Towards an Enhanced Stochastic Simulation Model for Risk Analysis in Highway Construction

Anshu Manik, William G. Buttlar, Kasthurirangan Gopalakrishnan

Abstract—Over the years, there is a growing trend towards quality-based specifications in highway construction. In many Quality Control/Quality Assurance (QC/QA) specifications, the contractor is primarily responsible for quality control of the process, whereas the highway agency is responsible for testing the acceptance of the product. A cooperative investigation was conducted in Illinois over several years to develop a prototype End-Result Specification (ERS) for asphalt pavement construction. The final characteristics of the product are stipulated in the ERS and the contractor is given considerable freedom in achieving those characteristics. The risk for the contractor or agency depends on how the acceptance limits and processes are specified. Stochastic simulation models are very useful in estimating and analyzing payment risk in ERS systems and these form an integral part of the Illinois’s prototype ERS system. This paper describes the development of an innovative methodology to estimate the variability components in in-situ density, air voids and asphalt content data from ERS projects. The information gained from this would be crucial in simulating these ERS projects for estimation and analysis of payment risks associated with asphalt pavement construction. However, these methods require at least two parties to conduct tests on all the split samples obtained according to the sampling scheme prescribed in present ERS implemented in Illinois.

Keywords—Asphalt Pavement, Risk Analysis, Stochastic Simulation, QC/QA.

I. INTRODUCTION

Quality control (QC) is a procedure or set of procedures intended to ensure that a manufactured product adheres to a defined set of quality criteria or meets the requirements of the client or customer, whereas quality assurance (QA) is intended to ensure that a product under development (before work is complete, as opposed to afterwards) meets specified requirements [1]. QA specifications are an important component of an organization’s commitment to overall quality management, and consist of several activities, including: process control, acceptance, and sometimes, independent assurance of a product [2].

To promote the construction of better-quality asphalt pavements, over the years, a number of state highway agencies have moved from using traditional method specifications to using statistically based QC/QA specifications [3]. In many QC/QA specifications, the contractor is primarily responsible for quality control of the process, whereas the highway agency is responsible for testing the acceptance of the product. These specifications are typically statistics based, in which methods such as stratified random sampling and lot-by-lot testing are used, allowing contractors to ensure that their operations are producing an acceptable product [4]. There is considerable literature discussing the various steps involved in developing a new QA specification for asphalt pavements [5]-[10].

Specifications for the construction of asphalt pavements can generally be classified into method-related specifications (MRS), end-result specifications (ERS), performance-related specifications (PRS), or combinations thereof. Method specifications give a set of procedures, that if followed by the contractor, will result in full payment for the constructed facility. This places a great deal of responsibility and testing burden on the agency rather than the contractor. End-result and performance-related specifications, as their names imply, require a contractor to achieve specified as-produced or as-constructed quality levels, which are ideally linked to the attainment of good future performance. These types of specifications shift most or all of the responsibility for producing a high quality product to the contractor, and should ideally offer the contractor complete freedom in the methods used to arrive at these quality levels [2].

Reduced construction and material variability is one measure of improved “quality” of construction. Although decreased variability may be desirable, there is considerable debate and confusion about the cost-effectiveness of the QC/QA programs [11].

A common provision in quality control/quality assurance (QC/QA) construction contracts is the adjustment of the pay that a contractor receives on the basis of the quality of the construction. It is important to both the contractor and the contracting agency to examine the amount of pay that the contractor can expect to receive for a given level of construction quality [12]. It has been shown that computer
simulations can provide a better, more detailed examination of the pay schedule than is possible by simply determining the expected pay. In particular, the simulation process can provide an indication of the variability of pay at various quality levels and can identify the factors most responsible for pay adjustments [12]. Using such simulation models, it is possible to analyze both pass/fail and pay adjustment acceptance procedures, construct operating characteristic curves, plot control charts, experiment with computer simulation, perform statistical comparisons of data sets, demonstrate the unreliability of decisions based on a single test result, and explore the effectiveness of stratified random sampling [13]-[14].

