A Novel Method to Evaluate Line Loadability for Distribution Systems with Realistic Loads

K. Nagaraju, S. Sivanagaraju, T. Ramana, and V. Ganesh

Abstract—This paper presents a simple method for estimation of additional load as a factor of the existing load that may be drawn before reaching the point of line maximum loadability of radial distribution system (RDS) with different realistic load models at different substation voltages. The proposed method involves a simple line loadability index (LLI) that gives a measure of the proximity of the present state of a line in the distribution system. The LLI can use to assess voltage instability and the line loading margin. The proposed method also compares with the existing method of maximum loadability index [10]. The simulation results show that the LLI can identify not only the weakest line/branch causing system instability but also the system voltage collapse point when it is near one. This feature enables us to set an index threshold to monitor and predict system stability on-line so that a proper action can be taken to prevent the system from collapse. To demonstrate the validity of the proposed algorithm, computer simulations are carried out on two bus and 69 bus RDS.

Keywords—line loadability index, line loading margin, maximum line loadability, system stability, radial distribution system

I. INTRODUCTION

Considerable voltage instability-related outage events have occurred around the world and resulted in major system failures (blackouts) in recent years. Voltage stability has become an important concern for utilities. Static voltage stability can be assessed using continuation power flow calculations [1–3]. Many static voltage assessment methods have been proposed so far, such as the minimum singularity value method, mode analysis method and sensitivity method [4–6]. The main disadvantages of the continuation power flow-based methods include considerable computational efforts making implementation difficult in on-line applications, possible premature divergence due to numerical instability in power flow calculations, inconsistency between real life situation and off-line model, and incapability of identifying buses that cause system collapse or the weakest lines.

It is well known that unlike angle instability, voltage instability often starts in a local network and gradually extends to the whole system. This feature makes the process of system losing voltage stability much slower (a few seconds or even longer) compared to that of losing angle stability, and also enables us to predict static voltage stability using local measurements. There are two types of local evaluation techniques for voltage stability: line-based and node (bus)-based techniques. Conceptually, if a line or a node in the system is critically voltage-instable, the whole system approaches a collapse point [7–15]. Refs. [8, 9] presented voltage stability assessment techniques using local bus phasors whereas Refs. [10–12] derived different line-based voltage stability indices. A common demerit of the existing line-based indices is the fact that impacts of the rest of system outside the line have been ignored and this will lead to inaccurate or even incorrect results in some cases.

Ref. [13] proposed an internal and external impedance method using Thevenin theorem whereas Ref. [14] introduced the limit of node voltage into the method. Ref. [15] proved the concept of the internal and external impedance method using Tellegen’s theory. The main disadvantage of this method is the assumption that the equivalent Thevenin voltage and impedance are constant in the two or more system states. If the two system states are far apart, this assumption is obviously invalid but if they are too close, it may result in a large calculation error in the estimate of equivalent Thevenin impedance since the estimation process may be associated with an extremely small value in the denominator. This causes inaccuracy and difficulties in the actual implementation.

A new line loadability Index or voltage stability index, LLI is proposed for radial distribution systems in this paper. The LLI index can be used to identify voltage instability and assess the line loading margin. Therefore, a threshold of line loading margin can be easily set up to trigger an emergency remedial action scheme to protect the system from voltage collapse. The major strength of proposed method is to calculate LLI when the power factor angle (\(\phi\)) equals the line impedance angle (\(\alpha\)).

II. MATHEMATICAL FORMULATION FOR LOAD FLOWS

Mathematical formulation of the load flow is based Ref. [16]. Consider an equivalent circuit model of typical branch between nodes p and q of the radial distribution system as shown in fig. 1.
In fig. 1, $|V_p|\angle \delta_p$ and $|V_q|\angle \delta_q$ are the voltage magnitudes and phase angles of two nodes p and q respectively and current flowing through the branch pq is $I_{pq}$. The substation voltage is assumed to be (1+j0) p.u.

