

Simulating and Forecasting Qualitative Macroeconomic Models using Rule-Based Fuzzy Cognitive Maps

Spiros Mazarakis, George Matzavinos, and Peter P. Groumpos

Abstract—Economic models are complex dynamic systems with a lot of uncertainties and fuzzy data. Conventional modeling approaches using well known methods and techniques cannot provide realistic and satisfactory answers to today's challenging economic problems. Qualitative modeling using fuzzy logic and intelligent system theories can be used to model macroeconomic models. Fuzzy Cognitive maps (FCM) is a new method been used to model the dynamic behavior of complex systems. For the first time FCMs and the Mamdani Model of Intelligent control is used to model macroeconomic models. This new model is referred as the Mamdani Rule-Based Fuzzy Cognitive Map (MBFCM) and provides the academic and research community with a new promising integrated advanced computational model. A new economic model is developed for a qualitative approach to Macroeconomic modeling. Fuzzy Controllers for such models are designed. Simulation results for an economic scenario are provided and extensively discussed.

Keywords—Macroeconomic Models, Mamdani Rule Based-FCMs(MBFCMs), Qualitative and Dynamics System, Simulation.

I. INTRODUCTION

A MACROECONOMIC model is an analytical tool designed to describe the dynamic behavior of the economy of a region or a country. It is a means of collating research on the economy in a systematic and policy-relevant way, and depends on the availability of such research. The goal of a macroeconomic model is to replicate the main mechanisms of an entire economic system, which may consist of a region, of a country or a union of countries. The only requirement is that the entity being modeled is large enough to display the distinctive properties that are the thematic area of macroeconomics.

Today more than ever before given the world economic crisis and the peripheral economic inequalities, there is the need to search and develop new models and methodologies in order to study economy. It is needed to find new solutions to our everyday economic problems. Today most modeling approaches cannot always provide realistic and acceptable solutions. The economic environment in the whole world is complex dynamic and uncertain. Macroeconomic systems are complex and fuzzy [1]. Decision makers, especially economists, usually face serious difficulties when approaching

significant, real-world dynamic systems. Fuzzy Cognitive Maps (FCMs), as introduced by Kosko [3], were developed as a qualitative alternative approach to model dynamics of complex systems. FCMs are Causal Maps (a subset of Cognitive Maps that only allows basic symmetric and monotonic causal relations). In most applications, a FCM is a man-trained Neural Network that is not Fuzzy in a traditional sense and does not explore usual Fuzzy capabilities. They do not share the properties of other fuzzy systems and the causal maps end up being quantitative matrices without any qualitative knowledge. Even then the FCMs have problems to provide the necessary environment for expressing the knowledge of a system. To anticipate solutions to these problems new models and methods are needed and improved FCM models are required.

In this paper a new computational method that is called Mamdani Based Fuzzy Cognitive Maps(MBFCMs) is developed. The new method is used for modeling and the simulation of a macroeconomic model. Mamdani controllers are designed. The method is applied on a process simulation and general on the control problem in the field of economy, with promising results.

The paper is organized as follows: The overview of Fuzzy Cognitive Maps and Rule-Based is represent in Section II, the new method Mamdani Based-FCM is presented in Section III. In Section IV the Construction of a qualitative Macroeconomic Model is presented. In Section V the simulation of an economic scenario is described and the simulation results are reported and extensively discussed. The Section VI consists of Conclusion and Future Research.

