Abstract—The purpose of this paper is to present the design and instrumentation of a new benchmark multivariable nonlinear control laboratory. The mathematical model of this system may be used to test the applicability and performance of various nonlinear control procedures. The system is a two degree-of-freedom robotic arm with soft and hard (discontinuous) nonlinear terms. Two novel mechanisms are designed to allow the implementation of adjustable Coulomb friction and backlash.

Keywords—Nonlinear control, describing functions, Adjustable Coulomb friction, Adjustable backlash.

I. INTRODUCTION

THERE are a number of methods that allow design of controllers for use with multivariable nonlinear systems [1-5]. It is desirable to experimentally verify and compare these methods with one another. For this reason, nonlinear control benchmark problems have been set up [6]. The primary disadvantage of the available benchmark problems to date is that they generally of an academic nature; they do not represent practical and/or industrial settings. Methods that account for real-life nonlinear terms such as saturation, Coulomb friction, and backlash are limited and very new. These methods must be experimentally verified. The primary contribution of this research is the design and instrumentation of a new bench mark laboratory for testing controller design methods that are applicable to highly nonlinear plants with or without discontinuous nonlinear terms.

Advancements of nonlinear control design methods, which account for discontinuous nonlinear terms, reduce the manufacturing costs of goods. This is because the plants may be manufactured without imposing tight tolerances. In a linear control frame work, tight manufacturing tolerances are imposed to make sure that plant is linear; hence, the manufacturing costs would be increased.

In 1983, a new describing function based platform was introduced that would allow design of single-range, dual-range, and multi-range controllers for plants that include discontinuous nonlinear terms such as saturation, Coulomb friction and backlash [7]. That platform has been developing over the past twenty years (e.g., [8-13] and references therein). However, these methods need to be experimentally verified, and the results must be compared with other controller design methods.

In [14], a novel distributed architecture for building Web-enabled remote robotic laboratories is presented. The presented solution is concentrated on three aspects: easy and efficient network communication in client-server applications, internet access of networked sensors, and portable architecture based on Java 2 platform. In [15], a design approach based on approximation of input-output linearization of non-linear ball and the beam system is presented. In [16], a design approach based on considering a reduced order model of a nonlinear plant obtained by application of a time-scale separation principle is presented. The procedure is applied to a helicopter problem. In [17], current trends in building of a control laboratory that would support control engineering and control theory courses are presented. In that work, emphasis is on the laboratory equipment, the scale models, and the software environments used for supporting the theoretical content of lectures; experiments are run in real-time. In [18], a laboratory for undergraduate automatic control courses at the Politecnico di Milano is presented. The laboratory serves the basic controls course with the aim of providing experimental counterparts for the abstract concepts introduced; also, the aim is for students to gain experience in a real control problem as early as possible. It also serves advanced courses where more advanced concepts are taught. The apparatus is a simple temperature control, and the control activities for students include experiment-based modeling, single loop control, PID tuning, multivariable linear control, decentralized control, feed forward compensation, cascade control and decoupling control. In [19], the development of a remote-access control system which allows users to perform control experiments through internet is presented. A dc motor module is used as an illustration of design, and the system is composed of an internal distributed system. An application system is linked by a data acquisition interface card. In [20], the experience of its author in developing and teaching a senior level “automatic control laboratory” course is presented; the corresponding experiments are concentrated on plant identification, determination of open-loop and closed-loop characteristics, understanding of the effect of damping in design, and implementation of PID control schemes. In [21], a simple and inexpensive laboratory to make students better engineers and practitioners is presented. The laboratory is to convince
students that theory indeed makes sense; it also serves as a tool to make the controls course a more interesting subject. In [22], the development of an interdisciplinary intelligent control laboratory is presented. Simple, robust and effective implementation of soft computing techniques for several educational and industrial applications including servo systems, heat trainers, inverse pendulum, twin rotor systems and visual component inspection stations are addressed in the project. The authors’ internet search on typical existing control laboratories follow.

i) Ball beam system: The beam is made to rotate in a vertical plane by applying a torque at the center of rotation and the ball is free to roll (with one DOF) along the beam. The relative degree of the system is not well defined.

ii) Laboratory helicopter: The laboratory helicopter is a twin rotor system. Since the rotor blades present fixed angle of attack, control is achieved by using the rotor speeds as control variables. This mechanical device features strongly coupled nonlinear dynamics.

