Abstract — This paper features the trajectory planning design of a indigenously developed 4-Axis SCARA robot which is used for doing successful robotic manipulation task in the laboratory. Once, a trajectory is being designed and given as input to the robot, the robot’s gripper tip moves along that specified trajectory. Trajectories have to be designed in the work space only. The main idea of this paper is to design a continuous path trajectory model for the indigenously developed SCARA robot arm during its maneuvering from one point to another point (during pick and place operations) in a workspace avoiding all the obstacles in its path of motion.

Keywords — SCARA, Trajectory, Planning.

I. INTRODUCTION

DIRECT kinematics gives the position and orientation of the robot arm in the 3DE space. Inverse Kinematics gives the sets of joint variables that will satisfy the same manipulator position and orientation. In between these two problems, robot motion comes into the picture. This robot motion consists of paths and trajectories. These paths and trajectories are nothing but the various possible routes that are taken by the robot to move from the source (pick point) to the goal / destination (place point) and traversed in a specified amount of time. We use this direct kinematics and inverse kinematics problem to solve a higher-level problem, i.e., to plan trajectories in the tool configuration space. Trajectory planning schemes helps us to interpolate / approximate between the points using a smooth motion.

Trajectory planning schemes generally interpolate or approximate the desired path by a class of polynomial functions and generate a sequence of time based control set points for the control of the manipulator from the initial location to the destination. Path end points can be specified either in joint coordinates or in Cartesian coordinates. However, they are usually specified in Cartesian coordinates, because it is easier to visualize the correct end effector configurations in Cartesian coordinates rather than in joint coordinates. Furthermore, joint coordinates are not suitable as a working coordinate system, because the joint axes of most of the robots are not orthogonal. Further, trajectory planning involves how to design and plan an obstacle collision free trajectory from the pick point to the place point in the work space of the robot. There exists an ‘n’ number of trajectories between the two given end - points, i.e., the source and the destination (goal), since the space is 3D in nature.

The main goal of trajectory planning is to describe the motion of the robot arm as a time sequence of joint or end-effector locations and its derivatives of the joints or end-effector locations, which are generated by interpolating or approximating the desired path by polynomial functions and to achieve a smooth motion of the end-effector. If temporal information or time information is added to the tool path by specifying the times at where the tool / gripper should be at various points along the path; then, the path gets converted into a trajectory. Hence, a trajectory is defined as a path with time information or temporal information, i.e., trajectory is both spatial as well as temporal as shown in Fig. 1.

In this work the mathematical modelling of the trajectory design is done for a unique 4 axes system, which was designed and fabricated with indigenous components starting from scratch. The kinematically modeled and designed robot is used for some PNP operations and was named as a Selective Compliance Assembly Robot Arm (SCARA). The primary motive behind this work was to develop a modular educational robotic system, the CRUST 2002 (Computerized Robotic Unit with Selective Tractability system) with the help of locally available components and sub-systems.

The paper is organized as follows. In Section 2, a brief introduction about the constructed robot is given. Next the mathematical model of the trajectory planning is presented. Simulations results are presented next followed by the conclusions and the references.

Fig. 1 Trajectory description

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II. HARDWARE DESIGN OF THE ROBOTIC SYSTEM

The robot is a 4 DOF stationary one having base, elbow, vertical extension and tool roll and consisting of both rotary and prismatic joints as shown in Fig. 2. There is no tool yaw and tool pitch (only tool roll) [1]. There are 4 joints, 4 axes (3 major axes - base, elbow, vertical extension and one minor axis - tool roll).

The 4 DOF’s are given by Base, Elbow, Vertical Extension and Tool Roll (Fig. 3), i.e., there are 3 rotary joints and 1 prismatic joint. Since \( n = 4 \); 16 KP’s are to be obtained and 5 RHOCF’s are to be attached to the various joints [2] as shown in the LCD [1] in Fig. 6.

