, then, there holds:

$$X_{n+1} + \delta X_{n+1} = F(X_n + \delta X_n)$$  \hspace{1cm} (11)

By forming the Taylor expansion and applying the linear approximation, the following mapping is obtained for the small displacement $\delta X_i$ at $X_i$:

$$\delta X_{n+1} = DF(X_n) \delta X_n$$  \hspace{1cm} (12)

where $DF(X)$ is the Jacobian matrix of $F$ at point $X$. The mapping is a time dependent linear mapping. Defining $\delta X_0$ as the initial value and executing $N$ time mappings by DF, there results:

$$\delta X_N = DF(X_{N-1})DF(X_{N-2}) \ldots DF(X_0) \delta X_0$$  \hspace{1cm} (13)

The Jacobian matrix $DF_N$ at the $N$-th mapping is defined as:

$$DF_N = DF(X_{N-1})DF(X_{N-2}) \ldots DF(X_0)$$  \hspace{1cm} (14)

Further a linearized map $DF$ of Eq. (9) in the tangent space is estimated from the experimental data according to the method by Sano and Sawada [26]. Using the eigenvalues $\sigma_i(N)$, the Lyapunov spectrum $\lambda_i$ ($i = 1, 2 \ldots m$) is derived as followed:

$$\lambda_i = \lim_{N \rightarrow \infty} \frac{1}{N} \log |\sigma_i(N)|$$  \hspace{1cm} (15)

The Lyapunov exponent, $\lambda_i$ is defined as a divergence rate in bits per unit time (bits/sec) i.e. logarithm of base two. In the case of experimental time series, the Jacobian matrix of Eq. (9) cannot directly be used since the dynamical system of the object of measurement is unknown. The Jacobian matrix of Eq. (9) in the tangent space is estimated from the experimental data according to the method by Sano and Sawada [35]. This method was applied in many application fields such as EEG and vocal sound [5]-[8] but newly applied in NIRS signal in the research.

The Lyapunov spectrum is closely related to the fractional dimension of the associated strange attractor. The Lyapunov dimension ($D_L$) conjectured by Kaplan and York [39] can be calculated from the Lyapunov spectrum by the following equation.

$$D_L = \frac{\sum_{i=1}^{j} \lambda_i}{\left|\lambda_{j+1}\right|}$$  \hspace{1cm} (16)

where $j$ is the maximum integer such that the sum of the Lyapunov exponents in descending order is positive.

III. EXPERIMENTAL RESULTS

The analysis would operate on the time series of oxy-Hb, deoxy-Hb and total-Hb at C4 channel of the NIRS system of imagery of left-hand, real left movement and relax condition.

For the analysis of NIRS signal, it was started by presenting the most traditional forms of data analysis, direct inspection of the time series, and preprocessing by PCA. Then, evolved into techniques of chaos theory, which will allow a distinction on whether the irregularities in the NIRS signals are noise or possibly chaos. In the latter case, a quantitative characterization of the complexity of the dynamics is provided in terms of the Lyapunov spectrum and Lyapunov dimension $D_L$ for time series of NIRS. In other words, the validity of nonlinear

Fig. 7 Two dimensional portrait of the PCA applied NIRS signal (a) Oxy-Hb (b) Deoxy-Hb and (c) Total-Hb concentration of imagery of left hand movement
analysis in this study is verified. The results of analysis of one of the subjects at motor imagery task are shown in this paper.

The Lyapunov spectrum of the trajectory was estimated by using the algorithm proposed by Sano and Sawada method [35]. In order to get good statistics on the Lyapunov spectrum, it was analyzed for not only single run but also 1000 runs of each signal with random initial conditions and their average value and standard deviation. The convergences of Lyapunov spectrum for single run of original NIRS signal of Fig. 2(a)-(c) are shown in the left panel of Fig. 8(a)-(c). Fig. 9 (a) shows the convergence of Lyapunov dimension of that NIRS signal. The estimated largest Lyapunov exponent is ~7 bits/sec and Lyapunov dimension is ~3. The convergences of Lyapunov spectrum for single run of original NIRS signal for Figs. 4(a)-(c) are shown in the right panel of Fig. 8(a)-(c). Fig. 9 (b) shows the convergence of Lyapunov dimension of NIRS signal. The estimated largest Lyapunov exponent is ~1.6 bits/sec and Lyapunov dimension is ~3. Table I shows the result for 1000 runs of chaotic dynamical systems of experimental original NIRS data for the Lyapunov spectrum (\(\lambda_i\)), Kaplan and Yorke dimension (D_L). Table II shows the result for 1000 runs of chaotic dynamical systems of experimental PCA filtered NIRS data for the Lyapunov spectrum (\(\lambda_i\)), Kaplan and Yorke dimension (D_L). Fig. 10 (a) and Fig. 11(a) show the largest Lyapunov exponents and Lyapunov dimension for imaginary of left hand movement of original NIRS signal of single subjects for different recording day. Fig. 10 (b) and Fig. 11(b) show that the largest Lyapunov exponents for imaginary of left hand movement of PCA applied NIRS signal of single subjects for different recording day. The total-Hb has smallest Lyapunov exponent and dimension for both signal.

![Image](image_url)
Fig. 9 (a) Convergence of Lyapunov dimension for single runs of original NIRS signal shown in Fig. 2(a)-(c), and (a) Convergence of Lyapunov dimension for single runs of PCA applied NIRS signal shown in Fig. 5(a)-(c)

### Table I
Lyapunov spectrum and estimated Lyapunov dimension for imagery task of original NIRS signal. The error bars are calculated from the standard deviation of 1000 runs.