iii) Robot joint mechanism: Backlash is introduced in the connection of the harmonic drive to the shaft (second link). The first link is kept fix while the second one is made to oscillate over a certain range; this link is driven by a servomotor through a teeth belt and a harmonic drive. The vibration responses are measured with two rotary encoders. The first encoder measures the angular motion input to the harmonic drive, and the second one measures the relative oscillation between the first link and second link.

iv) Flexible joint: The apparatus uses a dc motor to drive a rigid beam with springs to emulate flexible joint effects. Beam angle is measured with an encoder. The control problem is positioning of the tip position to overcome oscillations due to flexible joint. Springs can be attached in different ways, and different springs can be used to represent different flexibility effects. Also, a weight can be added to the beam.

v) Flexible link: The apparatus uses a dc motor to move a flexible link. The tip deflection is measured with a strain gauge at the motor end of the link. The problem is to control the tip position and to damp vibrations. Weights can be added to link to change vibration modes.

vi) Inverted pendulum: The apparatus uses a dc motor to drive an arm with a rotating bar at the end. An encoder is used to measure deflection of the bar. The problem is to control the balancing of rotating bar in the inverted position. There are two modes of operations requiring different types of controllers. Weights can be added to the link.

vii) Tanks: The pumps drive water through orifices of different diameters. Flow from first tank goes to a second tank, and flow from second goes to a basin. The outflows can be manipulated. The problem is to regulate the water level in tanks. This control problem includes delays and a nonminimum phase behavior of the single-input single-output plant.

What makes the presented research laboratory different from the above mentioned laboratories and literature is the presence of adjustable hard (discontinuous) nonlinear terms such as Columb friction and backlash in a typical multivariable plant of the sort encountered in robotics.

II. THE LABORATORY

The newly designed laboratory consists of a two degree-of-freedom robotic arm with a newly designed gripper as shown in Fig. 1. The two links of the robot arm are ac motor driven, and the gripper is stepper motor driven. The ac motors at the required power rating are more economical, and this the main reason for not selecting dc motors. The arm and gripper motors are driven by appropriate drivers, and these drivers, motor encoders, and gripper force sensor communicate with appropriate electronic devices provided by National Instruments. The main components of the laboratory are given in Table I.

![Fig. 1 The designed robot arm and gripper](image)

III. MATHEMATICAL MODEL

The dynamic equations of motions of both arms may be derived using Lagrangian mechanics [23]; the results follow.

\[
T_1 = \left\{ (M_1 + M_2) L_1^2 + 2 M_2 L_1 L_2 \cos \theta_2 \right\} \dot{\theta}_1 \\
+ \left\{ M_2 L_2^2 + M_2 L_1 L_2 \cos \theta_2 \right\} \ddot{\theta}_2 \\
- M_2 L_1 L_2 \sin \theta_2 \dot{\theta}_1 \dot{\theta}_2 + T_{c1}
\]

\[
T_2 = \left\{ M_2 L_2^2 + M_1 L_1 L_2 \cos \theta_2 \right\} \dot{\theta}_1 + M_2 L_2^2 \dot{\theta}_2 \\
+ M_2 L_1 L_2 \sin \theta_2 \dot{\theta}_1^2 + T_{c2}
\]

where

- \( T_1 \) = torque required to drive the first link
- \( T_2 \) = torque required to drive the second link
- \( M_1 \) = the mass of first link and servo drive
- \( M_2 \) = the mass of second link and motorized gripper
- \( L_1 \) = the length of first link
- \( L_2 \) = the length of second link
\( \theta_1, \dot{\theta}_1, \ddot{\theta}_1 \) = angular position, velocity and acceleration of first link
\( \theta_2, \dot{\theta}_2, \ddot{\theta}_2 \) = Angular position, velocity and acceleration of second link
\( T_{c1} \) = coulomb friction term at first link
\( T_{c2} \) = coulomb friction term at second link

The relationship between the drive torque and the motor torque is given below:

where, 
\( \eta \) = gear efficiency
\( a \) = number of contacting gears in gear train linking first servo drive to first link
\( b \) = number of contacting gears in gear train linking second servo drive to second link
\( N_1 \) = the speed reduction ratio between first servo drive and first link
\( N_2 \) = the speed reduction ratio between second servo drive and second link
\( J_{M1} \) = rotor inertia of first servo drive
\( J_{M2} \) = rotor inertia of second servo drive
\( J_{G1} \) = moment of inertia of gear driving the first link
\( J_{G2} \) = moment of inertia of gear driving the second link
\( \dot{\theta}_{M1} \) = Angular acceleration of first servo drive
\( \dot{\theta}_{M2} \) = Angular acceleration of second servo drive