From fig. 1, current flowing through branch between nodes p and q is given by

$$I_{pq} = \frac{|V_p|\angle \delta_p - |V_q|\angle \delta_q}{r_{pq} + jx_{pq}}$$

or

$$I_{pq} = \frac{P_q - jQ_q}{(V_q\angle \delta_q)}$$

Where, $P_q$ is the sum of the real power loads of all the nodes beyond node q plus the real power load at node q itself plus sum of real power losses of all branches beyond node q.

$Q_q$ is the sum of the reactive power loads of all the nodes beyond node q plus the reactive power load at node q itself plus sum of reactive power losses of all branches beyond node q.

From eqns. (1) and (2)

$$P_q - jQ_q = \frac{|V_p|\angle \delta_p - |V_q|\angle \delta_q}{r_{pq} + jx_{pq}}$$

i.e.

$$|V_p|\|V_q|\cos(\delta_p - \delta_q) + j|V_q|\sin(\delta_p - \delta_q) - |V_q|^2 = (P_q - jQ_q)(r_{pq} + jx_{pq})$$

Separating real and imaginary parts of above equation

The real part is

$$|V_p|\|V_q|\cos(\delta_p - \delta_q) = |V_q|^2 + P_q r_{pq} + Q_q x_{pq}$$

and the imaginary part is

$$|V_p|\|V_q|\sin(\delta_p - \delta_q) = P_q x_{pq} - Q_q r_{pq}$$

From eqn. (4)

$$P_q = \frac{|V_p|\|V_q|\sin(\delta_p - \delta_q) + r_{pq} Q_q}{x_{pq}}$$

Substituting $P_q$ value from eqn (5) in eqn (3) and rearrange the equation than

$$|V_q|^2 + |V_q|\left(\frac{r_{pq}}{P_{pq}^2 + Q_q^2}\right)\sin(\delta_p - \delta_q) - \cos(\delta_p - \delta_q)$$

$$+ Q_q\left(\frac{r_{pq}^2}{x_{pq}^2} + x_{pq}^2\right) = 0$$

This quadratic equation when solved gives the solution for $|V_q|$, and $|V_q|$ has got two solutions. One of the solutions gives the negative voltage and hence it can be discarded. The possible solution for $|V_q|$ is given by

$$|V_q| = -0.5 \left[\frac{|V_p|\left(\frac{r_{pq}}{P_{pq}^2 + Q_q^2}\right)\sin\delta - \cos\delta\right]}{2Q_q\left(\frac{r_{pq}^2}{x_{pq}^2} + x_{pq}^2\right)}$$

Where $\delta = \delta_p - \delta_q$.

The corresponding phase angle of $|V_q|$ can be calculated from eqns. (3) and (4) as

$$\delta_q = \delta_p - \tan^{-1}\left(\frac{P_q x_{pq} - Q_q r_{pq}}{|V_q|^2 + P_q r_{pq} + Q_q x_{pq}}\right)$$

1) 2) The active and reactive power losses in branch ‘pq’ are given by

$$LP_{pq} = \frac{r_{pq}^2 + Q_q^2}{|V_q|^2}$$

$$LQ_{pq} = \frac{x_{pq}^2 + Q_q^2}{|V_q|^2}$$

III. MATHEMATICAL FORMULATION FOR LINE LOADABILITY INDEX (LLI)

By eliminating the angles from eqns. (3) and (4) and rearrange the equation than

$$|V_q|^4 + 2\left(r_{pq} P_q + x_{pq} Q_q - \frac{|V_p|^2}{2}\right)|V_q|^2$$

$$+ \left(r_{pq}^2 + x_{pq}^2\right)|P_q^2 + Q_q^2| = 0$$

When the discriminant of eqn. (10) is greater than or equal to 0, that is,
Maximum loadability is reached when $P_q + jQ_q$ is increased to make the left term of eqn. (11) equal to zero. In order to determine that point, the power flow $P_q + jQ_q$ is replaced by the term $LLI \times (P_q + jQ_q)$ assuming a constant load power factor, where $LLI$ is a real number factor, we obtain

