Abstract—Considering the numerous applications of the study of the flow due to leakage in a buried pipe in unsaturated porous media, finding a proper model to explain the influence of the effective factors is of great importance. There are various important factors involved in this type of flow such as: pipe leakage size and location, burial depth, the degree of the saturation of the surrounding porous medium, characteristics of the porous medium, fluid type and pressure of the upstream. In this study, the flow through unsaturated porous media due to leakage of a buried pipe for up and down leakage location is studied experimentally and numerically and their results are compared. Study results show that Darcy equation together with BCM method (for calculating the relative permeability) have suitable ability for predicting the flow due to leakage of buried pipes in unsaturated porous media.

Keywords—Buried, Leaking pipe, Porous media, Unsaturated

I. INTRODUCTION

Study of the fluid flow due to leakage in a buried pipe have various applications like leakage location detection in oil and gas pipelines, subsurface irrigation, nuclear disposal, landfills and etc. Among these applications, subsurface irrigation due to the shortage of water resources is of great importance in today’s world.

In this method of irrigation, the leaking pipes are laid near the roots of the plants so the required amount of the water for the plants shall be directed to them. The most important advantage of this method is preventing evaporation of a significant proportion of the water. Volume of the water penetration through the soil and the pattern of the penetration are two factors which affect the design of this type of irrigation system. Initially, Philip in 1968 presented an analytical solution for a buried source of mass in unsaturated soil for both two and three dimensional geometry [1]. In 1986, Schwartzman investigated the role of leakage distance on flow propagation and volume of the wetted soil [2].

With progress in computer abilities for numerical solutions, the first numerical solution in this subject was presented by Cho in 1996 [3].

More extensive numerical and experimental studies are performed in this field [4-8]. Base on the knowledge of the authors all the above studies reviewed the problem from agricultural point of view and for numerical solutions commercial software (such as HYDRUS) for irrigation systems is used. Yet, the problem was not studied in the context of fluid dynamics.

In this paper both experimental and numerical methods have been used.

Initially, the way the simulation was conducted is outlined and then experimental method is described and at finally the results of the two methods are compared together

II. PROBLEM DESCRIPTION

For Modeling a pipe in a semi-infinite porous media the configuration (figure 1) is considered. The dimensions for representing the semi-infinite porous media are selected base on [9] which state that the proper ratio of width to radii is 1/30 which represents the semi-infinite porous media. The geometry considered is a horizontal pipe with a radius of ri embedded in an unsaturated porous medium at a depth of d (d/ri = 5 for the present study) beneath the top surface. A crack developed on the pipe is assumed to have an angular span of 90° to produce leakage from the pipe. Two locations of the leakage are considered in the present study: one is on the top and another at the bottom of the pipe. The fluid is assumed to discharge from the horizontal pipe at a radial velocity of UR.

III. GOVERNING EQUATIONS

Considering the numerous factors which are important and affect this type of flow, it is necessary to reduce the desired factors to study the problem and with some simplifications. In this regard, the assumptions below are considered:

The porous media is homogeneous, isotherm and unsaturated. No reaction occurs within the porous medium. The flow is two dimensional, laminar and incompressible.

In the current study for flow study of a leaking buried pipe in an unsaturated porous media, it is necessary to use multiphase relationships. The geometry of the problem is shown in figure 1.

At the beginning of the flow, it is supposed that: there is no water in the void volumes of porous media and it is filled with air (similar to a real condition), and gradually the water exits from the leakage and fill the void volumes. In this paper, the following dimensions are selected: \( \frac{d}{ri} = 5 \), \( \frac{w}{ri} = 30 \).

S.M.Hosseinalipour is the associate professor of Iran University of Science and Technology, Tehran, IRAN (phone: 21-73912929; e-mail: alipour@iust.ac.ir).

H.Aghakhani, is with Iran University of Science and Technology, Tehran, IRAN. (phone: 912-459-2389; e-mail:aghakhani63@gmail.com).
For these conditions, the following equations will be used to study the problem.

