Feature Preserving Nonlinear Diffusion for Ultrasonic Image Denoising and Edge Enhancement

Shujun Fu, Qiuqi Ruan, Wenqia Wang and Yu Li

Abstract—Utilizing echoic intension and distribution from different organs and local details of human body, ultrasonic image can catch important medical pathological changes, which unfortunately may be affected by ultrasonic speckle noise. A feature preserving ultrasonic image denoising and edge enhancement scheme is put forth, which includes two terms: anisotropic diffusion and edge enhancement, controlled by the optimum smoothing time. In this scheme, the anisotropic diffusion is governed by the local coordinate transformation and the first and the second order normal derivatives of the image, while the edge enhancement is done by the hyperbolic tangent function. Experiments on real ultrasonic images indicate effective preservation of edges, local details and ultrasonic echoic bright strips on denoising by our scheme.

Keywords—anisotropic diffusion, coordinate transformation directional derivatives, edge enhancement, hyperbolic tangent function, image denoising.

I. INTRODUCTION

ULTROSONIC imaging extends its application to many fields of medical diagnosis, with its natures of low cost, portability, noninvasion and real time image formation, compared with other imaging techniques. Because ultrasonic image not only can observe shapes of human viscera, but also can examine their functions and blood stream states, it has become an important part of medical imaging. However, ultrasonic image may be contaminated by the speckle noise in its formation process, specially when the ultrasonic wavelength corresponds to the coarseness of the irradiated object surface, which can be interpreted by the stochastic scatter model. The presence of speckle noise will degrade image quality, and even conceal image details, which affects following image segmentation, feature extraction and recognition, quantitative analysis, and most importantly disease diagnosis. Thus, to compress speckle noise and to improve image quality are the main step of ultrasonic image pretreatment.

Denoising and edge detection on ultrasonic image lie on the understanding of the statistics of speckle noise. According to the scatterer number density and space distribution in the ultrasonic scan range, and the nature of one ultrasonic imaging system, we can categorize speckle noise into one of three classes, which can be modeled by the Rayleigh, the K or the Rician distribution [1] respectively. So various image regions should be processed differently on denoising. At the same time, the ultrasonic imaging system often compress the ultrasonic echoic signal to adapt it to the display, because of its limited dynamic range, which has varied the probability density function of the signal and has transformed the multiplicative speckle noise into the additive one [2].

A number of methods have been proposed to address the problem of removing speckle noise including temporal averaging [3], median filtering[4], adaptive speckle reduction (ASR)[5,6] and wavelet shrinkage(WS)[7]. However, above methods could not succeed to balance between speckle suppression and feature preservation due to the complexity of speckle statistics. Therefore, a technique that relies on a more accurate model for removing speckle noise while preserving image feature well would be rather valuable for practical use.

In section II, we discuss the differential nature of a typical edge, and then we put forward an edge enhanced anisotropic diffusion (EEAD) scheme, where we design the local diffusion matrix using the first and the second order normal derivatives of the image, and we enhance image edges employing a hyperbolic tangent function. In section III, we implement the scheme using the explicit Euler format with the central differences scheme, and test it on real ultrasonic images. The validity of our scheme and future work are presented in section IV.

II. ANISOTROPIC DIFFUSION AND EDGE ENHANCEMENT FOR ULTRASONIC IMAGE

In a medical ultrasonic image, edges and local details between heterogeneous organs are the most interesting part for clinicians. Therefore, to preserve and to enhance edges and local details on denoising are very important.

A. Edge enhanced anisotropic diffusion

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Use of partial differential equations（PDEs）in image processing has grown significantly over the past years [8]. Its basic idea is to deform an image, a curve or a surface in a partial differential equation framework, and to approach the expected result as a solution to this equation.

P.Perona and J.Malik [9] put forward an anisotropic diffusion (AD) equation to smooth a noisy image:

$$\frac{\partial u(x,y,t)}{\partial t} = div(g(|\nabla u(x,y,t)|)\nabla u(x,y,t)) \text{,} \qquad (1)$$

where $u(x,y,t):\Omega\times[0,+\infty)\rightarrow R$ is a scale image, $g(|\nabla u|)$ is a decreasing function of the gradient. Their work made an important influence on this field.

In order to understand the diffusion action of above equation clearly, we analyze the diffusion filtering on edges. At point **o**, $n=\nabla u/|\nabla u|$, $t=\nabla u^{\perp}/|\nabla u|$ are the unit tangent and normal vectors, $t\perp n$ (see fig.1) .