$$\frac{LLI}{2} = \frac{V_p}{2} \left( r_{pq} P_q + x_{pq} Q_q \right) + \sqrt{r_{pq}^2 + x_{pq}^2} \left( P_q^2 + Q_q^2 \right) \geq 0$$  \hspace{1cm} (11)

$$V_p = \frac{\sqrt{r_{pq}^2 + x_{pq}^2} \left( P_q^2 + Q_q^2 \right)}{2 \left( r_{pq} P_q + x_{pq} Q_q \right)} \geq 1$$  \hspace{1cm} (12)

$$LLI = \frac{V_p}{2} \left( r_{pq} P_q + x_{pq} Q_q \right) + \sqrt{r_{pq}^2 + x_{pq}^2} \left( P_q^2 + Q_q^2 \right) \geq 1$$  \hspace{1cm} (13)

If $x_{pq} P_q - r_{pq} Q \neq 0$, it follows that $\varphi = \alpha$, where $\alpha = \arctg \left( \frac{Q_q}{P_q} \right)$ and $\varphi = \arctg \left( \frac{Q_q}{P_q} \right)$. In this case, the MLI index has no meaning, as its denominator is zero. This is an essential deficiency of MLI. Although this case may not happen very often, the MLI index loses its robustness mathematically. If $x_{pq} P_q - r_{pq} Q = 0$, that is $\varphi = \alpha$, we can have following derivation:

$$\text{MLI} = \frac{\sqrt{r_{pq}^2 + x_{pq}^2} \left( P_q^2 + Q_q^2 \right)}{2 \left( r_{pq} P_q + x_{pq} Q_q \right)}$$  \hspace{1cm} (14)

In other words, MLI = LLI when $\varphi \neq \alpha$. It can be seen that LLI has a more general and simpler expression than MLI. Particularly, LLI is suitable for any case, whereas MLI cannot be used when $\varphi = \alpha$. Also, MLI may result in a large error when $\varphi$ is close to $\alpha$.

IV. MODELLING OF REALISTIC LOADS

In conventional load flow studies, it is presumed that active and reactive power demands are specified as constant values, regardless of the amplitude of voltages at the bus. In actual practice, different types of loads might be present. The nature of these types of loads is such that their active and reactive powers are dependent on the voltage of the system and are investigated in different planning scenarios (test cases).

Realistic loads, i.e., residential, industrial, and commercial given in [17] have been adopted for investigations. The load models can be mathematically expressed as

$$PL = PL_0 \left( \frac{V}{V_o} \right)^\alpha$$  \hspace{1cm} (15)

$$QL = QL_0 \left( \frac{V}{V_o} \right)^\beta$$  \hspace{1cm} (16)

Where $\alpha$ and $\beta$ stand for load exponents $PL_0$ and $QL_0$, stand for the values of the active and reactive powers at the nominal voltages $V$ and $V_o$ stand for load node voltage and load nominal voltage.

The values of the real and reactive exponents used in the present work for industrial, residential, and commercial loads are given in Table I. For practical application, the evaluation of coefficients $\alpha$ and $\beta$ requires use of parameter estimation techniques.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>VALUES OF THE EXONENTS OF DIFFERENT REALISTIC LOADS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load component</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Constant Power</td>
<td>0.00</td>
</tr>
<tr>
<td>Industrial</td>
<td>0.18</td>
</tr>
<tr>
<td>Residential</td>
<td>0.99</td>
</tr>
<tr>
<td>Commercial</td>
<td>1.31</td>
</tr>
</tbody>
</table>

While investigating the effect of residential load, the system is assumed to be supplying only residential consumers. Similarly, for industrial and commercial loads, it is assumed...
that all the loads are industrial and commercial type respectively. But, in practice, load will be of composite type. Hence, 40% Constant power + 30% Industrial + 20% Residential + 10% Commercial is considered for the analysis.

