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Origami Theory and its Applications:A Literature Review

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***Abstract*—**This paper presents the fundamentals of Origami engineering and its application in nowadays as well as future industry. Several main cores of mathematical approaches such as Huzita-Hatori axioms, Maekawa and Kawasaki's theorems are introduced briefly. Meanwhile flaps and circle packing by Robert Lang is explained to make understood the underlying principles in designing crease pattern. Rigid origami and its corrugation patterns which are potentially applicable for creating transformable or temporary spaces is discussed to show the transition of origami from paper to thick material. Moreover, some innovative applications of origami such as eyeglass, origami stent and high tech origami based on mentioned theories and principles are showcased in section III; while some updated origami technology such as Vacuumatics, self-folding of polymer sheets and programmable matter folding which could greatlyenhance origami structureare demonstrated in Section IV to offer more insight in future origami.

***Keywords*—**Origami, origami application, origami engineering, origami technology, rigid origami.

I. Introduction

ORIGAMI is the art of paper folding originated from Japan and has been commonly practiced worldwide. It derived its name from the Japanese word 'oru' as in 'to fold', and 'kami' as in 'paper'. Whenever origami is mentioned, first thing comes in people mind might usually be 'paper crane' or even 'children activity'. It conventionally perceived as just a kind of art rather than something offer practical usage. Over the years, origami has intrigue many artists and scientist to investigate and reveal its underlying principles. These ultimately lead to the transition of origami from art to mathematical world.Since then origami technique has been utilized in many industry applications and proven to beuseful in areas such as architecture and packaging design.

II. Origami Science

A. The Fundamentals

Origami originated as a trial-and-error art design for making paper(s) to appear like real object by folding them. Later on several mathematical approaches were developed to understand the phenomena on the paper generated by the folding and also to estimate the outlook of the origami (folded paper).

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Prior to the theories, there are some common notations or terms in origami which should be acknowledged. These have been standardized by scientists, to name a few, including Lang, Huffman, Clowes, Waltz, Takeo Kanade, and Akira Yoshizawa [1] & [2]. In summary, the notations being widely used in the Origami practice are Mountain fold (fold the paper behind), Valley fold (fold the paper toward you), Crease (location of an earlier fold, since unfolded), X-ray line (hidden edge or crease), and Crease pattern (all the crease lines, since unfolded). While the basic folding patterns in the industry are the Cupboard Base, Windmill Base, Waterbomb Base, and Preliminary Fold [1].

Mathematical approaches for Origami, to name a few, are Geometry, Topology (explained by Thomas Hull [1]), Robert Lang's Tree Theorem [2] and Maekawa's String-to-beads method [3].

The mentioned mathematical approaches are surrounding three main fundamentals: Huzita-Hatori axioms, Maekawa and Kawasaki's theorems.

Firstly discovered by Jacques Justin in 1989, the Huzita-Hatori axioms consist of 7 axioms were improved by HumiakiHuzita in 1991, and finalized by KoshiroHatori, Justin and Robert Lang in 2001. The axioms [4] are:

1. Given two points P_1 and P_2, there is a unique fold that passes through both of them.
2. Given two points P_1 and P_2, there is a unique fold that places P_1 onto P_2.
3. Given two lines l_1 and l_2, there is a fold that places l_1 onto l_2.
4. Given a point P_1 and a line l_1, there is a fold that places P_1 onto l_2.
5. Given two points P_1 and P_2 and a line l_1, there is a fold that places P_1 onto l_1 and passes through P_2
6. Given two points P_1 and P_2, and two lines l_1 and l_2, there is a fold that places P_1 onto l_1 and P_2 onto l_2.
7. Given one point P_1 and two lines l_1 and l_2, there is a fold that places P_1 onto l_1 and is perpendicular to l_2.