The equations for the synchronous permanent servo motors are given in the Matlab documentations as shown below:

The deflection equations for the system, derived from Euler’s differential equation follow:

\[
\frac{d}{dt} i_d = \frac{1}{L_d} v_d - \frac{R}{L_d} i_d + \frac{L_q}{L_d} p\omega_r i_q
\]

\[
\frac{d}{dt} i_q = \frac{1}{L_q} v_q - \frac{R}{L_q} i_q - \frac{L_d}{L_q} p\omega_r i_d - \frac{L_p}{L_q} (L_d - L_q)i_d i_q
\]

\[
T_e = 1.5p[\lambda i_q + (L_d - L_q)i_d i_q]
\]

where, 
\( L_d \) = d-axis inductance
\( L_q \) = q-axis inductance
\( i_d \) = d-axis current
\( i_q \) = q-axis current
\( v_d \) = d-axis voltage
\( v_q \) = q-axis voltage
\( \omega_r \) = angular velocity of motor
\( \lambda \) = amplitude of flux induce by the permanent magnet of the rotor in the stator phase (Wb)
\( p \) = Number of pole pairs (related to permanent magnets)
\( T_e \) = Electromagnetic torque

IV. MECHANICAL DESIGN

In designing machine elements, it is noteworthy that deflection of element due to its own weight cannot be neglected. The deflection of arm is modeled analytically; the arm is modeled as a combination of two fixed-end bending system as shown in Fig. 2.

The deflection equations for the system, derived from Euler’s differential equation follow:

\[
T_{M1} = \frac{1}{n^{a-1}N_1} T_1 + (J_{M1} + J_{P1}) + \frac{J_{G1}}{n^{a-1}N_2} \dot{\theta}_{M1}
\]

\[
T_{M1} = \frac{1}{n^{b-1}N_2} T_2 + (J_{M2} + J_{P2}) + \frac{J_{G2}}{n^{b-1}N_2} \dot{\theta}_{M2}
\]

where, 
\( T_{M1} \) = motor torque of first servo drive
\( T_{M2} \) = motor torque of second servo drive

The relationship between the drive torque and the motor torque is given below:

\[
T_{M1} = \frac{1}{n^{a-1}N_1} T_1 + (J_{M1} + J_{P1}) + \frac{J_{G1}}{n^{a-1}N_2} \dot{\theta}_{M1}
\]

\[
T_{M1} = \frac{1}{n^{b-1}N_2} T_2 + (J_{M2} + J_{P2}) + \frac{J_{G2}}{n^{b-1}N_2} \dot{\theta}_{M2}
\]

TABLE I
MAIN COMPONENTS OF THE LABORATORY

<table>
<thead>
<tr>
<th>Component</th>
<th>Manufacturer</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Servo motor No1</td>
<td>ElectroCraft</td>
<td>24V AC, Power output 2 kW, R 19.1Nm, Stall Torque 57.3Nm, RPM 2000, Rotor Inertia 98.5 Kg/m², weight 27.4Kg</td>
</tr>
<tr>
<td>Servo motor No2</td>
<td>ElectroCraft</td>
<td>24V AC, Power output 300W, R 8.66Nm, Stall Torque 8.86Nm, RPM 2000, Rotor Inertia 4.04 Kg/m², weight 5.5Kg</td>
</tr>
<tr>
<td>Stepper Motor</td>
<td>National Instruments</td>
<td>0.4A, Holding torque 1.27Nm, Rotor inertia 0.0248kg/m²x10⁴, Phase inductance 209mH, Phase resistance 42.9Ω, Maximum speed 3000rpm, Weight 0.7Kg</td>
</tr>
<tr>
<td>Force Sensor</td>
<td>Parallax Inc.</td>
<td>Thickness 0.127mm, Length 203mm, Width 14mm, Active sensing area 9.53mm Ø, Connector – 3pin post connector, Operating Temperature -9°C - 60°C</td>
</tr>
<tr>
<td>Linking Shafts 1</td>
<td>From the university work shop</td>
<td>Shaft from Hub to shoulder, Length 500mm, Material-Steel, Ø inner 20mm – outer 35mm</td>
</tr>
<tr>
<td>Linking Shafts 2</td>
<td>From the university work shop</td>
<td>Shaft from shoulder to gripper, Length 500mm, Material-Aluminium, Ø inner 25mm – outer 35mm</td>
</tr>
<tr>
<td>Gears</td>
<td>From the university work shop</td>
<td>8 spur gears, key locking system for necessary gears, 2 bevel gears for the gripper mechanism</td>
</tr>
</tbody>
</table>
\[ y(x) = \begin{cases} \frac{1}{E_1 I_1} \left( \frac{W_1 x^4}{24} - \frac{R_1 x^2}{6} + \frac{BM_1 x^2}{2} \right), & \text{if } 0 \leq x < L_1 \\ A + B, & \text{if } 0 \leq x < L_2 \end{cases} \]