A cooperative investigation was conducted in Illinois over several years to develop a prototype ERS for asphalt pavement construction. The work was conducted under the Illinois Cooperative Highway Research Program (ICHRP), Project R23, and includes researchers from the University of Illinois at Urbana–Champaign and the Illinois Department of Transportation (IDOT), along with task force representatives from FHWA and the Illinois Asphalt Paving Association [15]. Early in the planning stages, the task force assigned top priority to understanding and balancing the risks between the contractor and the agency as part of the specification development process. Recently, a new computer simulation program, called ILLISIM, was developed which would enable detailed assessment of agency and contractor risks and assist in selecting sampling and measurement methods, specification limits, retest provisions, pay scales, and pay caps in such a manner that the trade-offs between number of samples and payment risks, and hence disputes, will be balanced [16].

Stochastic simulation models are very useful in estimating and analyzing payment risk in ERS systems. One of the key requirements for the simulation is generation of synthetic QC/QA data which would be representative of actual ERS projects. Air voids and asphalt content data have been shown to have normal probability distribution. In-situ Hot-Mix Asphalt (HMA) density data, if collected in a completely randomized fashion would also follow a normal distribution [17]. However, if the data is collected at certain fixed offsets, the data in entirety may not show normal probability distribution. Analysis of ERS data from Illinois pavements shows that there is some kind of a trend present in the density data and it seems probable that it is composed of a combination of normally distributed sub datasets combined in some specific fashion.

The normal distribution (a bell-shaped curve) represents a theoretical frequency distribution of measurements. Any normally distributed data can be characterized by its mean and standard deviation. But it is also possible that the standard deviation or variability is a combination of more than one variability. For example, analysis of ERS data from Illinois HMA pavements indicates that air voids data would most likely have production variability as well as measurement variability components determining its characteristics. In addition, the manner in which different variabilities combine may also vary from one quality characteristic to other. This paper presents innovative methodologies which can be implemented to identify and estimate different variabilities present in ERS project data.

II. VARIABILITIES IN IN-SITU HMA DENSITY MEASUREMENTS

Analysis of ERS project in-situ HMA density measurement data indicates that it might have a longitudinal as well as a transverse variability component. During hot mix asphalt pavement construction, after placement of the hot mix, a roller is run over it in several passes in a predefined pattern to compact the mix and obtain a rigid, stable and smooth surface. It is possible, however, that the mix may not be placed fully uniform along the length as well as in the transverse direction across the pavement. In addition to that, since only a portion of the pavement width is rolled in one pass, it is possible that there is some variation in compacting effort across the width. These phenomena would result in longitudinal as well as transverse variability in the pavement. Therefore, in summary, in-situ density data can be expected to have following variability components:

1. Longitudinal construction variability
2. Transverse variability and
3. Measurement device variability (for each party doing the measurements)

<table>
<thead>
<tr>
<th>Sublot #</th>
<th>Offset</th>
<th>N</th>
<th>Contractor (ppg)</th>
<th>District (ppg)</th>
<th>Third Party (ppg)</th>
<th>Pay (ppg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>249</td>
<td>93</td>
<td>92.1</td>
<td>92.1</td>
<td>92</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>95.3</td>
<td>95.4</td>
<td>95.6</td>
<td>95.6</td>
<td>95.6</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>96.1</td>
<td>96.8</td>
<td>96.8</td>
<td>96.8</td>
<td>96.8</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>93.8</td>
<td>94.1</td>
<td>93.8</td>
<td>93.8</td>
<td>93.8</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>92.5</td>
<td>92.6</td>
<td>92.6</td>
<td>92.6</td>
<td>92.6</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>94.5</td>
<td>94.5</td>
<td>94.5</td>
<td>94.5</td>
<td>94.5</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>95.2</td>
<td>95.2</td>
<td>95.2</td>
<td>95.2</td>
<td>95.2</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>94.8</td>
<td>94.8</td>
<td>94.8</td>
<td>94.8</td>
<td>94.8</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>93.9</td>
<td>93.6</td>
<td>93.6</td>
<td>93.6</td>
<td>93.6</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>93.9</td>
<td>93.0</td>
<td>93.0</td>
<td>93.0</td>
<td>93.0</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>93</td>
<td>92.8</td>
<td>92.8</td>
<td>92.8</td>
<td>92.8</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>92.8</td>
<td>92.5</td>
<td>92.5</td>
<td>92.5</td>
<td>92.5</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>90.8</td>
<td>89.9</td>
<td>89.9</td>
<td>89.9</td>
<td>89.9</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>88.6</td>
<td>88.6</td>
<td>88.6</td>
<td>88.6</td>
<td>88.6</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>94.4</td>
<td>94.6</td>
<td>94.6</td>
<td>94.6</td>
<td>94.6</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>94.8</td>
<td>94.8</td>
<td>94.8</td>
<td>94.8</td>
<td>94.8</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>94.5</td>
<td>94.9</td>
<td>94.9</td>
<td>94.9</td>
<td>94.9</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>92.4</td>
<td>92.1</td>
<td>92.1</td>
<td>92.1</td>
<td>92.1</td>
</tr>
</tbody>
</table>