As simple as the axioms above, Maekawa's theorem takes action in any origami model crease pattern as long as the paper folds flat. Looking at a single vertex (a point where multiple creases line come across) in the paper's interior of a flat origami crease pattern, the difference between the number of mountain creases (M) and valley creases (V) must always be 2 as shown in (1).

$$|M - V| = 2 \quad (1)$$

There is another famous result that relates the angles between the creases surrounding vertex and is named after Toshikazu Kawasaki. The Kawasaki Theorem has proven that for a given crease pattern is foldable only if all the sequences of angles surrounding each (interior) vertex can be summed to 180° as given in eq. (2) and eq. (3) and represented in Fig. 1 [5].

$$\alpha_1 - \alpha_2 + \alpha_3 - \alpha_4 + \cdots - \alpha_{2n} = 0° \quad (2)$$

Or another way of saying

$$\alpha_1 + \alpha_3 + \cdots + \alpha_{2n-1} = \alpha_2 + \alpha_4 + \cdots + \alpha_{2n} = 180° \quad (3)$$

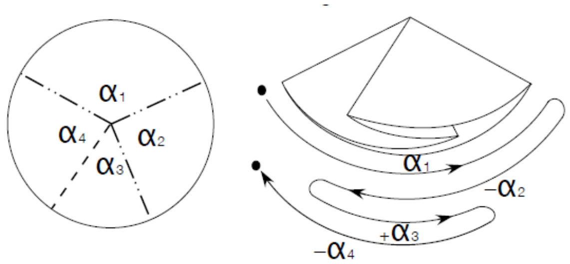


Fig. 1 Application of Kawasaki's Theorem in a foldable paper crane (figure adopted from [6])

According to Lang [2], there are 3 types of flaps: corner, edge and middle flaps as given in Fig. 2. The names of the flap come after the location of the tip of the flap falls on the square. Flaps are distinguished to determine the amount of material used in folding. Middle flap consumes the most part of a material as compared to corner flap which consumes the least amount of material. To prove this, Lang proposed that each the tips (of the flaps) forms the centre of a circle on a flat material.

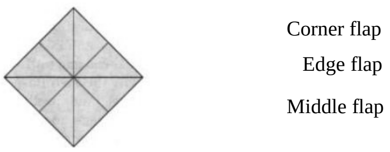


Fig. 2 Types of flap

Lang suggested that all circles surrounding the tips, may it be quarter-circle (for corner flap), semi-circle (for edge flap), or full circle (for middle flap), the radius of the circle is determined by the length of the flap, L. Lang commented, "The amount of paper consumed doesn't depend on the angle of the tip of the flap, only its length and location." Thus, circles-packing where circles do not overlap each other, is used in determining the sizes of the circles – the size of the flap. This approach is seem to be essential in Rigid Origami, as hard materials in engineering applications are not capable to recover from the crease formed by folding.

B. Rigid Origami

Rigid origami applies in folding materials with thickness. Car airbags, large solar panel arrays for space satellites (using Miura-fold), paper shopping bags are amongst the studies in Rigid origami.

Having thickness in the material has disabled some fundamentals of origami in Rigid Origami such as Huzita-Hatori axioms. However, Kawasaki's Theorem and Haga's Theorem are still valid in real material folding [7].

Tomohiro Tachi mentioned in his research that Rigid origami consists of rigid panels connected by hinges constrained around vertices [8]. The origami configuration is represented by fold angles between the adjacent panels [9]. Intensive Mathematical model for 3D folding was presented in his paper [8] and it was used in his written software RigidOrigami.

Britney Gallivan has developed a loss function for origami folding with thick material and the function sounds like eq. (4).

$$L = \frac{\pi t}{6}(2^n + 4)(2^n - 1) \quad (4)$$

Where L is the minimum length of the material, t is the thickness of the material, and n is the number of folds possible [9].

It appears that there is no direct connection between Lang's Tree Theorem and Tachi'sRigid Origami. However, the former could be used in designing the folding based on a desired final outlook of an origami where else the latter could provide calculations on the folding mechanisms with materials with thickness.

III. Origami Applications

A. Solar Panels

Origami concepts were used to pack and deploy a solar power array in the research vessel called Space Flight Unit (SFU) as shown in Fig. 3. The method of folding is called "Miura-ori" which had been introduced in last section.