II. OVERVIEW OF FUZZY COGNITIVE MAPS AND MAMDANI RULE-BASED

Overview of Fuzzy Cognitive Maps

FCMs have been introduced by Kosko in 1986 [3] assigned directed graphs for representing causal reasoning and computational inference processing, exploiting a symbolic representation for the description and modeling of a system. Concepts are utilized to represent different aspects of the system, as well as, their behavior. The dynamics of the system are implied by the interaction of concepts. FCM structures are used to represent both qualitative and quantitative data. The construction of an FCM requires the input of human experience and knowledge on the system under consideration. Thus, FCMs integrate the accumulated experience and

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knowledge concerning the underlying causal relationships amongst factors, characteristics, and components that constitute the system. An FCM consists of nodes—concepts, C_i , $i=1, \dots, N$, where N is the total number of concepts. The value A_i of a concept C_i , expresses the quantity of its corresponding physical value and is derived by the transformation of the fuzzy values assigned by the experts, to numerical values. Having assigned values to the concepts and the weights, the FCM converges to a steady state, through the interaction process subsequently described.

At each step, the value A_i of a concept is influenced by the values of concepts—nodes connected to it, and is updated according to the scheme [6]:

$$A_i(k+1) = f \left(A_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^n W_{ji} A_j(k) \right), \quad (1)$$

where k stands for the iteration counter and W_{ji} is the weight of the arc connecting concept C_j to concept C_i . The function f is the sigmoid function:

$$f(x) = \frac{1}{1 + e^{-\lambda x}} \quad (2)$$

where $\lambda > 0$ is a parameter that determines its steepness in the area around zero. A new method for developing FCMs is developed [6].

Mamdani Rule Based

The most commonly used fuzzy inference technique is the so-called Mamdani method. In 1975 Professor Ebrahim Mamdani of 1975, London University built one of the first fuzzy systems to control a steam engine and boiler combination. He applied a set of fuzzy rules supplied by experienced human operators. The operation of the Mamdani rule base can be broken down into four parts: 1) mapping each of the crisp inputs into a fuzzy variable (fuzzification). 2) determining the output of each rule given its fuzzy antecedents. 3) determining the aggregate output(s) of all of the fuzzy rules. 4) mapping the fuzzy outputs to crisp outputs (defuzzification).

Fuzzification

Since the Mamdani rule base models a crisp system, it has crisp inputs and outputs. The rules, however, are given in terms of fuzzy variables. The membership of each fuzzy input variable is evaluated for the given crisp input, and the resulting value is used in evaluating the rules.

Knowledge Basic-Inference

The control protocol is stored in the form of if-then rules in a rule base which is a part of the knowledge base. While the rules are based on qualitative knowledge the membership function defining the linguistic terms provide a smooth interface to the numerical process variables and the set-points. The inference mechanism combines this information with

knowledge stored in the rules and determines what the output of the rules-based system should be. In general, this output is again a fuzzy set.

Defuzzification

After the previous step, we have a fuzzy output defined for the rule base. We need to convert this output into a crisp output. To do this, the centroid of the fuzzy output is used.

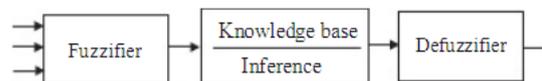


Fig. 1 The map is formed by knowledge base, inference mechanism and fuzzification and defuzzification

III. MAMDANI BASED FUZZY COGNITIVE MAPS (MBFCM)

Mamdani based Fuzzy Cognitive Maps (MBFCM), is a flexible and as simple as possible computational method. It is able to include complex systems and situations where the human reason, for any process, includes uncertain and fuzzy descriptions of a system. MBFCMs originated as a combination of ideas and methods from the theories of Fuzzy Cognitive Maps and mainly from Mamdani's Model. The MBFCMs is a simple way to describe the structure of a system and its behavior. MBFCMs incorporates the accumulated knowledge, experience and intelligence for the operation of the system using a set of fuzzy rules (IF-THEN). The MBFCMs is a modeling method that reflects the specific characteristics of a system with the nodes (variables) and the relations between them with fuzzy inference Mamdani.

A. General Description of Mamdani Based Fuzzy Cognitive Maps (MBFCMs)

The representation of Mamdani Based Fuzzy Cognitive Maps (MBFCMs) is possible with a network consisting of nodes that are interconnected. Each group of connections has a fuzzy Mamdani controller, which contains a set of rules (IF node A... THEN Node B...). The nodes of MBFCMs represent the concepts, which are used to describe the system's behavior and are related to Mamdani controllers that represent the causal connections that exist between, using nodes-concepts (Fig. 3).