The vector of joint variables is a combination of \( \theta \) and \( d \), i.e., \( q = \{ \theta, d \}^T \). The joint variable vector \( q \), vector of joint distances \( d \), vector of link lengths \( a \), vector of link twist angles \( \alpha \) are

\[
q = \{ \theta_1, \theta_2, d_3, \theta_4 \}^T.
\]

\[
d = \{ d_1, 0, d_3, d_4 \}^T = \{500, 0, 200\}^T \text{ mm}.
\]

\[
a = \{ a_1, a_2, 0, 0 \}^T = \{300, 250, 0, 0\}^T \text{ mm}.
\]

\[
\alpha = \{ \alpha_1, \alpha_2, 0, \alpha_4 \}^T = \{\pm \pi, 0, 0, 0\}^T.
\]

All the 4 joint axes are vertical in nature (all the \( z \)-axes can be pointing down or up) as shown in Fig. 3. The first 3 (B, E, VE) axes are called as the major axes and are used for positioning the wrist, while the last one, the minor axes (TR) is used to orient the tool in the direction of the object [13]. The first 3 major axes determine the shape and size of work envelope. It consists of an L shaped structure, to the end of which, the second link is attached. There are 2 links \( a_1 \) and \( a_2 \) which move parallel to work surface; the vertical extension \( d_3 \) is variable and moves in a direction \( \perp \) to the work surface; length of the gripper / tool / EE is \( d_4 \) and its maximum protrusion is 300 mm [2].

The tool / gripper / EE is permanently pointing down as shown in Fig. 5 and can rotate in a plane \( \perp \) to the work surface plane \( x^0y^0 \). The approach vector \( r^3 \) is fixed, i.e., \( r^3 \perp x^0y^0 \) (work surface) plane; \( r^3 = -z^0 \). Because of this reason, the designed and kinematically modeled SCARA robot can do robotic manipulation directly from above the object when exact perpendicularity is required. The SCARA robot is a minimal representation of any robot [3] as shown in Fig. 4.
Our SCARA robot is a special type of polar / spherical coordinate robot in which the major axes are R R P [14].

III. INTRODUCTION TO MATHEMATICAL MODELLING OF THE TRAJECTORY DESIGNING

The tool configuration vector of the robot [1] is given by

\[
\mathbf{w}(q) = \begin{bmatrix}
\begin{array}{c}
\ldots \\
\vdots \\
\theta_1 \\
\ldots \\
\mathbf{p}_1 \\
\mathbf{p}_2 \\
\mathbf{p}_3 \\
\mathbf{R}_4 \\
\mathbf{R}_5 \\
\mathbf{R}_6
\end{array}
\end{bmatrix},
\]

(1)

The output of inverse kinematics problem for the four axes Adept - 1 SCARA robot is given by the following four inverse kinematic equations [1]:

The base angle (rotary in nature) is given by

\[
q_1 = \tan^{-1}\left(\frac{\mathbf{a}_1 + \mathbf{a}_2 \mathbf{C}_2}{\mathbf{w}_1 + \mathbf{a}_2 \mathbf{S}_2 \mathbf{w}_1}\right),
\]

(3)

The elbow angle (rotary in nature) is given by

\[
q_2 = \tan^{-1}\left(\frac{\mathbf{a}_4}{\mathbf{w}_2 - \mathbf{a}_2 \mathbf{S}_2 \mathbf{w}_2}\right).
\]

(4)
\[ q_2 = \pm \cos^{-1} \left( \frac{w_2^2 + w_2^2 - a_1^2 - a_2^2}{2a_1a_2} \right) . \]  

The vertical extension (prismatic in nature) is given by

\[ q_3 = d_1 - d_4 - w_3. \]

The tool roll angle (rotary in nature) is given by

\[ q_4 = \pi \ln |w_4|. \]

Consider two intermediate variables \( b_1 \) and \( b_2 \). These intermediate variables [9] are used to remove the coupling between the joints and to extract the joint rates, i.e., the rate at which a particular joint is driven or moved [1].

\[ b_1, b_2 \rightarrow \text{Obtained from } q_2 \text{ and } w(q) \text{ as follows:} \]

\[ b_1 = (a_1 + a_2 C_2) w_1 - (a_2 S_2) w_2 \quad (8) \]

\[ b_2 = (a_1 + a_2 C_2) w_2 + (a_2 S_2) w_1 \quad (9) \]

From the TCV, we get,

\[ w_1 = a_1 C_1 + a_2 C_{1-2} = a_1 C_1 + a_2 (C_1 C_2 + S_1 S_2) \quad (10) \]

\[ w_2 = a_1 S_1 + a_2 S_{1-2} = a_1 S_1 + a_2 (S_1 C_2 - C_1 S_2) \quad (11) \]

\[ w_3 = d_1 - q_3 - d_4 \quad (12) \]

\[ w_4 = w_5 = 0 \quad (13) \]

\[ w_6 = -\exp \left( \frac{q_4}{\pi} \right) \quad (14) \]

The rate at which the joints are driven can be found out by differentiating the expressions for joint angles from equations (3) to (7).