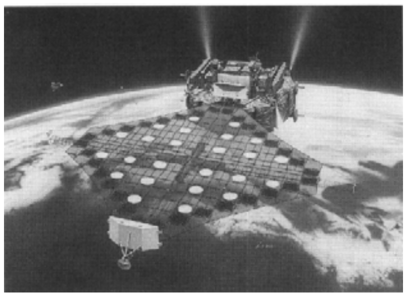


Fig. 3 Foldable Solar Panel designed by Koryo Miura (figure adopted from [10])

B. Space Telescope, Eyeglass

The Eyeglass is a foldable telescopic lens designed by Robert Lang which can be easily packed into a space shuttle and deployed when in space by utilizing origami principals and techniques.

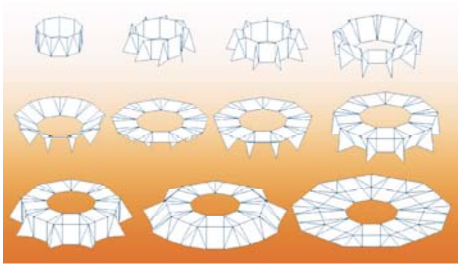


Fig. 4 The Eyeglass can be folded from a flat disk (bottom right) to a small flanged cylinder (top left) (figure adoptedfrom[11])

The origami structure shown in Fig. 4 is called the "Umbrella" structure after its resemblance to a collapsible umbrella, which was scalable and had mass-producible parts. In 2002, a 5-meter prototype, as shown in Fig. 4, comprising 72 segments patterned with binary Fresnel arcs in photoresist was completed and shown to concentrate light as expected [11].

It was believed that a 25-meter or larger version diffractive space telescope could be deployed within a decade [11] while the ultimate goal might be folding a 100-meter lenses into 3-meter diameter cylinders [12].

C. Origami Stents for Medical Purposes

An origami stent was developed which may be used to enlarge clogged arteries and veins. The design followed waterbomb base origami style which enables it to be collapsed into a smaller size while travelling in veins/arteries, and expanded to a larger diameter at the clot site to serve its purpose (Fig. 5). The patterns are made from three type of folds: two sets of helical folds orthogonal to each other and cross folds. The existence of long helical folds enables a highly synchronized deployment process [13].

a b

Fig. 5 Origami stent designed by Zhong You and Kaori Kuribayashi: (a) in collapse form, (b) in expended form (figure adopted from [14])

D. High Tech Origami

Researchers from University of Illinois have developed a technique to fold microscopically thin slice of silicon into different kinds of shapes (three-dimensional) which could beuseful in many areas, including solar energy. They intended to design a spherical solar cell which claimed to be able to absorb solar energy more efficiently than a typical flat solar cell.

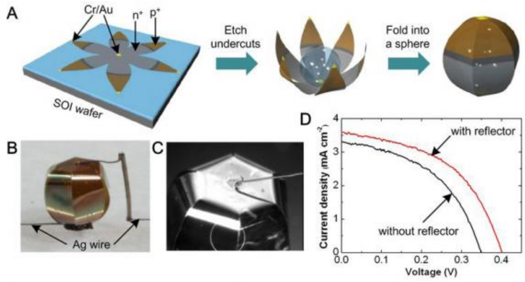


Fig. 6 Water droplets direct self-assembly process (figure adoptedfrom [15])

As shown in Fig. 6, a two-dimensional piece of silicon was cut into a flower shape with a small piece of glass planted in the center to help retaining the eventual desired shape. It was followed by placing a water droplet in the center of the silicon flower. As the water evaporates, capillary forces pulled the edges of the foil together to form the sphere shape [15][16].

The new self-assembly process is still in its early stage; however it shows a completely different approach to making three-dimensional structures.

E. Origami Grocery Bag

A foldable grocery bag from steel was built by Zhong You and Weina Wu using origami-inspired design as shown in Fig. 7. It allowed shopping bag built from a rigid material or an open-topped cardboard box could be folded flat without having its bottom opened. This approach could help speed up factory automated packaging processes. The ultimate dream of the designers is to make rigid building that could be reconfiguredin the future [17][18].