A MBFCMs method consists of a number of simple-formed systems Mamdani (Fig. 2). Thus we are able to combine subsystems so as to define a more complex system (Fig. 3).

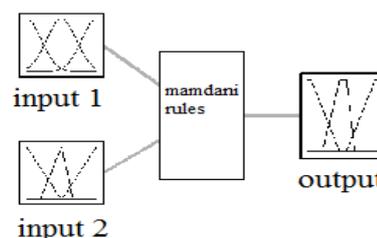


Fig. 2 A Simple form of a Mamdani controller with two inputs and an output

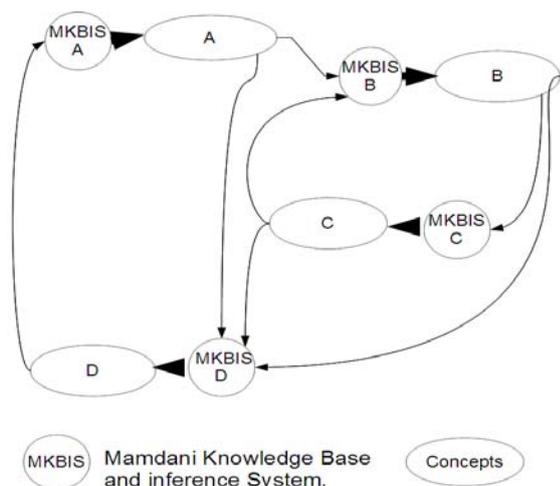


Fig. 3 A simple Form of MBFCMs

Each node-concept includes a MKBIS (Mamdani Knowledge Base and Inference System) used to reach through the knowledge base that contains the non-linear relationship between nodes, which consist of one or more inputs and always an output. More specifically, node A of Fig. 3 has a MKBIS where with one input received from the node D decides the value of the output. Node B has MKBIS with two inputs received from the A and C and decides for the output, the value of B, etc. The MBFCMs method allows the representation of the dynamics of complex qualitative systems with feedback, the simulation of events and their influence on the system. It is represented as a system of fuzzy directed graphs with feedback, having been designed using fuzzy concepts nodes and fuzzy Mamdani links. The nodes-concepts are fuzzy variables which are described by linguistic terms. Their relations are defined with the implication of Mamdani, which contains fuzzy rules (IF...THEN.). Each node-concept is defined by membership functions (mfs). The membership functions represent possible values of the node-concept. Any kind of relationship that can be represented by fuzzy rules allows "opposition", "similarity", "impact". The inputs can be combined using fuzzy operations (AND, OR, NAND and others.) A MBFCMs method allows the question (IF...THEN.) in Cognitive Networks. Moreover, this new method enables us to manage non-linear, non-symmetrical systems and the qualitative variables and the qualitative relations between them.

B. Mathematical Model

Each node of Fig. 3 represents a concept of the system and the value of each variable of the node is dependent on the interconnected nodes of the MBFCMs. The mathematical equation is represented for the calculation of the values of a node at each repetition is shown below (Link 3).

$$A_i^t = MKBIS \left\{ A \right\} \left[input_i^{t-1} - input_n^{t-1} \right] \quad (3)$$

Thus the value of the variable of the node i at time t will be $A_i(t)$ depends on the values of inputs, (input1 ...input n .) of the function MKBIS (input 1... input... n) of the interconnected nodes at time $t - 1$. The Mamdani Knowledge Base and Inference System MKBIS is a process of the fuzzy inference Mamdani Model which contains the fuzzyfication of values of inputs, (rule evaluation), the aggregation of the conclusions of the outputs and finally their defuzzification.