Fig. 1 Sample in-situ density data sheet from ERS project in Illinois

For testing, five positions are marked across the pavement as shown in Fig. 2 at 2-, 4-, 6-, 8- and 10-ft offset from the edge of the pavement. Two samples are cored from each position. These cores can be considered as split samples because there would be minimal variation in density within one foot of longitudinal distance. One core from each pair is tested by the contractor for QC/QA purposes. The second core may be tested by the district or the agency. The state of Illinois requires that the district conduct test on at least 20% of the split cores for quality assurance purposes.
Fig. 1 shows that all the split samples were tested by the district. This is a special case. In the year 2000 and 2002, ERS was still in initial development phase in Illinois and 100% QA tests were performed to obtain sufficient data for assessing the effectiveness of the newly developed specification. Also, there are only four sublots shown in the figure since this is just a sample of the data sheet. Actual project may have 20 to 50 or more sublots. Later sections describe the reasons for selecting this example for this study.

The sampling scheme shown in Fig. 2 allows for determination of in-situ density along the entire stretch of the pavement and across the mat. Pictorially, in-situ HMA density could be represented as shown in Fig. 3. The elevation of the surface is proportional to the density. The density values are actually known only at test locations. Elevations at other places have been interpolated for the purpose of representation only.

Fig. 4 pictorially presents this case where the transverse variability in in-situ HMA density has been eliminated. However, since the densities across the mat are still average of the original five densities, one fifth component of the transverse variance would still be present. Variance due to longitudinal variability however should be present as in the original data, because data along the pavement was not manipulated. In addition, measurement variability component is present in the data and would contribute to the total variance. Mathematically, this can be presented as shown in Eq. 5. The coefficients for the variance terms are being treated as unknowns and would need to be determined.

Can these variance components be separated using the information available from an ERS project density data sheet (see Fig. 1)? Traditionally, if the user wishes to analyze a database of historical measurements for which individual sources of variability cannot be separated, a single standard deviation is used to encompass the combined variability of the process. To the best of the authors’ knowledge, there is no standard method or published work available for separating the variability components. This will be the main focus of this paper.

Conceptually, first the transverse variability can be suppressed by processing the contractor density data as outlined in the following steps:

Determine average density in each sublot:

\[
\mu_{\text{cont,sublot},i} = \frac{\sum_{j=1}^{5} d_{\text{cont},i,j}}{5}
\]  

(2)

Where,

- \( i = 1, 2, 3... n \)
- \( n = \) number of sublots
- \( j = 1, 2, 3, 4, 5 \) (five densities across the mat)
- \( d_{\text{cont},i,j} = \) in-situ contractor density measurement

Define new density at each location as the average density for that sublot:

\[
d_{\text{cont,avg},i,j} = \mu_{\text{cont,sublot},i}
\]  

(3)

Determine variance of the averaged density data:

\[
\sigma_{\text{cont,avg}}^2 = \frac{\sum_{i=1}^{n} \sum_{j=1}^{5} \left( d_{\text{cont,avg},i,j} - \mu_{\text{cont,sublot},i} \right)^2}{(n * 5 - 1)}
\]  

(4)
mathematically.

\[ a_1 \sigma^2_{\text{trans}} + b_1 \sigma^2_{\text{long}} + c_1 \sigma^2_{\text{cont, meas}} = \sigma^2_{\text{cont, avg}} \]  

(5)

Where,

\( a_1, b_1, c_1 \) are coefficients which need to be determined.