A. Mass Conservation Equation

For the multiphase flow in a porous media, the mass conservation equation would be as equation (1); each term is expressed in a list of symbols at the end of the paper. Calculation method for \( r_{ik} \) will be discussed in the next section.

\[
\frac{\partial (\phi S_i)}{\partial t} - \nabla \cdot (k \rho g Z) = -k_{\text{eff}}(\tilde{V}P_\text{e} - \rho g Z) + \rho q_i
\]  

(1)

B. Momentum Equation

For modeling the fluid flow through porous media there are a number of relations each of which has its own restrictions and range of validations. Although Darcy equation which presented in 1856 is the first equation for modeling fluid flow through porous media, is still being used by researchers in their studies, the validation range of Darcy equation is for Reynolds number much smaller than one, but it should be noted that the definition of Reynolds number which is used for validation of Darcy equation is different with its usual definition. The Darcy equation and the Reynolds number which is defined for validation range of Darcy equation is:

\[
\tilde{V} = -\frac{k}{\mu} (\tilde{V}P_\text{e} - \rho g \tilde{V}Z)
\]  

(2)

\[
\text{Re} = \frac{\rho \tilde{V}k}{\mu}
\]  

(3)

In the equation (3), the \( k \) term is the permeability of porous media. Generally the permeability is an anisotropic property therefore a matrix is needed to represent it, but whenever the porous media is isotropic (as we supposed here) its matrix decrease to a scalar.

With a little change, modified Darcy equation could be used for multiphase flow through porous media. Equation (4) shows the modified Darcy equation for multiphase flow:

\[
\tilde{V}_i = -\frac{k_{\text{eff}}}{\mu_i} (\tilde{V}P_\text{e} - \rho_i g \tilde{V}Z)
\]  

(4)

In the above equation, \( k_{\text{eff}} \) is the relative permeability for i'th phase, and it is a local function of fraction of i'th phase which has occupied the void volume of porous media. The quantity of relative permeability always is between zero and one. There are different relations for calculating the relative permeability, but for the present study the BCM model has been used [10]. Base on this model the relation of the relative permeability with uniform distribution of pores shall be:

\[
K_{\text{eff}} = S_w^n_w
\]  

(5)

\[
K_{\text{eff}} = (1-S_w)^n_o
\]  

(6)

In the above equations \( K_{\text{eff}} \) and \( K_{\text{eff}} \) are relatively the relative permeability of wetting phase (water) and relative permeability of the non wetting phase(air), and \( S_w \) is the saturation degree of wetting phase which is equal to:

\[
S_w = \phi - S_o
\]  

(7)

\( n_w \) and \( n_o \) are BCM powers for wetting and non-wetting phases, relatively which both of them for the present study are equal to 2.

For calculating the permeability in this paper Kozeny-Carman relation has been used:

\[
K = \frac{D_p^2 \rho^3}{150(1-\rho)^2}
\]  

(8)

In the above equation \( D_p \) is the effective diameter of the porous particles.

IV. EXPERIMENT

Although in the recent years, numerical methods help scientists to solve many problems in various fields of science, but these methods are incomplete as long as their results are not validated by the experimental or analytical results. For the present study because of the lack of proper analytical or previous experimental results, the required experiments are
accomplished and numerical results are compared with the experimental results.

For experimental part, a configuration set up which its schematic diagram is shown in the figure 2 is assembled.

The procedure of the experiment has been described below. The water is accumulated in a tank and the level of the water is maintained constant by a level control valve during the experiment. A constant velocity of water in the leakage location is maintained through controlling the level of the water in the tank. Two gate valve was used for controlling the flow of the water after the tank. By opening these valves the water enters the cell by a pipe which has a leakage equal to 9° on its wall. The location of the leakage for the two cases of leakage at the top and at the bottom has been changed. Before and after the cell, two flow meters were used for calculating the flow velocity of the water that exits from the leakage. The leakage velocity was calculated by equation 9 which outlet and inlet flows are obtained by the flow meters and leakage area has been measured previously.

$$v_{\text{leakage}} = \frac{Q_{\text{out}} - Q_{\text{in}}}{A_{\text{leakage}}}$$

For processing and analyzing the experimental results, a 12 mega pixel video recorder camera is used and all the experiment period is recorded and then this recorded film is analyzed by image processing software. Figure 3 represents how an image is processed to obtain advancing front of the water through the porous media.
V. NUMERICAL SIMULATION

For numerical simulation finite difference base control volume method with non-uniform structured mesh for better convergence and accuracy was used as figure 5, the mesh density near the leakage was adjusted so high because on those locations the gradients of flow specifications is relatively more that other locations so more accuracy is required. With spreading the water flow through the domain gradually the gradients decrease so coarser meshes could be used. The result independency from grid is checked with more fine grids.

Fig. 5 The generated mesh that used for solving the problem for leakage from the top

The boundary conditions have been illustrated in figure 6; because of the symmetry of the problem, only a half of the domain has been simulated. For the left and down boundaries, the velocity gradient set to zero, at the leakage, the velocity inlet conditions is selected and for the top boundary and the semicircle representing the pipe, zero velocity is used as the boundary condition, and finally for the leakage, the velocity of the water which leaves the pipe is specified as its boundary condition.