C. Variations-Levels

In the Macroeconomic model there is a need to further divide the Concept Variable into two classes, Variation and Level. Variation represents a change in a value (node) system for a certain period of time (e.g. variation of inflation increased very dramatically). The second class of nodes that is constructed is the Level, which represents the absolute values of the corresponding nodes (e.g. Inflation is Good). Fig. 4 shows a simple example of the relationship between these two main classes [8].

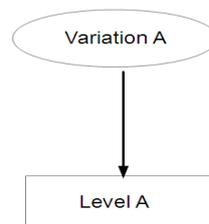


Fig. 4 Relationship of the two main classes

Level A is accepted as an input of the current value of the Variations A and together with the previous value Level A($t-1$), divided by t , calculates the present value of A. That is, the absolute value of a concept, is calculated by its initial value and the variation at time t . The relationship is defined as follows:

$$LevelA(t) = LevelA(t-1) + VariationA(t) * \Delta t \quad (4)$$

D. The Time in Mamdani Based Fuzzy Cognitive Maps

Time is very important when we have to handle quantitative variables and is expressed by mathematical functions. However, when our system contains qualitative variables, time must be expressed and taken under consideration when is built the rules base. Without this process it is impossible to guarantee a useful correct simulation.

It is important to choose a time as a basis which will represent each repetition of the system. When knowledge base (rules) is defined, the time base must be taken into account. The rules that represent the causal relationship are tightly connected to by the time base. If the time base is a short space, then the rules that reflect the influence of a particular value indicate a small variation e.g. months. If it is one year the rules indicate a bigger variation e.g. one year. The time base depends on the system that we have to study. However,

shorter time bases require a more detailed definition which has as a result complex rules bases. In the MBFCMs we can choose different time spaces per node. This is because some nodes-concepts are defined by the system at different space. In particular, in the macroeconomic system, the economic indicators are published every 3 months, 6 months or 1 year. This difference is represented by the causal relation that describes them and by the spaces of the membership functions in the MBFCMs.

IV. CONSTRUCTION OF A QUALITATIVE MACROECONOMIC MODEL

Economists face serious difficulties when trying, to model a macroeconomic system due to its high complexity and fuzziness. However with a simplified model we can have useful conclusions. The model used in this paper takes into account the most important economic notions and could lead us into useful conclusions concerning economical problems and targets. The initial hypothesis is that the financial system is characterized by instability and imbalances. The concepts and the relations were constructed after an economical analysis. The Macroeconomic model (Fig. 5) that was constructed uses only the most important economic factors concepts (inflation, consumption, wealth, growth, unemployment). Even though, there are such few nodes, a realistic model is difficult to be represented due to the complexity of the relations between the concepts-nodes. The Fig. 5 represents the connections of concepts of a simplified Macroeconomic model.

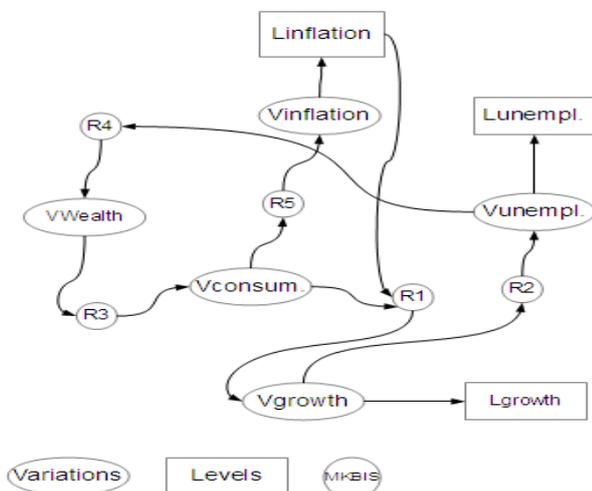


Fig. 5 A Simplified Macroeconomic model

A. Design of Fuzzy Controllers

After the construction of the Macroeconomic model the next step is the design of the Mamdani controller using the Fuzzy editor. The fuzzy files R1 R2 R3 R4 R5 are created for the presentation of the economic model with the MBFCMs. Firstly, we used the triangle membership function(trimf), for both input and output, the operators AND(min), OR(max), the implication min (mamdani), the aggregation max and the defuzzifier centroid. The Fig. 6 represents the R1 fuzzy

controller.