On similar terms it may be possible to suppress the longitudinal variability. The following steps can be performed to achieve this:

1. Take the original densities along and across the pavement.
2. Determine the mean of all the contractor density data, say \( \mu_{\text{cont, lot}} \): 

\[ \mu_{\text{cont, lot}} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{5} d_{\text{cont, i, j}}}{n \times 5} \]  

(6)

3. Determine mean of the five densities for each sublot, say \( \mu_{\text{cont, sublot}} \):

\[ \mu_{\text{cont, sublot}} = \frac{\sum_{j=1}^{5} d_{\text{cont, i, j}}}{5} \]  

(7)

4. Shift each density measurement in a given sublot by the difference between average of total lot and average of that particular sublot i.e.:

\[ d_{\text{cont, shifted, i, j}} = d_{\text{cont, i, j}} + (\mu_{\text{cont, lot}} - \mu_{\text{cont, sublot, i}}) \]  

(8)

5. Determine variance of the shifted data:

\[ \sigma^2_{\text{cont, shifted}} = \frac{\sum_{i=1}^{n} \sum_{j=1}^{5} \left( d_{\text{cont, shifted, i, j}} - \frac{\sum_{i=1}^{n} \sum_{j=1}^{5} d_{\text{cont, shifted, i, j}}}{n \times 5} \right)^2}{n \times 5 - 1} \]  

(9)

The data manipulation proposed above basically changes the density profile and brings each sublot density to the same level as any other. This would happen on a project where there is no longitudinal variability although transverse variability is present. Pictorially the density profile would look as shown in Fig. 5. Compared to the original density profile (Fig. 3), the transverse variation is the same in this case but the peaks and troughs look clearer and distinct. It would be expected that variance due to transverse variability would contribute fully to the variance for this profile. However, longitudinal variability has been completely suppressed. Therefore, coefficient for the longitudinal variability term in Eq. 10 should be zero. Contribution of measurement variability also should remain intact. However, these hypotheses would need to be verified mathematically. Therefore, a generic expression, as shown in Eq. 10 is recommended for further analyses.

\[ a_2 \sigma^2_{\text{trans}} + b_2 \sigma^2_{\text{long}} + c_2 \sigma^2_{\text{cont, meas}} = \sigma^2_{\text{cont, shifted}} \]  

(10)

Density data from ERS projects constructed in the year 2000 and 2002 were used in this analysis. An important observation that was made is that in all the cases the variances from the shifted and averaged profile added up exactly to the total variance of the original profile as shown below:

\[ \sigma^2_{\text{cont, avg}} + \sigma^2_{\text{cont, shifted}} = \sigma^2_{\text{cont, comb}} \]  

(11)

The steps outlined above give a set of equations with certain unknown terms. These simultaneous equations need to be solved. The set of equations are presented again in Eqs. 12 to 14.
\[ a_1\left(\sigma^2_{\text{trans}}\right) + b_1\left(\sigma^2_{\text{long}}\right) + c_1\left(\sigma^2_{\text{cont, meas}}\right) = \sigma^2_{\text{cont, avg}} \]  \hspace{0.5cm} (12)

\[ a_2\left(\sigma^2_{\text{trans}}\right) + b_2\left(\sigma^2_{\text{long}}\right) + c_2\left(\sigma^2_{\text{cont, meas}}\right) = \sigma^2_{\text{cont, shifted}} \]  \hspace{0.5cm} (13)

\[ \sigma^2_{\text{cont, avg}} + \sigma^2_{\text{cont, shifted}} = \sigma^2_{\text{cont, comb}} \]  \hspace{0.5cm} (14)

Therefore, there are three equations available with a total of nine unknowns. An explicit solution is mathematically impossible. However, if the magnitudes of the coefficients, namely \(a_1, b_1, c_1\) and \(a_2, b_2, c_2\) can be determined by an alternate method, then only three unknowns, the three variance components, need to be determined. Mathematically three unknowns with three equations may have a unique solution.