For another case which the leakage is at the bottom of the pipe, the boundary conditions are exactly the same.

The table below table contains the porous media specifications which have been used in numerical simulation:

<table>
<thead>
<tr>
<th>POROUS MEDIA SPECIFICATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity</td>
</tr>
<tr>
<td>Permeability (m2)</td>
</tr>
</tbody>
</table>

VI. RESULTS AND DISCUSSIONS

To generalize the obtained results, all the results are presented in dimensionless time scale that is displayed in equation 10:

$$T = \frac{V t}{d}$$

In the above equation $V$ is the leakage velocity which in previous section, its measurement was described, $t$ is the time elapsed from the start of the experiment and $d$ is the diameter of the pipe. The accuracy of this relation is confirmed by the numerous experiment results.

Experimental and numerical results are compared in figures 7, 8. Each contour displays the advancing front of the water for specific time that is specified in the legend of the figures.

As could be seen in the above figures in both scenarios (leakage from the top & bottom of the pipe), the consistency between the experimental and numerical results is quite satisfactory. According to these results at the beginning of the leakage, the experimental and numerical results have excellent compatibility but after spreading of the water through the porous media, this compatibility declines and the numerical and experimental results increasingly drift apart. In the leakage from the top of the pipe this incompatibility is more evident because by spreading the flow, water gets close to the boundaries of the problem so the effect of boundaries will appear sooner than when the leakage is from the bottom. The other cause of this less compatibility is that, although much efforts are done to create a complete homogenous porous environment for the cell with the proper porous media, but the reality is that the real porous materials have a distribution of particles sizes which make impossible the creation of a complete homogenous environment which is completely accessible in the numerical models. This difference between
numerical and experimental study is the reason of some incongruities in all results and could be seen in experimental results with some distortion in the related contours. The other reason of incompatibility in this section of study between numerical and experimental results is the effect of the pipe geometry in the problem. So, with elapsing time, the effect of this factor decreases and the numerical results and experimental results shall get closer together.

Fig. 7 The comparison of the numerical results (left section) and the experimental results (right section) for leakage from the top of the pipe

Fig. 8 The comparison of the numerical results (left section) and the experimental results (right section) for leakage from the bottom of the pipe
The other conclusion that can be obtained from comparison between the leakage of bottom and top of the pipe is that for this type of flow, the main factor of spreading the flow within the porous media is diffusion, because, for bottom leakage case, despite gravity force and initial velocity are in the same direction, but after a while the water reach the top boundary and surrounds the pipe and this fact shows the important effect of diffusion to spreading the water.

The comparison between top and bottom leakage results also shows that with time elapse, the flow spreading near the pipe is almost the same for both cases and the effect of the pipe geometry decreases in the problem. It is important to mention that for different properties of porous media, this similarity should be checked and these results obtained just for specific type of porous media that has been examined in this study.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>LIST OF SYMBOLS</th>
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</thead>
<tbody>
<tr>
<td>d</td>
<td>Pipe Diameter, m</td>
</tr>
<tr>
<td>D_p</td>
<td>Effective diameter of the particles, m</td>
</tr>
<tr>
<td>g</td>
<td>Gravity Acceleration, m/s^2</td>
</tr>
<tr>
<td>K</td>
<td>Permeability, m^2</td>
</tr>
<tr>
<td>l</td>
<td>Length, m</td>
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<tr>
<td>n</td>
<td>BCM Relation Power</td>
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<tr>
<td>P</td>
<td>Pressure, N/m^2</td>
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<tr>
<td>q</td>
<td>Mass Source</td>
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<tr>
<td>S</td>
<td>Saturation Degree</td>
</tr>
<tr>
<td>t</td>
<td>Time, s</td>
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<tr>
<td>T</td>
<td>Dimensionless Time</td>
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<tr>
<td>u</td>
<td>Inlet velocity of leakage, m/s</td>
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<tr>
<td>U</td>
<td>Flow, m^3/s</td>
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<tr>
<td>V</td>
<td>Velocity, m/s</td>
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<td>Z</td>
<td>Head, m</td>
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<tr>
<td>\phi</td>
<td>Porosity</td>
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<tr>
<td>\rho</td>
<td>Density, kg/m^3</td>
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<tr>
<td>\mu</td>
<td>Viscosity, N.s/m^2</td>
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<tr>
<td>i</td>
<td>Phase index</td>
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<tr>
<td>f</td>
<td>Fluid</td>
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<tr>
<td>w</td>
<td>Wetting Phase</td>
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<tr>
<td>o</td>
<td>Non-wetting Phase</td>
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<tr>
<td>r</td>
<td>Relative</td>
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REFERENCES