V. ILLUSTRATIVE EXAMPLES

A. Case Study I

In Figure 1, it is assumed that bus p is a bus with a given voltage and bus q is a PQ load bus. \( \frac{v_{pq}}{\omega_q} = 0.05 + j0.10 \text{ p.u.} \), \( p_q + jQ_q = 0.5 + j1.0 \text{ p.u.} \) and \( v_p \angle \delta_p = 1.0 \text{ p.u.} \). At this initial load state, \( \phi = \alpha = 63.4^\circ \). Tables II, III, and IV present voltage solutions at the load bus q i.e. \( V_q \); the two loadability or voltage stability indices LLI and MLI; and the maximum loading capacity \((\text{LLI} \times S_q, \text{ in p.u.})\) and loading margin of the line \((\text{LLI} - 1) \times S_q, \text{ in p.u.})\), estimated using the LLI index when the load is gradually increased. \( \lambda \) represents multiples of the increased load with regard to its initial value. Table II shows the results when both active and reactive loads are stressed with a constant power factor; Table III shows the results when only the active load is stressed; and Table IV shows the results when only the reactive load is stressed. It can be seen in all the cases that \( \text{LLI} > 1 \) before the maximum loadability point and \( \text{LLI} = 1 \) before the collapse point. In other words, the proposed LLI index is exactly consistent in determination of loadability or voltage instability in this simple distribution line system. The symbol “—” denotes that the MLI index cannot be calculated when \( \alpha = \phi \), as the denominator is zero in this case. In the cases shown in Tables III and IV, the increase in only active or reactive load makes \( \alpha \neq \phi \) and these cases, MLI equals LLI.

![Fig. 2 Single line diagram of 69-bus Radial distribution system](image)

B. Case Study II

A 69-bus, 12.66 kV radial distribution system shown in fig. 2 is considered, whose line and load data are given in [7]. The base MVA is 100. The total real and reactive power loads at nominal voltage are 3791.79 kW and 2683.40 kVAr respectively.

| Table II | Simulation results of one-line distribution system when increasing both active and reactive load with a constant power factor |
| \( \lambda \) | \( |V_q| \) p.u. | LLI | MLI | \( \text{LLI(S}_q\times) \) p.u. | \( (\text{LLI(S}_q\times-1)\times) \) p.u. |
| 1.0000 | 0.8536 | 2.0000 | — | 2.2361 | 1.1181 |
| 1.2953 | 0.7968 | 1.5442 | — | 2.2361 | 0.7880 |
| 1.4741 | 0.7564 | 1.3567 | — | 2.2361 | 0.5879 |
| 1.6777 | 0.7007 | 1.1922 | — | 2.2361 | 0.3605 |
| 1.9094 | 0.6604 | 1.0474 | — | 2.2361 | 0.1012 |
| 2.0000 | 0.5000 | 1.0000 | — | 2.2361 | 0.0000 |

| Table III | Simulation results of one-line distribution system when only increasing the active load |
| \( \lambda \) | \( |V_q| \) p.u. | LLI | MLI | \( \text{LLI(S}_q\times) \) p.u. | \( (\text{LLI(S}_q\times-1)\times) \) p.u. |
| 1.0000 | 0.8536 | 2.0000 | — | 2.2361 | 1.1181 |
| 1.3213 | 0.8045 | 1.5869 | — | 2.2419 | 0.8291 |
| 1.5188 | 0.7696 | 1.4059 | — | 2.2480 | 0.6490 |
| 1.7458 | 0.7224 | 1.2149 | — | 2.2553 | 0.4393 |
| 2.0000 | 0.6519 | 1.0977 | — | 2.2630 | 0.2014 |
| 2.2132 | 0.5037 | 1.0000 | — | 2.2690 | 0.0000 |