Bootstrapping is a method that is used to make inferences about a population, from the data contained in a sample that was drawn from that population [18]. It is a method for estimating the sampling distribution of an estimator by resampling with replacement from the original sample. It is distinguished from the jackknife procedure, used to detect outliers, and cross-validation, used to make sure that results are repeatable [19].

Several hypothetical, "bootstrapped" samples are created by randomly selecting values from the original sample. The advantage of this method is that as long as the sample size is over 30 or so, the transverse variability, longitudinal variability and measurement variability would remain very similar to those of the entire population. Also, the bootstrapping method can be applied on a synthetically generated population. The data can be generated with fixed coefficients can be determined. However because of the finite size of the samples, there would be slight variation in the magnitude of variance components and hence coefficients. In fact, it is because of this slight variation that the solution is actually possible. Otherwise, the set of equations would be singular and no solution can be derived.

Since it is expected that there would be some variation in the magnitude of the determined coefficients, a one-time determination of the values may not yield the right results. Therefore, bootstrapping method is applied and solution is generated multiple times. In this case, 150 solutions were generated. A small number of run results are presented in Table 1 as an example. The average of the 150 solutions generated could be taken as the magnitude of the coefficients in the equations being solved. The final coefficient values have been presented in the following set of equations.

\[ \frac{4}{5}\left(\sigma^2_{\text{trans}}\right) + 0\left(\sigma^2_{\text{long}}\right) + \frac{4}{5}\left(\sigma^2_{\text{cont, meas}}\right) = \sigma^2_{\text{cont, shifted}} \]  \hspace{0.5cm} (16)

<table>
<thead>
<tr>
<th>Run no.</th>
<th>(\sigma^2_{\text{trans}})</th>
<th>(\sigma^2_{\text{long}})</th>
<th>(\sigma^2_{\text{cont}})</th>
<th>(\sigma^2_{\text{shifted}})</th>
<th>(\sigma^2_{\text{avg, cont}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.79</td>
<td>0.00</td>
<td>0.79</td>
<td>0.20</td>
<td>1.03</td>
</tr>
<tr>
<td>2</td>
<td>0.79</td>
<td>0.01</td>
<td>0.77</td>
<td>0.20</td>
<td>0.99</td>
</tr>
<tr>
<td>3</td>
<td>0.79</td>
<td>0.01</td>
<td>0.79</td>
<td>0.19</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>0.80</td>
<td>0.01</td>
<td>0.78</td>
<td>0.20</td>
<td>1.01</td>
</tr>
<tr>
<td>5</td>
<td>0.79</td>
<td>0.00</td>
<td>0.77</td>
<td>0.21</td>
<td>0.98</td>
</tr>
<tr>
<td>6</td>
<td>0.79</td>
<td>0.00</td>
<td>0.81</td>
<td>0.20</td>
<td>0.98</td>
</tr>
<tr>
<td>7</td>
<td>0.76</td>
<td>0.17</td>
<td>0.64</td>
<td>0.19</td>
<td>1.01</td>
</tr>
<tr>
<td>8</td>
<td>0.80</td>
<td>-0.01</td>
<td>0.80</td>
<td>0.20</td>
<td>0.97</td>
</tr>
<tr>
<td>9</td>
<td>0.80</td>
<td>0.00</td>
<td>0.78</td>
<td>0.21</td>
<td>0.98</td>
</tr>
<tr>
<td>10</td>
<td>0.79</td>
<td>0.01</td>
<td>0.79</td>
<td>0.20</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Eqs. 15 and 16 have three unknowns and still cannot be solved. Fortunately, the data being used for this analysis has another set of measurements recorded by the agency (which is Illinois DOT in this case). The longitudinal and transverse variability components in the agency’s data would be exactly the same as those in the contractor data because the tests were performed on split samples by the contractor and the agency. However, the variance component because of measurement variability would be different for the two parties. Therefore, Eqs. 17 and 18 would hold good for the agency data.