where,

\[ A = \frac{1}{E_2 I_2} \left( \frac{W_2 x^4}{24} - \frac{R_2 x^2}{6} + \frac{BM_2 x^2}{2} + E_2 I_2 \frac{dy}{dx} \right) \]

\[ B = \frac{1}{E_1 I_1} \left( \frac{BM_1 L_1^2}{2} - \frac{R_1 L_1^2}{6} + \frac{W_2 L_4^2}{24} \right) \]

The analysis of design requirements for both the links is summarized in [24]. The main function of both the links is to carry the bending load. Minimizing the weight and the deflection are important objectives of the design. The length and the size (primarily outer diameters of circular links) are prescribed early in the design input phase. The free design variables are the thicknesses of both links; they are regarded as the inner diameters of the links.

Aluminum is chosen as the material for the second link as the second link supports a relatively light load (i.e., the motorized gripper). Aluminum exhibits good flexural rigidity. The first link, however, is made of steel to achieve a high strength to support large bending loads. The loads include both of the motorized gripper and elbow compartment at the free end of the link. This choice of material tends to minimize the deflection of the rigid robotic arm.

Computing software (e.g., Matlab) may be used to compute the overall deflection of the robotic arm. The analytical discontinuity equation stated above is also utilized. The input variables of the program are the dimensions of inner diameters of the links. Flow chart of determining suitable values of inner diameters is shown in Fig. 3.

![Flow chart of determining optimal inner diameters of links](image)

In the following treatment, the selection of driver units for the links is discussed. AC servo motors are adopted in our system to drive the links. The main criterion of selecting the motor is the matching of inertia ratio between the load inertia and the rotor inertia of motor. Matching of inertia ratio is crucial as if the inertia ratio exceeds the prescribed value, it will overload the servo motor although it could still provide the desirable torque. As a result, the motor will overheat, and its service life will be reduced drastically.

Weight is a main concern at the elbow compartment. Larger weight of motor implies that more materials will be needed for the elbow design. Subsequently the accumulated weight will cause the whole arm to deflect more, and this is not desirable. Although smaller weight of the motor tends to give more allowance on allowable payload weight, it complicates the design of gear train. In this case, the efficiency of transmitting torque from motor to link is a concern. Compromise has to be made between the weight of the motor and number of gears in the gear train.

Meanwhile, for the shoulder compartment, gear train is used to transmit the motor torque. In this case, a few degrees of gear backlash could be adjusted to the gear train by a special mechanism as discussed in the following section. Sizes of the gears depend on the size of the chosen driver and the number of gears must be limited.
The speed reduction ratio can be expressed in terms of moment of inertias for the computation of suitable reduction ratio as follows:

\[ N_i^2 = \frac{(J_{e1} + J_{o1})SF}{J_{m1}R - J_{p1}} \]

\[ N_i^2 = \frac{(J_{e2} + J_{o2})SF}{J_{m2}R - J_{p2}} \]

where,

SF = Safety factor
R = Allowable inertia ratio, specified by the motor manufacturer
and the rest of the parameters are defined as before.

Sizing of components is discussed next. Combined loading technique is used while determining whether the sized components can withstand the load. Three types of loading are considered. Those are: bending, torsional, and axial loadings. Their respective stresses are calculated with respect to their cross section as follow:

\[ \sigma_b = \frac{M_y}{I} \]

\[ \sigma_a = \frac{F}{A} \]

\[ \tau = \frac{Tr}{J} \]

where,

\( \sigma \) = bending/axial stress (may be subscripted for clarity)
\( \tau \) = torsional stress
\( I \) = moment of inertia
\( M \) = moment
\( F \) = applied force
\( A \) = cross-section area
\( r \) = radius
\( T \) = torque

Superposition of stresses is applied, and this depends on the loading condition of components. Principal stresses and maximum shear stresses are then calculated based on a selected plane-stress element at a cross section of the component [25] using the following equations:

\[ \sigma_{12} = \frac{1}{2}(\sigma_x + \sigma_y) \pm \frac{1}{2}(\sigma_x - \sigma_y)^2 + \tau_{xy}^2 \]

\[ \tau_{\text{max}} = \frac{1}{4}(\sigma_x - \sigma_y)^2 + \tau_{xy}^2 \]