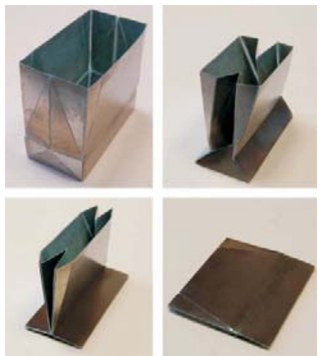


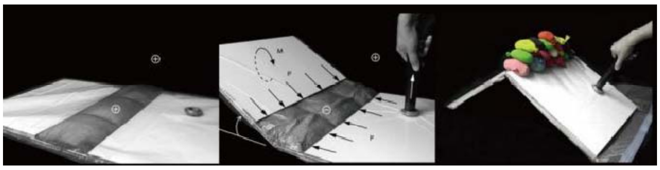
Fig. 7 A prototype made from a number of stainless steel plate (figure adopted from [17])

IV. Advancement of Origami

A. Vacuumatics

Multi-DOF rigid foldable structures based on triangle panels is able to follow the change in the environment and human activities due to its flexibility. Vacuumatics is a solution to make the 3D form becomes geometrically stable by stiffening the dihedral hinges. The structural system is a negative pressured double membrane containing aggregate particles. It is initially plastic or viscoelastic, but by introducing negative pressure, the induced friction force between compressed particles makes the structure stiff [19].

Another interesting aspect of the structure is the particles can reconfigure themselves to be in another equilibrium state which therefore enables the strength of hinge to be controlled regarding the amount of vacuum.The example of Vacuumatics is demonstrated in Fig. 8.



a b c

Fig. 8 Hinge design: (a) Particles are placed in between panels. (b) Self-folding moment is induced bt the vacuum (c) The vacuum keeps the hinge stiff (figure adopted from [19])

B. Self-Folding of Polymer Sheets

A technique was developed by Michael Dickey where polymer sheets self-fold when exposed to light as shown in Fig. 9. Polymer sheets were run through a desktop printer to get a pattern of black lines, or crease pattern in origami, and they automatically fold along the black lines when exposed to light. The concept is such that black absorbs more energy than other colours and thus black lines will shrink faster than other areas.

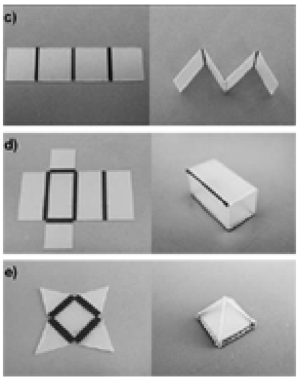


Fig. 9 Polymer sheets with black lines (figure adopted from [12])

C. Programmable Matter Folding

Robert Wood and his team at Harvard University and the Massachusetts Institute of Technology have combined a thin sheet with programmable joints which flex in response to electrical warming [20].The sheet is a kind of variable 'programmable matter' which developed by chemists specialized in molecular self-assembly and by robotics engineers working on 'self-reconfiguring' robots.Above all, Wood's team has to first create an algorithm for folding with a dictated set of creases based on the desired end shapes. Then the hinged 'actuators' were located at each crease that open and close on command. The hinges are made of Nitinol, a 'smart' metal alloy of nickel and titanium which has the ability to return to its initial shape after being warmed and then bent.

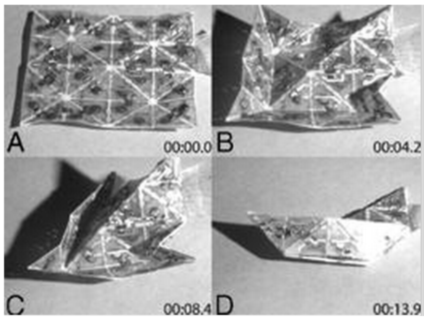


Fig. 10 A self-folded experimental 'boat' (figure adopted from [21])

Their current prototype, shown in Fig. 10, used one-way switch as their hinges which must be bent back by hand. The hinges are stapled into a sheet made from triangular panels of stiff fiberglass where their edges are joined by flexible silicone rubber. There is a magnet sit in the middle of each panel which holds them firmly but not irreversibly together when they are folded face to face.