\[ \frac{1}{5}\left(\sigma^2_{\text{trans}}\right) + \left(\sigma^2_{\text{cont}}\right) + \frac{1}{5}\left(\sigma^2_{\text{agency, meas}}\right) = \sigma^2_{\text{agency, avg}} \]  \hspace{0.5cm} (17)

\[ \frac{4}{5}\left(\sigma^2_{\text{trans}}\right) + 0\left(\sigma^2_{\text{long}}\right) + \frac{4}{5}\left(\sigma^2_{\text{agency, meas}}\right) = \sigma^2_{\text{agency, shifted}} \]  \hspace{0.5cm} (18)

Eqs. 15 to 18 form a set of simultaneous equations. There are four equations and four unknowns. Therefore, a unique solution may be possible for each dataset. In matrix form, this set of equations can be presented as follows.

\[
\begin{bmatrix}
0.2 & 1 & 0.2 & 0 \\
0.8 & 0 & 0.8 & 0 \\
0.2 & 1 & 0 & 0.2 \\
0.8 & 0 & 0 & 0.8 \\
\end{bmatrix}
\begin{bmatrix}
\sigma^2_{\text{cont}} \\
\sigma^2_{\text{long}} \\
\sigma^2_{\text{meas, cont}} \\
\sigma^2_{\text{meas, long}} \\
\end{bmatrix}
= 
\begin{bmatrix}
\sigma^2_{\text{cont, avg}} \\
\sigma^2_{\text{cont, shifted}} \\
\sigma^2_{\text{avg, cont}} \\
\sigma^2_{\text{avg, shifted}} \\
\end{bmatrix}
\]  \hspace{0.5cm} (19)

An attempt to solve the above set of equations using matrix method revealed that the coefficient matrix is singular!
This means that this set of equations still does not have a closed-form solution. The singularity comes because of the relationship shown in Eq. 20. Because of this relationship, the equations are not independent.

\[ \sigma_{\text{cont,avg}}^2 + \sigma_{\text{cont,shifted}}^2 = \sigma_{\text{cont,comb}}^2 \]  

(20)

A deeper understanding of the characteristics of such data set may have clues to solve this problem. The in-situ HMA density, air voids (AV) and asphalt content (AC) data can be mathematically modeled as shown in Eqs. 21 and 22.

\[ (d / AV / AC)_{\text{cont}} = \mu + \sigma_{\text{prod}} + \sigma_{\text{cont,meas}} \]  

(21)

\[ (d / AV / AC)_{\text{agency}} = \mu + \sigma_{\text{prod}} + \sigma_{\text{agency,meas}} \]  

(22)

Where

\[ (d / AV / AC) \] represents Normal distribution of AV and AC content data

\[ \sigma_{\text{prod}} \] represents production variability

Therefore, the difference of paired measurements can be modeled as shown in Eq. 23. It is interesting to note that the difference in paired measurements is because of measurement variability only. All other terms in the model get cancelled because the paired measurements are performed on split samples which are almost identical in quality.

\[ (d / AV / AC)_{\text{cont}} - (d / AV / AC)_{\text{agency}} = \sigma_{\text{cont,meas}} - \sigma_{\text{agency,meas}} \]  

(23)

Measurement variability is due to error in measurement arising from the influence of equipment, personnel, lab, environment etc. on the measurement. Error can be either in the form of bias or random error. Bias refers to consistently measuring either lower or higher than the actual value. When comparing measurements, it is easy to determine if at least one of the parties have bias. Bias has been dealt with in great detail by Aurilio et al. [20]. For the sake of simplicity, it is assumed that bias is not present in the proposed model. Then the measurement variability is because of random error alone. The nature of random instrument or human error is that it is centered around a mean and is generally normally distributed. It is also a special characteristic of normal populations that the difference between two normally distributed populations is itself another normal population with a mean (equal to zero) and standard deviation as shown in Eq. 24. This property can be used to characterize the paired difference values here. Eq. 25, therefore, gives another equation but without introducing any unknowns.

\[ \sigma_{\text{cont,meas}} - \sigma_{\text{agency,meas}} = N(\mu, \sigma_{\text{diff}}) \]  

(24)

\[ \mu = 0 \]

\[ \sigma_{\text{diff}} = \sqrt{\sigma_{\text{cont,meas}}^2 - \sigma_{\text{agency,meas}}^2} \]  

