Modeling of Random Variable with Digital Probability Hyper Digraph: Data-Oriented Approach

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Abstract—In this paper we introduce Digital Probability Hyper Digraph for modeling random variable as the hierarchical data-oriented model.

Keywords—Data-Oriented Models, Data Structure, Digital Probability Hyper Digraph, Random Variable, Statistic and Probability.

I. INTRODUCTION

DATA-ORIENTED modeling is a theory that models the concepts with data structure (Organized Data). In this paper we introduce digital probability hyper digraph for modeling random variable as the hierarchical data-oriented model. Data-oriented modeling is a useful and applied method which we have created and applied it as follows:

- The basic structure of data-oriented modeling has been introduced [1].
- Discrete structures like Probability Digraph, probabilistic language, complete tree walk and n-complete tree walk have been presented for many statistical concepts adaptation with computer. In other words we provide requirement tools, definition and important mathematical theorems for these models [2].
- We have utilized these structures [2] for modeling and then using this model for simulating uniform distribution [3].
- We have presented Data Oriented Models of population and sample named classified image and then provided an algorithm to estimate the distribution of a statistical population based on data-oriented model [4].

- New data oriented modeling of uniform random variable well-matched with computing systems have been presented [6].
- A data oriented model of image has been presented [7].
- Data-oriented modeling of fuzzy controller for controlling the Anti-lock braking system has been introduced [8].
- We have introduced a novel method for improving the uniformity of random number generator named uniformity improving method, or UIM in short, and data-oriented model of uniform random variable named UDPD is simulated by this approach [9].

Our contribution in this paper is to introduce a method for data-oriented modeling of any random variable that conforms with modern computer’s structure. By using this method we can utilize computers in statistical inference and probability with higher speed and efficiency.

The fundamental structures the digital hyper digraph of are defined in section two. Random variable $X$ is modeled with digital hyper digraph in section three. The discussion which is explanatory of the performance of this model and also usage of this model in computer systems is proposed in section four.

II. FUNDAMENTAL STRUCTURES

We define the following concepts of the digital hyper digraph:

Definition:

Let $G= (V, E)$ be a weighted directed graph with nonempty and finite set of vertices $V$ and set $E$ as edges. The weight of each edge is the probability of transition. For example, weight of edge $a \rightarrow b$ is the probability of transition from $a$ to $b$. It is called transition probability and is denoted by $P_{ab}$. $G$ is called probability digraph or prodigraph in short if and only if for any vertex $a \in V$ we have:

$$\sum_{b \in V} P_{ab} = 1.$$ 

Note that $P_{ij} = 0$ if and only if the edge $i \rightarrow j$ is not in digraph then there exists if $P_{ij} > 0$. It is obvious that we can display probability digraph with probability matrix $[P_{ij}]$. 

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**Definition:**

Let \( G = (V, E) \) be a prodigraph. Then \( G \) is a digital prodigraph, diprodigraph (DPDG) in short, if and only if any vertex in \( V \) is a digit. Fig. 1 shows a diprodigraph in binary base.

**Definition:**

Let \( G = (V, E) \) be a diprodigraph, \( G \) is a vertex-diprodigraph if and only if it contains a null vertex as entrance vertex to diprodigraph. This vertex has no digit and determines initial distribution at the moment of entrance to this vertex (or diprodigraph). Figure 2 represents diprodigraph of figure 1 with entrance vertex (null vertex) as a vertex-diprodigraph.

**Definition:**

\( G = (V, E) \) is a Digital Probability Hyper Digraph, diprohydigraph or DPHDG in short, if and only if at least one member of \( V \) be a vertex-diprodigraph and other members of \( V \) are digit or vertex-diprodigraph.

By using diprohydigraph, random variable \( X \) is modeled in the next section.

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**III. RANDOM VARIABLE MODELING**

Suppose random variable \( X \) has probability density function \( f(x) \) which is distributed in \([0, 1]\). The main contribution of this paper is to introduce a diprohydigraph, \( G_X \) as a data oriented model of \( X \).

We define \( G_u = (V_u, P_u) \) as a uniform vertex-diprodigraph, where:

\[
V_u = \left[ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 \right]^T
\]

And

\[
P_u = \begin{bmatrix}
0 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\
0 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\
0 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\
0 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\
0 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\
0 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\
0 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\
0 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\
0 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\
0 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1
\end{bmatrix}
\]

The diprohydigraph \( G_X \) is made by \( G_u \) as shown in Fig. 3 where:

\[
p_i = \int_{0.0}^{0.1+i} f(x) dx.
\]

The obtained Value of Walk from \( G_X \), VOW, [6] is matched with the value of \( X \) and the probability of it conforms to probability density function, \( f(x) \).

**IV. FURTHER WORKS**

We claim that data-oriented models can be used for statistical and probabilistic inferences with greater efficiency in special way. Data-oriented models of uniform variables have been introduced [4,5,6,9], research in the following areas by using the diprohydigraph \( G_X \), will have some valuable benefits:

- Estimation of population distribution
- Simulation of random variables
- Calculation of probabilistic parameters
V. CONCLUSION

In this paper we presented new data-oriented model of random variable $X$ which is named digital probability hyper digraph, diprohydigraph in short. All statistical inferences are made by computing $f(x)$, can be made by data processing with $G_X$. Because adaptation of $G_X$ with computing system, it can utilize in statistical inference by using computer efficiently.

REFERENCES


