Medical Image Segmentation Using Deformable Models and Local Fitting Binary

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Abstract—This paper presents a customized deformable model for the segmentation of abdominal and thoracic aortic aneurysms in CTA datasets. An important challenge in reliably detecting aortic aneurysm is the need to overcome problems associated with intensity inhomogeneities and image noise. Level sets are part of an important class of methods that utilize partial differential equations (PDEs) and have been extensively applied in image segmentation. The rest of the paper is organized as follows. Section II reviews the level set method, edge based and region-based active contours and introduces the proposed computational methods. Section III describes the numerical implementation, conclusions are presented in section V.

I. INTRODUCTION

Image segmentation commonly utilizes one of two principal approaches to classify pixels belonging to a particular object or region, either edge-based or region-based. Edge-based segmentation looks for discontinuities in image intensity [4-6], whilst region-based methods look for uniformity within an image sub-region, based on some consistent property such as intensity, colour or texture [2, 6, 9, 10]. Active contours methods, also referred to as deformable models, evolve an image contour from an initial guess using image forces derived from region properties to drive the search to locate the boundaries of the desired objects. Level sets provide an implementation of an active contour method based on regions or edges. A local energy functional has been defined in terms of a contour and two fitting functions that locally approximate the image intensities on either side of the contour. An important characteristic of active contour methods is to identify the appropriate stopping condition for the curve evolution. In this paper we have used the Courant Friedrichs Levy (CFL) condition to establish the necessary conditions for numerical convergence of the level set PDE’s, which also satisfies a criterion for algorithmic stability.

The level set methodology developed by Osher and Sethian [6] has been used in the formulation of both region and boundary based approaches for image segmentation. They offer a robust and accurate technique for tracking boundaries moving under complex motions. Level set segmentation involves solving the energy-based active contour minimization problem by computation of geodesics or minimal distance curves [6, 7] The main idea of the level set method is to represent a closed curve on the plane as the zero level set of a higher dimensional function. The motion of the curve is then embedded within the motion of the higher dimensional surface. This means that the closed curves on a two-dimensional surface are regarded as a continuous surface of a three-dimensional space.
The definition of a smoothing function $\phi(x,y,t)$ represents the surface while the set of definitions $\phi(x,y,t) = 0$ define the curves. Thus the evolution of a curve can be transformed into the evolution of a three-dimensional level set function. Given a level set function $\phi(x,y,t)$ whose zero level set corresponds to a curve, with the curve as the boundary, the whole surface can be divided into an internal region and an external region of the curve.

The evolution (motion) formula of the level set is:

$$\phi_t(x,y,t) + V \nabla \phi(x,y,t) = 0$$

where $V$ denotes a constant speed term that moves the curve either outwards or inwards in search of the contour. A special case is when the motion is determined by the mean curvature [4] where $V = \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right)$ is the curvature of the level-curve of $\phi$ passing through $(x,y)$.

A common edge attraction function can be defined by a positive and decreasing function of the image gradient given by

$$g = \frac{1}{1 + |\nabla G_\sigma * I|^2}$$

Where $G_\sigma$ is a Gaussian kernel with standard deviation $\sigma$ and $G_\sigma * I$, a smoother version of $I$, is the convolution of the image with a Gaussian. The level set method based on the local fitting binary function (LBF) is given by equation [6]:

$$E(\phi, f_1, f_2) = \sum_{i=1}^{2} \lambda_i \int \left[ K_\sigma(x-y) |f_i(y) - f_i(x)|^2 M(\phi(y)) dy \right] dx + V \int |\nabla H(\phi(x))| dx + \mu \frac{1}{2} \int |\nabla \phi(x)|^2 dx$$

$K_\sigma$ is a kernel function (Gaussian in our case) that decreases and approaches zero as $|x-y|$ increases: $f_1(x)$ and $f_2(x)$ approximate the image intensities inside and outside the contour.

To handle topological changes, the contour in the local energy model converts to a level set formulation $\phi$.

For minimizing the LBF model, first the functional form of the model is conformed to the level set method; next in order to solve the level set equation, an implicit finite difference scheme is applied. Gradient descent is employed to minimize the energy functional with respect to the level set function $\phi$ which is as follows:

$$\frac{\partial \phi}{\partial t} = -\delta_{\epsilon} (\phi)(\lambda, e_1 - \lambda, e_2) +$$

$$w \delta_{\epsilon} (\phi) \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) + \mu (\nabla^2 \phi - \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right))$$

where $\delta_{\epsilon}$ is the smooth Dirac delta function, $\nu, \mu$ are constants and $e_1, e_2$ are the functions as follows:

$$e_i(x) = \int K_\sigma(y-x) |f_i(y) - f_i(x)|^2 dy$$

The term $-\delta_{\epsilon} (\phi) (\lambda, e_1 - \lambda, e_2)$ drives the active contour toward the object’s boundary and coefficients $\lambda_1$ and $\lambda_2$ are the weights of the two integrals. The second term has a length shortening (arc length) term [7, 8]. The third term is a regularization term [9], which maintains the regularity of the level set function. Changing the edge indicator function in (1) to region-based is found to be more effective than other approaches in coping with intensity inhomogeneities in the image. The evolution equation with the edge-function is as follows:

$$\frac{\partial \phi}{\partial t} = \alpha g \delta (\phi) +$$

$$w \delta_{\epsilon} (\phi) \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right) + \mu (\nabla^2 \phi - \text{div} \left( \frac{\nabla \phi}{|\nabla \phi|} \right))$$

The coefficient $\alpha$ can be positive or negative, depending on the relative position of the initial contour to the object of interest.