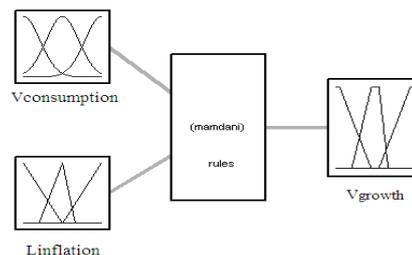


Fig. 6 R1 Mamdani controller

if Linflation is deflation then Vgrowth is Large decrease
 if Vconsumption is large decrease and Linflation is very low then Vgrowth is large decrease
 if Vconsumption is large decrease and Linflation is low then Vgrowth is large decrease
 if Vconsumption is maintain and Linflation is low then Vgrowth is maintain
 if Vconsumption is maintain and Linflation is very low then Vgrowth is small decrease
 if Vconsumption is maintain and Linflation is normal then Vgrowth is small increase
 if Vconsumption is large increase and Linflation is very low then Vgrowth is increase
 if Vconsumption is large increase and Linflation is low then Vgrowth is large increase
 if Vconsumption is large increase and Linflation is normal then Vgrowth is increase
 if Vconsumption is large increase and Linflation is zero then Vgrowth is increase

Fig. 7 Example of some rules-level of inflation -variation of growth

B. References

At first seven memberships functions are used for the node of level of inflation (deflation, zero, very low, low, normal, high, very high). Seven memberships functions are used for the nodes of level of unemployment and level of growth (extremely low, very low, low, zero, high, very high, extremely high). Seven memberships functions for the nodes of variation of consumption, variation of growth, variation of wealth, variation of consumption, variation of unemployment (large decrease, small decrease, decrease, maintain, small increase, increase, large increase).

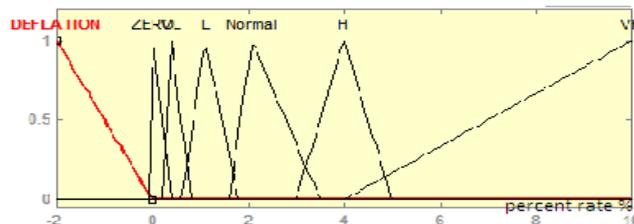


Fig. 8 The triangle Membership function, Level "inflation" linguistic terms {deflation, zero, very low positive, low positive, normal, high, very high}.