Aiming at different noise models of various domains in an ultrasonic image, we hope that an isotropic diffusion is practiced in homogeneous domains, while an anisotropic diffusion is done in domains of edges and local details, which diffuses along the tangent direction of edges, and does not diffuse across edges. Therefore, we can simply design a diffusion matrix using local coordinate transformation.

In fig.1, the coordinates relation between (n,t) and (x,y) is

$$\begin{pmatrix} n \\ t \end{pmatrix} = \frac{1}{|\nabla u|}\begin{pmatrix} u_x & u_y \\ -u_y & u_x \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}.$$

Further, we have

$$\begin{pmatrix} \frac{\partial}{\partial n} \\ \frac{\partial}{\partial t} \end{pmatrix} = \frac{1}{|\nabla u|}\begin{pmatrix} u_x & u_y \\ -u_y & u_x \end{pmatrix}\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{pmatrix}.$$

We adopt $f_1(x,y,t)$ and $f_2(x,y,t)$ as the diffusion coefficients along n and t. Then, the diffusion equation becomes:

$$\frac{\partial u}{\partial t} = div(D\bullet\nabla u) \text{,} \qquad (2)$$

where

$$\begin{aligned} D &= \frac{1}{|\nabla u|^2}\begin{pmatrix} u_x & u_y \\ -u_y & u_x \end{pmatrix}^{\mathrm{T}}\begin{pmatrix} f_1 & 0 \\ 0 & f_2 \end{pmatrix}\begin{pmatrix} u_x & u_y \\ -u_y & u_x \end{pmatrix}. \\ &= \frac{1}{|\nabla u|^2}\begin{pmatrix} f_1u_x^2+f_2u_y^2 & (f_1-f_2)u_xu_y \\ (f_1-f_2)u_xu_y & f_1u_y^2+f_2u_x^2 \end{pmatrix} \end{aligned} \qquad (3)$$

Therefore, we put forward the following edge enhanced anisotropic diffusion (EEAD) model:

$$\begin{cases} \frac{\partial u}{\partial t} = \alpha(x,y,t)(div(D\bullet\nabla u)) - \beta(x,y,t)f_3(x,y,t)th(lv_{nn})|u_n| \text{,} \\ v = G_t * u \end{cases} \qquad (4)$$

initial condition: $u(x,y,0)=u_0(x,y)$,

boundary condition: $u_n = 0$,

where $\alpha(x,y,t)$ and $\beta(x,y,t)$ are the control coefficients of the anisotropic diffusion and the edge enhancement respectively; D is the diffusion matrix of v (smooth version of u); $f_3(x,y,t)$ is the edge enhancement coefficient; $th(x)=(e^x-e^{-x})/(e^x+e^{-x})$ is a hyperbolic tangent function, with l constant to control its gradient; G_t is a Gaussian smoothing function.
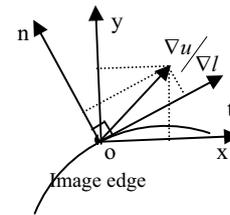

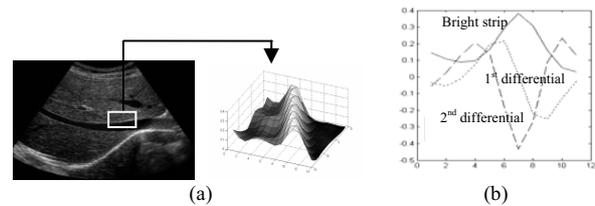Fig.1 Decomposition of the directional derivative $\nabla u/\nabla l$ and the coordinate transformation on image edge.

(a)　　　　　　　　　　(b)

Fig.2 The ultrasonic echoic bright strip (a)　and its differentials (b).

B.　Analysis of our model and adoption of its parameters
1)　Anisotropic　diffusion

In the isotropic area (corresponds to a small gradient variety) we need an isotropic diffusion along all directions, so we adopt $f_1 \approx f_2$; in the area of edges and local details (corresponds to a big gradient variety), we need an anisotropic diffusion along the tangent directions of edges in order to preserve edges, so we adopt $f_1 < f_2$, $f_1,f_2 \rightarrow 0$ ($|v_n| \rightarrow +\infty$).

From the principle of medical ultrasonic imaging system, we know that ultrasonic images catch useful medical information utilizing different intensities and distributions of echoic signals from various organs and local details. There may appear ultrasonic echoic bright strips with different intensities on edges and local details, which are important medical diagnostic message, and we should preserve them by all means in image processing (see fig.2).

When examining the differential nature of the echoic bright strip, we find that at its center the first order normal derivative of its profile approximates to 0, while its second order normal derivative reaches a minimum (see fig.2). In order to stop an excess polish at the echoic bright strip during smoothing, we can add a second order normal derivative term to the diffusion coefficients.