**Computation of Elbow Rate \( \dot{q}_2 \)**

It is the rate at which the elbow joint is driven and depends on \( b_1, b_2 \) and its derivatives and is directly \( \alpha \) to the output speed of elbow motor [1].

\[ q_2 = \pm \cos^{-1} \left( \frac{w_2^2 + w_2^2 - a_1^2 - a_2^2}{2a_1a_2} \right) . \]  

Differentiating the expression for \( q_2 \) expression w.r.t. \( w \), we get

\[ \dot{q}_2 = \frac{d}{dw} (q_2) = \frac{1}{\sqrt{1 - \left( \frac{w_2^2 + w_2^2 - a_1^2 - a_2^2}{2a_1a_2} \right)^2}} \cdot \frac{d}{dw} \left( \frac{w_2^2 + w_2^2 - a_1^2 - a_2^2}{2a_1a_2} \right) \quad (15) \]

\[ \dot{q}_2 = \frac{1}{\sqrt{\left( \frac{2a_1a_2}{a_2} \right)^2 - \left( \frac{w_2^2 + w_2^2 - a_1^2 - a_2^2}{2a_1a_2} \right)^2}} \cdot \frac{2a_1a_2}{a_2} \left( w_1^2 + w_2^2 - a_1^2 - a_2^2 \right) \quad (16) \]

\[ \dot{q}_2 = \frac{1}{\sqrt{\left( \frac{2a_1a_2}{a_2} \right)^2 - \left( \frac{w_2^2 + w_2^2 - a_1^2 - a_2^2}{2a_1a_2} \right)^2}} \cdot \frac{2a_1a_2}{a_2} \left( w_1^2 + w_2^2 - a_1^2 - a_2^2 \right) \quad (17) \]
\[ \frac{b_1 b_2 - b_2 b_1}{b_1^2 + b_2^2} \]  

Substitute the value of \( b_1 \) and \( b_2 \) from Eqs. (23) and (25) in (30) and compute \( q_1 \). Note that \( b_1 \) and \( b_2 \) are the intermediate variables and nothing but the derivatives of \( b_1 \) and \( b_2 \).

**Computation of Vertical Extension Rate** \( q_3 \)

It is the rate at which the vertical extension moves along the vertical extension / tool- roll axis (perpendicular to work surface, \( x^0 y^0 \) plane) and is directly proportional to the output speed of the vertical extension motor.

It is not coupled with other three joints and is easiest to recover, since prismatic in nature [1].

\[ q_3 = d_1 - d_4 - w_3 \]  

(31)

\[ q_3 = -w_3 \]  

(32)

**Computation of Tool Roll Rate** \( q_4 \)

It is the rate at which tool roll joint is varied and is proportional to the output speed of the tool roll motor and is obtained from last component [1] of TCV, \( w(q) \).

\[ q_4 = \pi \ln |w_6| \]  

(33)

\[ q_4 = \pi \frac{w_6}{w_6} \]  

(34)

The rate at which the 4 joints of the robot rotate is given below in Eqs. (35) - (38). By giving all these four equations as the input to the trajectory planner block diagram as designed in Fig. 2, a suitable trajectory is defined in the 3 dimensional euclidean space [10] and the robot moves along that designed trajectory.

\[ \dot{q}_1 = \frac{b_1 b_2 - b_2 b_1}{b_1^2 + b_2^2} \]  

(35)

\[ \dot{q}_2 = \alpha \frac{2 (w_1 w_3 + w_2 w_2)}{\sqrt{2 (a_a) - \sqrt{w_1^2 + w_2^2 - a_1^2 - a_2^2}}} \]  

(36)

\[ \dot{q}_3 = -w_3 \]  

(37)

\[ \dot{q}_4 = \pi \frac{w_6}{w_6} \]  

(38)

Note that the robot in our case is controlled at the joint level rather than at the gripper tip level. This is because; there is no direct control over the gripper tip. The only thing we can control in a robot is the speed of the motor, which is connected to the joint.