V. SIMULATION OF AN ECONOMIC SCENARIO -SIMULATION RESULTS

The first economic scenario that was considered is an economic system that starts from a realistic welfare state with given initial condition [7]. In particular, we consider a variation of growth (Vgrowth) of about 1%, and a variation of Vunemployment) of about -1%. Also, an increase of variation of consumption of a rate of 10% (V consumption) and an increase of inflation (Vinflation) at about 1%. Finally, was considered a value of growth (Lgrowth) of 3%, a value of unemployment (Lunemployment) of about 4% and a value of inflation (Linflation) at a rate of 2%. The system evolves in time as shown in Fig. 8 to 14. The Figs. 8, 9, 10 represent the Levels of nodes and the Figs. 11,12,13,14 represent the Variations of nodes. It is observed that a small positive Variation of growth leads to a decrease of Level of unemployment and also, a decrease of Variation of consumption leads to a decrease of Variation of inflation and a decrease of Level of inflation as it would be expected. Furthermore, it is observed that at the early stage until 15 years, the system has a huge growth (at about 10%) and a reduction of unemployment (at about -10%) as expected. This is because the inflation index remains in an appropriate range, from 1% to 5%, which also reflects the economic importance of inflation. A low and steady rate of inflation is desirable for economist while the Level of inflation is at this range the risk is low and consequently, people consume and invest more causing, economic growth. After this period (around 15 years) a decrease of Level of inflation out of the desirable range and also proportional decrease of Level of growth is observed as well as an increase of the Level of unemployment rate until the system collapses. This result can be explained by the fundamental law of economics as follows. Every factor-concept that affects the price of inflation must continuously demonstrate an increasing tense-tend in order to maintain its price. For example, if variation of consumption corresponds to a "small increase" then variation of inflation is "maintain". As a result of this fundamental law the system cannot preserve the above condition continuously. The macroeconomic policies created the credit economy in order to bring the system back to stability. Initially, this solution stabilized the system because of the fulfillment of the above fundamental law and the fact that when Variation of consumption corresponds to small increase then Variation of inflation is "maintain". But it is a given that this condition cannot be fundamental law cannot be on completion forever. As a result, the system is led to instability after a period of time. The instability-crisis will be deeper and deeper. This scenario resembles with the one of the Greek economic crisis where suddenly the input money will not feedback the system anymore. Finally this Macroeconomic system using the new method Rule-Based Mamdani Fuzzy Cognitive Maps seems to be realistic and perhaps it is able to simulate a chaotic simplified Macroeconomic system.