(25)

The set of available equations now are as shown below:

\[ \frac{1}{5} \sigma_{\text{trans}}^2 + 1 \sigma_{\text{long}}^2 + 1 \frac{4}{5} \sigma_{\text{cont,meas}}^2 = \sigma_{\text{cont,avg}}^2 \]  

(26)

\[ \frac{4}{5} \sigma_{\text{trans}}^2 + 0 \sigma_{\text{long}}^2 + 0 \frac{4}{5} \sigma_{\text{cont,meas}}^2 = \sigma_{\text{cont,shifted}}^2 \]  

(27)

\[ \frac{1}{5} \sigma_{\text{trans}}^2 + 1 \sigma_{\text{long}}^2 + 1 \frac{4}{5} \sigma_{\text{agency,meas}}^2 = \sigma_{\text{agency,avg}}^2 \]  

(28)

\[ \sigma_{\text{cont,meas}}^2 + \sigma_{\text{agency,meas}}^2 = \sigma_{\text{diff}}^2 \]  

(29)

These set of four equations can be put in the matrix form as shown below. The coefficient matrix is no more singular.

\[
\begin{bmatrix}
0.2 & 1 & 0.2 & 0 \\
0.8 & 0 & 0.8 & 0 \\
0.2 & 1 & 0 & 0.2 \\
0 & 0 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
\sigma_{\text{trans}}^2 \\
\sigma_{\text{long}}^2 \\
\sigma_{\text{cont,meas}}^2 \\
\sigma_{\text{agency,meas}}^2 \\
\sigma_{\text{diff}}^2
\end{bmatrix}
= \begin{bmatrix}
\sigma_{\text{trans}}^2 \\
\sigma_{\text{long}}^2 \\
\sigma_{\text{cont,meas}}^2 \\
\sigma_{\text{agency,meas}}^2 \\
\sigma_{\text{diff}}^2
\end{bmatrix}
\]  

(30)

Therefore, a unique solution can be derived for each dataset. Hence, the following components contributing to total variance can be estimated:

1. Transverse variability
2. Longitudinal variability
3. Measurement variability in contractor data
4. Measurement variability in agency data

Using the proposed method, all the four variability components were determined for datasets from ERS projects constructed in the year 2000. The results have been presented in Table 2.

<table>
<thead>
<tr>
<th>Project</th>
<th>( \sigma_{\text{trans}} )</th>
<th>( \sigma_{\text{long}} )</th>
<th>( \sigma_{\text{cont,meas}} )</th>
<th>( \sigma_{\text{agency,meas}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>D2 Binder-IL 38 E Dixon</td>
<td>0.68</td>
<td>0.79</td>
<td>0.33</td>
<td>0.64</td>
</tr>
<tr>
<td>D2 Surface-IL 38 E Dixon</td>
<td>0.98</td>
<td>0.91</td>
<td>0.43</td>
<td>0.76</td>
</tr>
<tr>
<td>D3 Binder- I 55 Shirley</td>
<td>0.58</td>
<td>0.79</td>
<td>0.51</td>
<td>0.42</td>
</tr>
<tr>
<td>D3 Surface- I 55 Shirley</td>
<td>0.88</td>
<td>1.15</td>
<td>0.36</td>
<td>0.38</td>
</tr>
<tr>
<td>D6 - IL 96   Shirley</td>
<td>1.13</td>
<td>0.55</td>
<td>0.15</td>
<td>0.55</td>
</tr>
<tr>
<td>D8 Surface - IL 140</td>
<td>1.05</td>
<td>1.10</td>
<td>0.52</td>
<td>0.36</td>
</tr>
</tbody>
</table>

TABLE II

ESTIMATED VARIABILITY COMPONENTS FOR ERS PROJECTS (YEAR 2000)
III. VARIABILITIES IN AIR VOIDS AND ASPHALT CONTENT MEASUREMENTS

Asphalt content and air voids data do not have the same sampling scheme as in-situ density data and therefore, the method for estimation of variability components also would have to be different. Production variability in this case would not have longitudinal and transverse components. The data can still be modeled as shown in Eq. 31.