Based on above consideration, we adopt the following diffusion coefficients:

$$f_1 = 1/(1+c_1|v_n|^2+c_2|v_{nn}|^2), \quad f_2 = 1/\sqrt{1+c_1|v_n|^2+c_2|v_{nn}|^2}, \quad (5)$$

where c_1 guarantees to preserve edges and local details, while c_2 to preserve echoic bright strips.

2)　Edge enhancement

Then, we analyze a typical slope edge (see fig.3). **a** is the profile of one-dimension slope edge, whose center is **o**, and **b**, **c** are its first and second differential curves. It is evident that **b** increases from 0 gradually, reaches its maximum at **o**, and then decreases to 0; while **c** changes its symbol at **o**, from positive to negative. So we can control the variety of gray

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levels of image beside the edge center softly using a hyperbolic tangent function, to enhance the edge by minishing its breadth (see fig.4). Here we adopt the edge enhancement coefficient as:

$$f_3 = 1 - 1/(1 + c_3|v_n|^2), \qquad (6)$$

where c_3 controls selectively the area to be enhanced.

3) The control coefficients

With evolving the anisotropic diffusion equation, speckle noise becomes less and less. Therefore $\alpha(x,y,t)$ should decrease slowly, while $\beta(x,y,t)$ should increase from 0 slowly in order not to magnify noise.

According to the scale space theory and the literature [10], we can get the optimum smoothing time by estimating the variance of the speckle noise (for example, using wavelet decomposition coefficients):

$$T_0 = \sigma^2/a , \qquad G_t = e^{-\frac{x^2+y^2}{2at}} / 2a\pi t , \qquad (7)$$

and then, we adopt the following control coefficients:

$$\alpha = \begin{cases} 1 + l_1(1 - e^{l_2 t^2}), & t \le T_0 \\ 0, & t > T_0 \end{cases} , \quad \beta = \begin{cases} 0, & t \le T_0 \\ 1 + l_1(1 - e^{l_2 t^2}), & t > T_0 \end{cases} , \qquad (8)$$

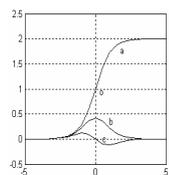where l_1, l_2 are constants (see fig.5 for their profiles).

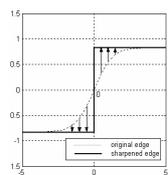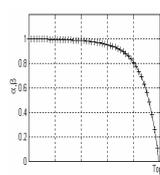Fig.3 A typical edge and its differentials. Fig.4 Edge enhancement. Fig.5 Control coefficients.

III. IMPLEMENTATION AND EXAMPLES

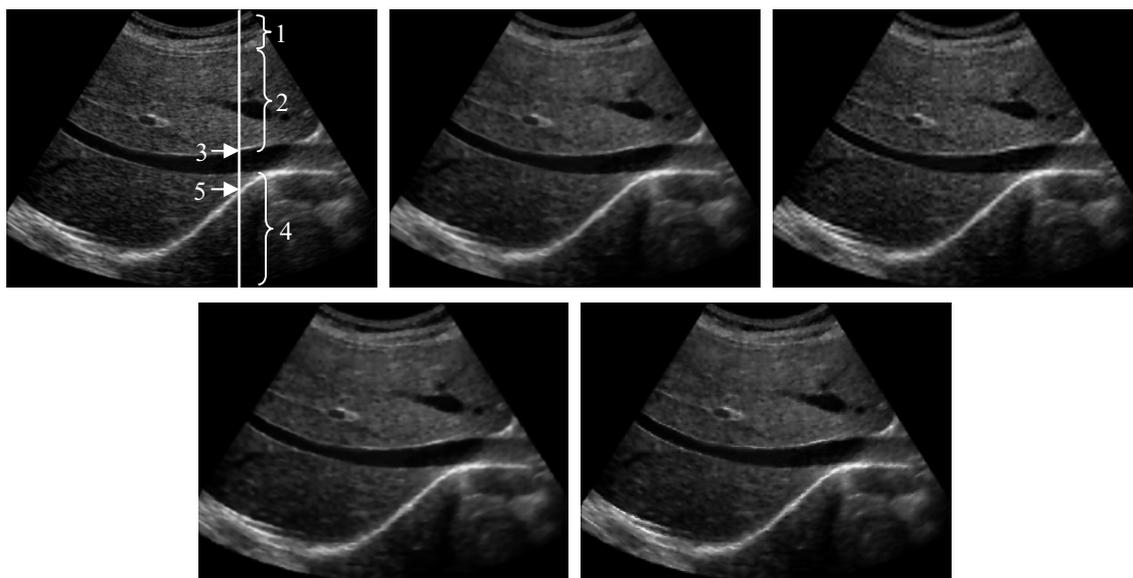We used the explicit Euler format with the central difference scheme for model (4).