Superposition of stresses is applied, and this depends on the loading condition of components. Principal stresses and maximum shear stresses are then calculated based on a selected plane-stress element at a cross section of the component [25] using the following equations:

The minimum allowable stress is then calculated using maximum shear stress (Tresca) criterion [26].

\[ \sigma_{\text{allowable}} = 2SF \tau_{\text{max}} \]

The grade of the material is chosen such that the yield stress of the material is higher than the minimum allowable stress of the component. Otherwise, the size of component is revised such that its calculated allowable stress is within the yield stress of materials in the list.

V. VARIABLE COULOMB FRICTION AND BACKLASH

The shaft assembly needs to be in contact with some other surface to generate coulomb friction. Therefore the friction is created in a way that one rotating surface is rubbing against a rigid surface. The rotating surface has a circular conical surface and as shown in the Fig. 5. When both the nuts at the two extreme ends of the pivoting rod are tightened, Coulomb friction is generated. The generated Coulomb friction can be changed by adjusting the two nuts.

Backlash plays the significant role in the hub assembly due to clearance between mating gear teeth surfaces. Therefore axial distance between gears in the hub gear train needs to be adjusted. It is done by adjusting the middle gear axis. When it is moved perpendicular to the line of the gears, the clearance between the gear teeth will be changed. Therefore backlash may be increased or decreased by adjusting the nut as shown in the Fig. 6.

VI. GRIPPER DESIGN

One of the aims of this project is to design a simple and low cost robotic gripper for a laboratory environment for research purposes. Fig. 5 is a 3D image of the designed gripper. This design is a modified version of a prior design at The University of Nottingham. Friction grip is chosen for the type of jaws in the robotic gripper design. Friction grip jaws rely totally on the force of the gripper to hold the part. The “squeeze” of the gripper does the gripping action. In addition, the object and the gripping surface are in contact with each other by means of surface friction. The reason friction grip is chosen is because of its flexibility in holding objects with various geometries.
A stepper motor is chosen to drive the gripper mechanism. A controller is required (as part of the stepper motor configuration) to control the motion of the motor. Steps and direction signals are sent to the motor driver, and the driver interprets these signals and moves the motor in discrete steps. The control system of stepper motor can be run in open loop configuration. The gripper that is being design for laboratory environment needs to be small with high gripping force. The motor is mounted horizontally and then directly connected to the center pivot through bevel gears. The center pivot is then connected with the other two links. The other two links are connected onto the arms. The normal force on the gripper acts in the opposite direction of the pivot. There is also the weight of the arms resisting the closure motion due to linear movement. The forces acting at both ends of the links are decomposed into horizontal and vertical components of the gripping force. The torque provided by the motor is converted into the torque required to drive the gripper arms. This will cause the gripper to be displaced in a linear horizontal direction. The main torque transmitting device in the mechanism is the bevel gear. A gear ratio with 1:4 is set to achieve the required torque. The torque plays an important role in maintaining the grip at the moment the load is being transferred. In order to maintain the grip, during the time that the object is held, the torque provided by motor must exceed the stall torque.

The movement of the arm of the gripper can be controlled by commands issued from the microprocessor of the National Instruments equipment. As was mentioned, the desired movement of gripper may be achieved in an open-loop manner. However, there are concerns regarding imposing a large gripping force that may damage a fragile object. As a solution, a force sensor is installed on the surface of the gripper arms that come in contact with the payload. The force sensor is used to generate the command for imposing adequate force on the payload. In this way, a fragile object may be handled properly. Also, the other advantage of using this force sensor is to monitor the gripping force that would prevent the loss or dropping of the payload once it is gripped. The gripper is capable of handling a payload of 1Kg.

VII. SUMMARY AND CONCLUSION

The goal of this research was to design a controls laboratory that could be used to verify the controller design methodologies that have been developed for multivariable plants that include both soft and hard (discontinuous) nonlinear terms. This goal is met. The need for such a benchmark problem is demonstrated in the Introduction section of the paper.

The governing nonlinear dynamic equations are presented, and these equations may be used to carry out the paper controller design of controllers according to the nonlinear control literature. In this way, various controller design procedures may be compared.

The robotic design presented herein allows for inclusion of adjustable amounts of Coulomb friction and backlash. The mechanisms that achieve different levels of nonlinearity are novel, and they are presented.

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REFERENCES


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