\[
(AV/AC) = \mu + \sigma_{\text{prod}} + \sigma_{\text{meas}}.
\]  (31)

Where,
\( (AV/AC) \) represents air voids, or asphalt content data
\( \mu \) represents mean
\( \sigma_{\text{prod}} \) represents the production variability and
\( \sigma_{\text{meas}} \) represents the measurement variability

It should be noted that the contractor’s, as well as the agency’s data, and third party data, would be expected to follow this model. Third party, as referred to here, is an independent agency employed by the agency for resolving disputes in test measurement results. Since all the parties test the same material using split samples, the mean and production variability is same for all of them. The difference observed in the test data between the contractor and the agency, for example, can be attributed to the measurement variability. Therefore, the model when applied to the contractor data would be:

\[
(AV/AC)_{\text{cont}} = \mu + \sigma_{\text{prod}} + \sigma_{\text{cont, meas}}
\]  (32)

And when applied to the agency, it would be:

\[
(AV/AC)_{\text{agency}} = \mu + \sigma_{\text{prod}} + \sigma_{\text{agency, meas}}
\]  (33)

Since each measurement is performed on the split samples of the same material, the two values modeled above would form paired data. Subtracting the second from the first would eliminate the mean and production variability terms.

\[
(AV/AC)_{\text{cont}} - (AV/AC)_{\text{agency}} = \sigma_{\text{cont, meas}} - \sigma_{\text{agency, meas}}
\]  (34)

Both the terms on the right side of the equation come from a normal population. Therefore, their difference also would be normally distributed. Therefore,

\[
\sigma_{\text{cont, meas}} - \sigma_{\text{agency, meas}} = N(\mu, \sigma_{\text{diff}})
\]  (35)

Where \( N(\mu, \sigma_{\text{diff}}) \) represents a normally distributed population with \( \mu \) as mean and \( \sigma_{\text{diff}} \) as combined standard deviation where:

\[
\mu = 0
\]

\[
\sigma_{\text{diff}} = \sqrt{\sigma_{\text{cont, meas}}^2 + \sigma_{\text{agency, meas}}^2}
\]  (36)

Further it can be assumed that the measurement variability for one type of test, like core density or asphalt content, would be fairly similar. Then, field data could be pooled in order to obtain a typical value for measurement variability, thus assuming:

\[
\sigma_{\text{cont, meas}} = \sigma_{\text{agency, meas}} = \sigma_{\text{meas}}
\]  (37)

Therefore:

\[
\sigma_{\text{meas}} = \frac{\sigma_{\text{diff}}}{\sqrt{2}}
\]  (38)

IV. CONCLUSIONS

To promote the construction of better-quality asphalt pavements, over the years, a number of state highway agencies have moved from using traditional method specifications to using statistically based QC/QA specifications. Reduced construction and material variability is one measure of improved quality of asphalt pavement construction. Over the years, researchers and highway agencies in Illinois have attempted to develop a prototype End-Result Specification (ERS) system for asphalt pavement construction. Stochastic simulation models are very useful in estimating and analyzing payment risk in ERS systems. One of the key requirements for the simulation is generation of synthetic QC/QA data which would be representative of actual ERS projects.

A new method has been developed to estimate the variability components in in-situ density, air voids and asphalt content data from ERS projects. The information gained from this would be crucial in simulating these ERS projects for estimation and analysis of payment risks associated with pavement construction. However, these methods require at least two parties to conduct tests on all the split samples obtained according to the sampling scheme prescribed in present end-result specifications implemented in Illinois.

ACKNOWLEDGMENT

This work is based on the results of ICHRP Project R23, Evaluation of Potential Applications of End-Result and Performance-Related Specifications. ICHRP Project R23 is sponsored by Illinois Department of Transportation (IDOT). The authors would like to acknowledge Laura Shanley at IDOT’s Bureau of Materials and Physical Research for...
providing the data and assistance in conducting this study. The contents of this paper reflect the views of the authors who are responsible for the facts and accuracy of the data presented within. The contents do not necessarily reflect the official views and policies of the IDOT. This paper does not constitute a standard, specification, or regulation.

REFERENCES