| Table IV | Simulation results of one-line distribution system when only increasing the reactive load |
| \( \lambda \) | \( |V_q| \) p.u. | LLI | MLI | \( \text{LLI(S}_q\times) \) p.u. | \( (\text{LLI(S}_q\times-1)\times) \) p.u. |
| 1.0000 | 0.8536 | 2.0000 | — | 2.2361 | 1.1181 |
| 1.3213 | 0.8045 | 1.5869 | — | 2.2419 | 0.8291 |
| 1.5188 | 0.7696 | 1.4059 | — | 2.2480 | 0.6490 |
| 1.7458 | 0.7224 | 1.2149 | — | 2.2553 | 0.4393 |
| 2.0000 | 0.6519 | 1.0977 | — | 2.2630 | 0.2014 |
| 2.2232 | 0.5037 | 1.0000 | — | 2.2690 | 0.0000 |

| Table V | Load flow results of 69-bus RDS for different types of realistic loads |
| Parameters | Constant Power load | Industrial | Residential | Commercial | Composite |
| Real power load in system (MW) | 3791.79 | 3761.26 | 3631.64 | 3584.47 | 3726.53 |
| Reactive power load in system (MVar) | 2683.40 | 2090.85 | 2278.67 | 2318.32 | 2377.37 |
| Total real power loss (kW) | 224.61 | 174.86 | 170.22 | 167.26 | 189.61 |
| Total reactive power loss (kVAr) | 101.98 | 80.57 | 78.62 | 77.35 | 0.91591 |
| Minimum Voltage (p.u.) | 0.90923 | 0.91878 | 0.92051 | 0.92150 | 0.98375 |
In table V was shown the system load flow solution total real load, total reactive load, total active power losses, reactive power losses and system minimum voltage for different types of realistic load models at substation voltage of 1.0 p.u. The 69-bus radial distribution system was used to test the proposed Line loadability index. Realistic load models with different substation voltages were considered in the test. It has been observed that the minimum Line loadability index is found at line 56 and the load between buses 56 and 57 was gradually increased with the constant power factor, Line 56 and Bus 57 were more sensitive to voltage instability and, thus, were identified as the weakest line and bus in the case of stressing loads at Bus 57. The minimum voltage observed at bus 65 by gradually increasing the load between the buses 56 and 57. Table 6 presents the line loadability index, line maximum loadability (MVA), and actual loading (MVA) of Line 56 at the critical loading conditions for realistic load models at three different substation voltage levels.

In all the realistic load models, minimum Line Loadability Index, Line Maximum Loadability, Margin line loading and voltage at 57th bus increasing while increase the substation operating voltage. The minimum LLI value at line 56 is the lowest for constant power load and highest for commercial load. Line Maximum Loadability is the lowest for constant commercial load and highest for constant power load. Line loading margin is the lowest for constant power load and highest for commercial load. At line maximum loadability voltage at 57th bus is the lowest for composite load and highest for commercial load.

VI. CONCLUSION

A new line loadability index LLI is presented to identify weak lines and buses due to voltage instability for radial distribution systems. The product of the LLI and apparent power (S) of the line at any operation state is the estimate of maximum loadability of the line due to voltage instability, and therefore, (LLI-I)×S can also be used to assess the line loading margin. Compared to the existing MLI loadability index, the proposed LLI has a more general and simpler expression. A deficiency of the MLI index is the fact that it cannot be calculated when the power factor angle (ψ) equals the line impedance angle (α). The simulation results of test systems demonstrate the feasibility and effectiveness of the proposed index and method. A system will lose voltage stability whenever the LLI of at least one branch is near one. This feature enables us to set an index threshold to monitor and predict voltage stability on-line so that a proper action can be taken to prevent the system from collapse. The method has been tested on two test radial distribution systems by considering the realistic loads with different substation voltages and the results are presented.

REFERENCES


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