To initiate the on-demand folding, the team intended to develop removable 'stickers' which contain the circuitry specific to a particular folded shape therefore granting a given sheet to fold without the need of computer control over the hinge-heating process.

V. Conclusion and Future Works

This paper has presented the fundamentals of Origami science, Rigidorigamiand some of its applications. It is in hope that one could gain more insight about origami technology and apply it in every possible area for sustainable utilization of space and weight.

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References

[1] Kanade, Takeo. "A Theory of Origami World." *Artificial Intelligence*, 1980: 280-311.
[2] Lang, Robert J. Origami Design Secrets - Mathematical Methods for an Ancient Art. Massachusetts: A K Peters Ltd, 2003.
[3] Lang, Robert J. "TreeMaker User Manual." *Langorigami*. 2004. www.langorigami.com (accessed Oct 15, 2009).
[4] *Huzita-Hatori Axioms.* Jan 31, 2009. http://en.wikipedia.org/wiki/Huzita%27s_axioms (accessed Aug 10, 2009).
[5] Barile, Margherita, and Margherita Barile. *"Kawasaki's Theorem." From MathWorld--A Wolfram Web Resource, created by Eric W. Weisstein.* http://mathworld.wolfram.com/KawasakisTheorem.html (accessed Aug 30, 2009).
[6] Thomas C. Hull. *Revisited and extended.* Western New England College http://courses.csail.mit.edu/ (accessed 10 September 2012)
[7] Tachi, Tomohiro. Rigid Origami Silmulation. *TT's Page Research Project*. [Online] 2009. [Cited: Nov 5, 2009.] http://www.tsg.ne.jp/TT/cg/index.html#rigid_origami
[8] *Simulation of Rigid Origami.* Tachi, Tomohiro. Tokyo : 4OSME, 2009.
[9] Kevin &Bobby, *Saint Joseph's College.* [Online] [Cited: Nov 5, 2009.] www.saintjoe.edu/~karend/m111/Origami.ppt.

[10] KTK Scientific Publishers. “Map Fold a La Miura Style, Its Physical Characteristics and Application to the Space Science”, in Research of Pattern Formation, edited by R. Takaki, pp. 77-90.

[11] Kitty Tinsley. 2003. *A Giant Leap for Space Telescopes.*Lawrence Livermore National Laboratory https://www.llnl.gov/(accessed 10 August 2012)

[12] Origami-Resource-Center.com. *Origami Science*. http://www.langorigami.com/science/technology/eyeglass/eyeglass.php (accessed 5 August 2012)

[13] Zhong You, and Kaori Kuribayashi. 2003. *A Novel Origami Stent*. Tulane University http://tulane.edu/ (accessed 10 August 2012)

[14] K. Kuribayashi and Z. You. *Innovative Origami Expandable Stents.* University of Oxford http://www-civil.eng.ox.ac.uk/ (accessed 12 August 2012)

[15] University of Illinois at Urbana-Champaign. 2009. *High-tech origami: Water droplets direct self-assembly process in thin-film materials*. http://www.sciencedaily.com/releases/2009/11/091123152222.htm (accessed 12 August 2012)

[16] University of Illinois at Urbana-Champaign. 2010. *High-Tech Origami*. http://www.las.illinois.edu/news/2010/solarcell/ (accessed 12 August 2012)

[17] Zeeya Merali. 2011. *Paper, Plastic, or Steel?*. http://news.sciencemag.org/sciencenow/2011/03/paper-plastic-or-steel.html?ref=hp (accessed 15 August 2012)

[18] Bob Yirka. 2011. *Origami solution found for folding steel shopping bags*. http://phys.org/news/2011-03-origami-solution-steel-bags.html (accessed 15 August 2012)

[19] T. Tachi, M. Masubuchi and M. Iwamoto. 2012. *Rigid Origami Structures with Vacuumatics: Geometric Considerations.* (accessed 5 August 2012)

[20] Hawkes, E. et al. Proc. Natl Acad. Sci. USA advance online publication doi:10.1073/pnas.0914069107 (2010).

[21] Philip Ball. 2010. *Origami that folds itself*. http://www.nature.com/news/2010/100628/full/news.2010.317.html (accessed 15 August 2012)