<table>
<thead>
<tr>
<th>NIRS signal at C4</th>
<th>τ_d</th>
<th>m</th>
<th>Lyapunov Exponent (λ_i) (bit/s)</th>
<th>Lyapunov Dimension (D_L)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Oxy-Hb</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N = 24800</td>
<td>20</td>
<td>5</td>
<td>7.229±0.395</td>
<td>1.715472±0.038</td>
</tr>
<tr>
<td>f_c = 40 Hz</td>
<td></td>
<td></td>
<td>-10.104±0.070</td>
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<td></td>
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<td>-13.389±0.074</td>
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<td>-17.835±0.064</td>
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<td>-27.369±0.158</td>
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<tr>
<td><strong>Deoxy-Hb</strong></td>
<td></td>
<td></td>
<td>7.092±0.3462</td>
<td>1.578229±0.035</td>
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<tr>
<td>N = 24800</td>
<td>20</td>
<td>5</td>
<td></td>
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</tr>
<tr>
<td>f_c = 40 Hz</td>
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<td>-12.275±0.172</td>
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<td>-19.531±0.138</td>
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<td>-29.07±0.283</td>
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<tr>
<td><strong>Total-Hb</strong></td>
<td></td>
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<td>4.308±0.054</td>
<td>1.759132±0.007</td>
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<tr>
<td>N = 24800</td>
<td>15</td>
<td>5</td>
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<tr>
<td>f_c = 40 Hz</td>
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<td>-5.675±0.074</td>
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<td>-18.099±0.090</td>
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<td>-28.28±0.102</td>
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### Table II
Lyapunov spectrum and estimated Lyapunov dimension for imagery task of PCA applied NIRS signal. The error bars are calculated from the standard deviation of 1000 runs.

<table>
<thead>
<tr>
<th>NIRS signal at C4</th>
<th>τ_d</th>
<th>m</th>
<th>Lyapunov Exponent (λ_i) (bit/s)</th>
<th>Lyapunov Dimension (D_L)</th>
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<tr>
<td><strong>Oxy-Hb</strong></td>
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<tr>
<td>N = 24800</td>
<td>12</td>
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<td>1.536±0.0468</td>
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<td><strong>Deoxy-Hb</strong></td>
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<td>N = 24800</td>
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<td>5</td>
<td></td>
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<tr>
<td>f_c = 40 Hz</td>
<td></td>
<td></td>
<td>-1.029±0.1147</td>
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<td>-3.649±0.1391</td>
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<td>-8.033±0.5081</td>
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<td></td>
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<td>-18.88±0.9726</td>
<td></td>
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<tr>
<td><strong>Total-Hb</strong></td>
<td></td>
<td></td>
<td>0.605±0.0139</td>
<td>2.153544±0.017</td>
</tr>
<tr>
<td>N = 24800</td>
<td>20</td>
<td>5</td>
<td></td>
<td></td>
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<tr>
<td>f_c = 40 Hz</td>
<td></td>
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<td>-0.379±0.0247</td>
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<td>-1.468±0.018</td>
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<td>-3.145±0.0395</td>
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<td>-7.227±0.0626</td>
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In this paper, first, the NIRS signals were analyzed by PCA to ensure the signal for preprocessing of high frequency component noise. The signal was reconstructed by using largest eigenvector component and preceded for further analyzing. The existences of the chaos in the NIRS signal are verified for real and imaginary of motor movement and relax condition. The Lyapunov exponent is a measure of separation of trajectories that diverge widely from their initial close positions. The larger the exponent the more chaotic/more sensitive to the initial condition the system is. The results from the nonlinear measures suggest that the NIRS signals are composed of chaos property and the attractor of NIRS is in systems with a strange attractor and thus the presence of deterministic chaos. Chaotic analysis showed that the Lyapunov exponent of brain waves increased during the imagery, and decreased during the relax condition. The present results indicated that the motor imagery promoted the neurochaos phenomenon based on the neuron activity and parasympathetic-division activity. All the quantifiers proposed to date require considerable effort to extract their numerical value from the data. The results suggest that fractal dimension may serve as a sensitive index for quantifying dynamical changes in NIRS signals during imaginary motor tasks. Moreover, the present approach has the potential to become a useful tool to characterize the patterns of motor-related cortical activities for the classification of biological research as well as optical brain-computer interface (BCI) research [16]-[18]

| Table III | \begin{tabular}{|c|c|c|c|}
| NIRS signal & Chaos parameter & Relax condition & Real Movement & Imagery Movement \\
| at C4 & & & & \\
| Oxy-Hb & \( \lambda_1 \) & 0.4438±0.0384 & 1.522±0.037 & 1.536±0.047 \\
| N = 24800 & \( \xi = 40 \text{Hz} \) & \( D_h \) & 1.270±0.0254 & 3.449±0.067 & 2.789±0.034 \\
| Deoxy-Hb & \( \lambda_1 \) & 0.121±0.015 & 1.709±0.065 & 1.063±0.15 \\
| N = 24800 & \( \xi = 40 \text{Hz} \) & \( D_h \) & 1.059±0.007 & 3.319±0.014 & 1.961±0.135 \\
| Total-Hb & \( \lambda_1 \) & 0.264±0.051 & 0.333±0.017 & 0.605±0.014 \\
| N = 24800 & \( \xi = 40 \text{Hz} \) & \( D_h \) & 1.216±0.047 & 2.336±0.018 & 2.154±0.017 \\
\end{tabular} |