Our arithmetic has been tested by different ultrasonic images using MATLAB. At the same time we have compared it with others: AWMF (Adaptive Weighted Median Filter)[4], WSTS (Wavelet Soft Thresholding Shrinkage)[7], AD (Anisotropic Diffusion)[9]. Below we discuss an example by denoising an ultrasonic liver image (376×507).

Adoption of parameters of four different methods: AWMF, a =0.05, 5×5 stencil; WSTS, Symlets wavelet, level=2; AD, k =0.08, Δt =0.1; EEAD, (c_1, c_2, c_3, l) = (0.15, 1.4, 0.015, 0.015), Δt =0.07, T_0 =1.4. All parameters have been optimized to approach best results.

From Fig.6 and Fig.7, it is clear that our method produces more promising results than others, both in denoising speckle noise (part **1**, **2**, **4**) and in preserving edges, local details and ultrasonic echoic bright strips (part **3**, **5**) (where broken lines are of original image, real lines are of the results obtained by four different methods respectively).

IV. CONCLUSIONS AND FUTURE WORK

A new nonlinear edge enhanced anisotropic diffusion model was proposed to reduce ultrasonic speckle noise while preserving the edges, local details and ultrasonic echoic bright strips. The new technique has the advantages of denoising and preserving important features and organ surfaces well, which has a large potential in ultrasonic imaging enhancement and in assisting automated segmentation/ calculation techniques.

For the future, we shall apply the model to special local pathological changes, where we can adopt model parameters better, and can hope preferable results.
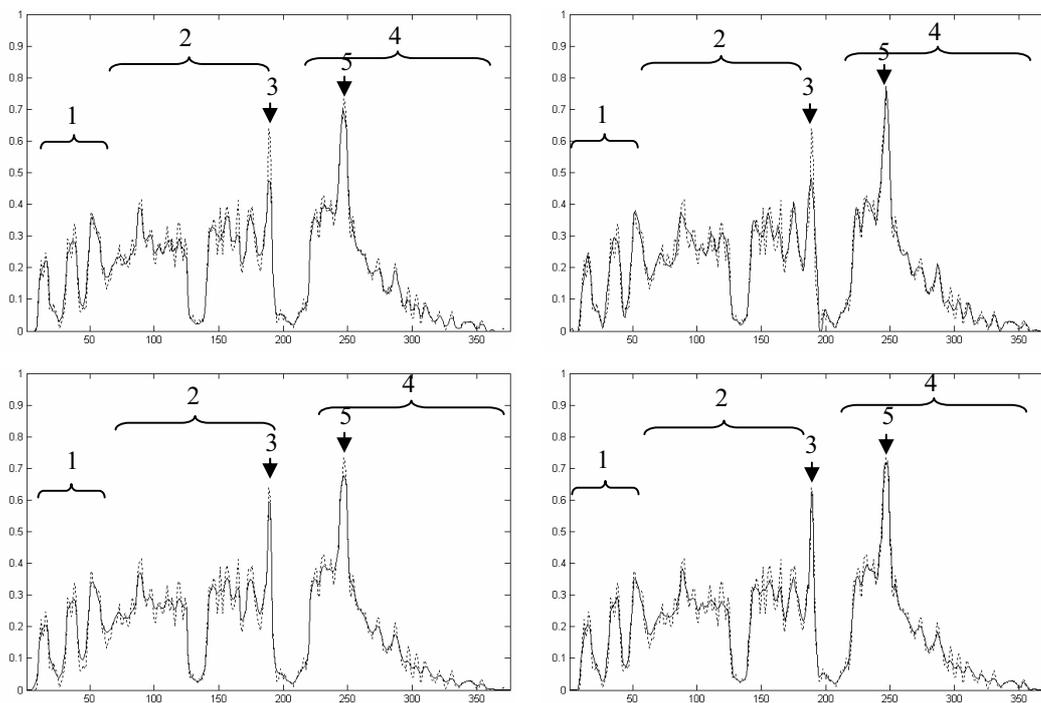

Fig.6 Denoising the liver image: original, AWMF, WSTS, AD and EEAD (from top left to bottom right).

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Fig.7 The 320th column profiles of above results: AWMF, WSTS, AD and EEAD (from top left to bottom right).

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