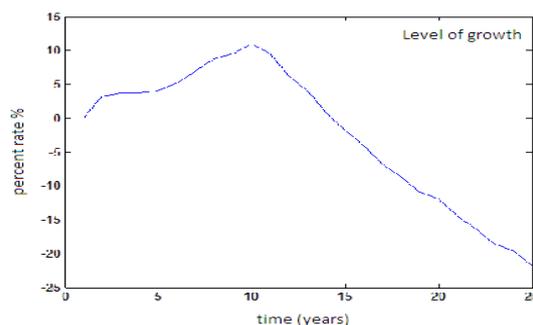


Fig. 9 Level of growth

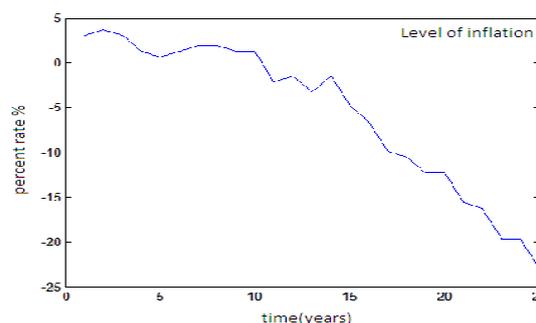


Fig. 10 Level of Inflation

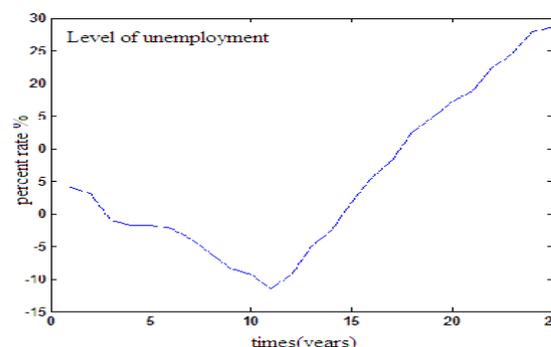


Fig. 11 Level of Unemployment

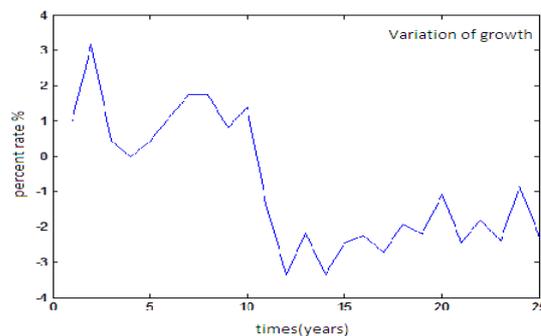


Fig. 12 Variation of growth

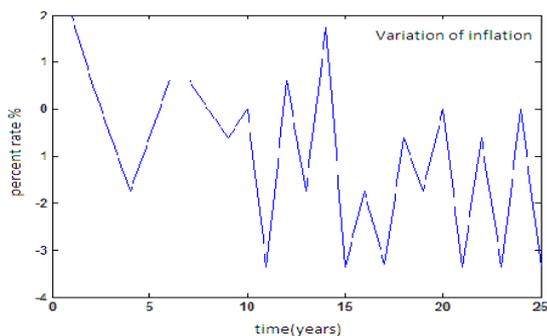


Fig. 13 Variation of Inflation

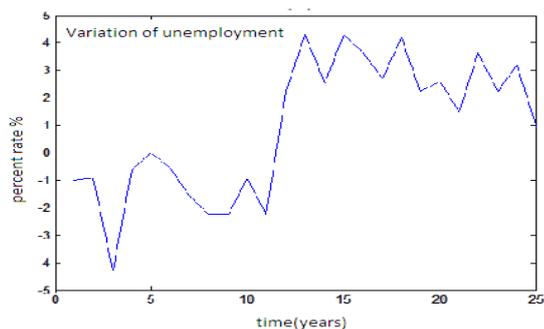


Fig. 14 Variation of Unemployment

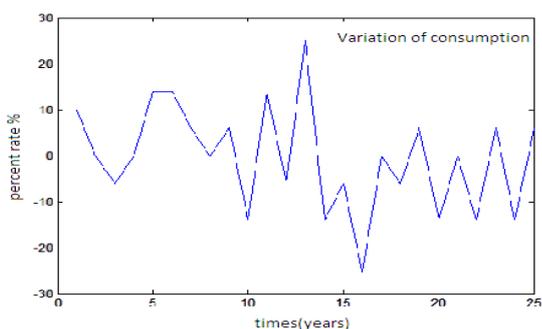


Fig. 15 Variation of Consumption

VI. CONCLUSIONS, FUTURE DEVELOPMENT AND APPLICATIONS

In this paper a new method-MBFCMs for modeling a macroeconomic system was developed. The development of mathematical techniques such as stochastic and non-stochastic processes, are two widespread methods for the modeling of economic systems. These methods model the economic factors by statistical observation in time. The method that was developed in this paper gave us the opportunity to observe and study the behavior of a macroeconomic model through basic economical principles. For the first time Mamdani controllers are combined with the Fuzzy Cognitive Maps, simplifying the procedure of modeling the system. During our effort to study the system's behavior we faced two basic problems:

The first problem was that FCMs do not share the properties of other fuzzy systems and the casual maps end up being quantitative matrices without any qualitative knowledge. The second problem was that the existing techniques had

difficulties in providing the necessary environment for free expression of knowledge of the system to an expert user. The new method MBFCMs was developed in order to solve the above-mentioned problems. The MBFCMs can manage non-monotonic and non-linear casual relations between the system's concepts, without the need of a quantitative weight matrix, in a similar way as observed in the relations of real systems. The MBFCMs can be chosen to accurately model a real system in which the relation between the inputs and outputs are known. The MBFCMs have the ability to construct any system which imitates the human brain and can include the uncertainties and fuzziness. It seems that it can deal with the non-linearity, the dynamics, the causality, the totally qualitative values and the totally qualitative relations of the system in a good way. Even though the MBFCMs are at an early stage, they have the potential of being further developed so that they can take advantage of the expert's experience without the use of programming methods. The MBFCMs present some difficulties in representing large macroeconomic models with accuracy, due to the fact that many-concepts imply many rules. On the other hand they can simulate easily and with accuracy a micro economic system with few rules. Also this new method can simulate a game theory-system and an industrial case study, which consist challenging future topics.

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