### III. IMPLEMENTATION

The LBF model has been implemented in Matlab. The Courant Friedrichs Levy (CFL) condition has been applied to ensure a stability criterion for our algorithm. The CFL number is:

$$\epsilon = \left( \frac{v \Delta t}{\Delta x} \right)$$

where $v$ is the velocity, $\Delta t$ is the time step, $\Delta x$ is the length interval. The CFL condition is a necessary condition for convergence while solving certain partial differential equations numerically. The main steps of the algorithm can be expressed as follow:

**I. Initialize the level set function $\phi$ to be binary function as follows:**

$$\phi(x,y,t) = \begin{cases} 
-c & x \in \Omega_0 - \hat{\Omega}_0 \\
0 & x \in \hat{\Omega}_0 \\
c & x \in \Omega - \hat{\Omega}_0
\end{cases}$$
Where $c>0$ is a constant, $\Omega_0$ is a subset of the image domain $\Omega$ and $\partial \Omega_0$ is the boundary of $\Omega_0$.

2. Evolve the level set function $\phi$ according to (1)

3. Check with iteration number whether the evolution is stationary; if not increment iteration number and repeat from step 2, else stop.

All partial derivatives can be discretized as central finite differences; also the temporal derivative is discretized as a forward difference. Therefore there are in total six convolutions, of which two ($K_x * I$ and $K_y * I$) need only be computed once before the iteration loop; the other four convolutions are computed at each iteration.

IV. EXPERIMENTAL RESULTS

To validate and assess the robustness of the proposed method we have applied the algorithm to 2D image slices of computed tomography angiography (CTA) datasets. The task was to detect an aortic thrombus from abdominal and thoracic scan sections. The datasets were collected at Lausanne University. The methodology has been tested with several image slices, and for images with intensity inhomogeneity, which are rather noisy and exhibit weak boundaries between the thrombus and surrounding tissue. The benefit of using a binary step function as the initial level set function (value -2 inside and value 2 outside the boundary) is that new contours can emerge readily and the curve evolution is considerably faster than the evolution from an initial function as a signed distance function. To compute the convolutions in $f_1(x)$ and $f_2(x)$ more efficiently, the smoothing kernel is a $\sigma \times \sigma$ mask, where $\sigma$ is the smallest odd number no less than $4\sigma$.

The methodology has been tested with the following set of parameters that were empirically determined:

$$\sigma = 4, \lambda_1 = 0.9, \lambda_2 = 2.0, \Delta t = 0.1, \mu = 1, \nu = 0.003$$

Using a larger time step ($\Delta t$) can speed up the evolution but may cause errors in the boundary location. Therefore the time step $\Delta t$ and coefficient $\mu$ must satisfy ($\Delta t \mu < 0.25$) in order to preserve a stable level set evolution. The algorithm uses a maximum of 300 iterations and figures 1-3 show the number of iterations required to generate a stable boundary (manually selected).

Traditional level set methods [2, 3] generally fail to segment images with significant intensity inhomogeneity. In these circumstances some part of the background/foreground is incorrectly identified as the foreground/background. The results presented in figure 1 show the delineation of the boundary of both the aorta (the bright region) and thrombus (the small darker region adjacent and below the bright region) using the edge indicator function. The weak boundary on the lower edge of the thrombus region has been reliably detected. Figure 2 shows an example where the thrombus covers the entire vessel wall. Again, the algorithm can reliably detect the aorta. Finally, figure 3 shows the operation of the region-based approach, detecting only the thrombus region in the vessel.

Fig. 1 Results of segmentation on two examples exhibiting thoracic aortal aneurism. The final contour computed by the edge attraction function.
V. CONCLUSIONS

We have presented an active contour model based on local binary fitting and an edge indicator which is better adapted to the problem of intensity inhomogeneities in the image. The method was demonstrated by segmenting the ascending and descending thoracic aorta thrombus and the abdominal aorta thrombus with intensity inhomogeneities and weak object boundaries. The time required for segmentation was significantly decreased using an effective convergence criteria. Finally, the effectiveness of the algorithm has been validated on a CTA dataset to assess its performance in terms of efficiency and accuracy. Further work will be to implement an automatic stopping criterion for the contour evolution, and to extend the level set algorithm to operate directly on 3D datasets.

REFERENCES