Hence, by controlling the joint, i.e., by varying the speed of the actuators, the robot tool tip is made to move along a particular path / trajectory. If \( q_k \) is the \( k^{th} \) joint variable, then \( \dot{q}_k \) will be the \( k^{th} \) joint velocity / speed or the rate at which the \( k^{th} \) joint will move and is directly \( \propto \) to the output speed of the \( k^{th} \) motor, where \( 1 \leq k \leq n \).

Note that a trajectory in our robot context is defined as, “if temporal information or time information is added to the gripper tip path by specifying the times at where the tool / gripper should be at various points along the path; then, the path gets converted into a trajectory.

![Fig. 7 Joint trajectories and tool trajectory](image)

![Fig. 8 A typical trapezoidal speed profile curve](image)

Hence, a trajectory is defined as a path with time / temporal information, i.e., a trajectory is both spatial as well as temporal [11]. The joint trajectories, the gripper tip trajectory and the trajectory speed profile curve is shown in Figs. 7 and 7 respectively. The simulation results of the position, velocity and the acceleration curves are shown in the Figs. 9 - 12 respectively.
IV. CONCLUSIONS

A brief investigation into the trajectory design of the manipulator arm was discussed. The mathematical analysis of the trajectory design is also presented in this paper along with the block-diagrammatic representation.

A numerical analysis is also done. This trajectory is inputted into the computer which is interfaced with the robotic system and the robot’s tip of the gripper moves along the desired / specified / designed trajectory. Here in this paper, the design of a continuous path trajectory is presented for the robot arm. It is also called as controlled path motion trajectory.

Once the user specifies the path, the robot moves continuously along the specified path. Hence the name continuous path motion and the corresponding trajectory traced by the gripper tip are known as a continuous path motion trajectory. The user explicitly specifies the path, the robot moves continuously along the specified path.

When the robot is moving continuously along the path, care has been taken to see that all the joint angles are controlled properly, otherwise it will not move along the specified path. Hence the name controlled path motion and the corresponding trajectory traced by the tool tip is known as a controlled path motion trajectory.

V. SIMULATION RESULTS

Fig. 9 Plot of position versus time (Matlab output)

Fig. 10 Plot of velocity versus time (Matlab output)

Fig. 11 Plot of acceleration versus time (Matlab output)

Fig. 12 Plot of position / velocity / acceleration versus time (Matlab output)
REFERENCES


T. C. Manjunath, born in Bangalore, Karnataka, India on Feb. 6, 1967 received the B.E. Degree in Electrical Engineering from the University of Bangalore in 1989 in First Class and M.E. in Automation and Control Engineering with specialization in Automation, Control and Robotics from the University of Gujarat in 1995 in First Class with Distinction, respectively. He has got a teaching experience of 18 long years in various engineering colleges all over the country. He is working as an Assistant Professor in the Department of Information Technology of Thakur College of Engineering and Technology, Kandivali (East), Mumbai, India since August 2006 and he is doing his Ph.D in the College Teachers category in the Department of Systems and Control Engineering, Indian Institute of Technology Bombay, Powai, Mumbai–400076, India, in the field of modeling, simulation, control and implementation of smart flexible structures using DSPACE and its applications. He has finished his research and submitted his Thesis and waiting for his defense. He has published 80 papers in the various national, international journals and conferences and published two textbooks on Robotics, one of which has gone up to the fourth edition and the other, which has gone up to the fifth edition (2 volumes) along with the CD which contains about 200 C / C++ programs for simulations on robotics. He has also published a monograph from springer-verlag based on this thesis, topic titled, “Modeling, Control and Implementation of Smart Structures”, Volume 350 in LNCIS, costing nearly 100 USD. He is a student member of IEEE since 2002, SPIE student member and IOP student member since 2004, life member of ISSS, India, a Life member of the ISTE, India, Life member of ISOI, India, Life member of SSI, India. He has visited Singapore, Russia, United States of America and Australia for the presentation of his research paper in various international conferences. His biography was published in 23rd edition of Marquis’ Who’s Who in the World in 2006 issue. He has also guided more than 2 dozen robotic projects. He had also served as a volunteer in the IEEE-ICIT-2006 International Conference on Industrial Technology held in Mumbai in Dec. 2006. His current research interests are in the area of Robotics, Smart Structures, Control systems, Network theory, Mechatronics, Process Control and Instrumentation, CT and DT signals and systems, Signal processing, Periodic output feedback control, Fast output feedback control, Sliding mode control of SISO and multivariable systems and its applications.