Abstract—In biomedical implant field, a new formula is given for the study of Radio Frequency power attenuation by simultaneous effects of side and angular misalignment of the supply/data transfer coils. A confrontation with the practical measurements done into a Faraday cage, allowed a checking of the obtained theoretical results. The DC supply systems without material connection and the data transmitters used in the case of biomedical implants, can be well dimensioned by taking into account the possibility of power attenuation by misalignment of transfer coils.

Keywords—Biomedical implant field, misalignment coils, power attenuation, transmitter and receiver coils.

I. INTRODUCTION

In order to communicate with the implanted electronic circuits in the biomedical field, a system of an external transmitter coil coupled with an implanted receiver coil is used. Often, the coil’s system is also used to directly supply for minimised implant from the electromagnetic field, without using accumulators [1], [2], [4], [5], [7], [11]. So, in data communication with implant or in DC current supply of this one by external field, an effective transfer of radio frequency energy is required. This ensures to the implant a sufficient supply power and a better signal on noise or small error rate of the received data.

The analysis of radio frequency (RF) power transfer, from the external transmitter coil towards the implanted receiver coil, enables us to evaluate the really received power. The RF power attenuation by simultaneous misalignment, in translation and rotation, of the transfer coils is expressed in a single formula. In this original equation, different diameters of coils with various combined side and angular misalignments can be considered. This helps in a better transfer coils systems design, which ensure energy fluidity without rupture of supply or transmitted data towards the implanted circuits.

II. MAGNETIC FIELD RADIATED BY A CIRCULAR COIL

The magnetic potential \( \Phi \) in any point \( P \) of space, around a coplanar coil of \( N_1 \) circular spires traversed by a variable current \( i \), is given as in [5], [6], [9], by:

\[
\Phi = \frac{\mu_0 \sqrt{a N_1 i}}{2 \pi} \frac{S(k)}{\sqrt{r \sin \theta}} \bar{u}_\phi = A_\delta \bar{u}_\delta
\]  

where \( \mu_0 = 4 \pi 10^{-7} \text{ H/m} \), is the permeability of the free air. \( a \), is the radius of the circular coil. \( r, \theta, \phi \) : are the spherical coordinates of the point \( P \). \( \bar{u}_\phi \) is the vector unit in \( \phi \) direction. The function \( S(k) \) is given by:

\[
S(k) = \left[ \left( \frac{2}{k} - k \right) F(\frac{\pi}{2}, k) - \frac{2}{k} E(\frac{\pi}{2}, k) \right]
\]  

where \( F(\frac{\pi}{2}, k) \) and \( E(\frac{\pi}{2}, k) \) are called elliptic integral of 1st and 2nd kind [10], with parameter \( k \):

\[
k = \frac{4a r \sin \theta}{\sqrt{r^2 + a^2 + 2a r \sin \theta}} \leq 1
\]  

III. COUPLED COILS WITH MISALIGNMENT

A misalignment of two coupled coils is shown in Fig. 1. It includes the two cases of misalignment: by translation \( \Delta \) and rotation \( \psi \).
Fig. 1 Coupled coils with misalignment. The C1 and C2 circular coils have a radius a and b respectively, with d as longitudinal inter-coils distance. The analyse uses the spherical and cartesian coordinates.

By using the magnetic potential $\Phi_1$ generated by the C1 coil (1), the magnetic flux through the C2 coil with $N_2$ spires is given by a curvilinear integral along his contour:

$$\phi_m = N_2 \int_{C_2} \Phi_1 \, d\vec{l} = \frac{\mu_0}{2\pi} \int_{C_2} \frac{S(k)}{r \sin \theta} \, u_\theta \, d\vec{l}$$

(4)

Projections of $\vec{r}$ (distance of any point H on C2 coil perimeter to C1 coil center) in the OXY plan and on the OZ axis, give us:

$$\begin{cases} r \sin \theta = \sqrt{(\Delta + b \cos \psi \cos \phi)^2 + (b \sin \phi)^2} = b_\phi \\ r \cos \theta = d - b \sin \phi \cos \phi \end{cases} \quad (5)$$

where $\phi$ is the angle which forms the point H with the C2 coil center P. The coefficient $k$ in (3) becomes:

$$k = \frac{4a b_\phi}{\sqrt{(a + b_\phi)^2 + (d - b \sin \psi \cos \phi)^2}} \quad (6)$$

IV. MUTUAL INDUCTANCE

The mutual inductance is used to evaluate the induced voltage in the receiving coil, and consequently the available power for the implant. The mutual inductance coefficient is obtained from (4) and (5):

$$M = \frac{\phi_m}{i} = \frac{\mu_0}{2\pi} N_1 N_2 b_\phi \frac{1}{\sqrt{b_\phi}} \int_0^{\pi/2} S(k) (\Delta \cos \phi + b \cos \psi) \, d\phi$$

(7)

In case without misalignment ($\Delta = \psi = 0$), we have $b_\phi = b$ and (6) becomes also a constant:

$$k = k_0 = \sqrt{\frac{4a b}{(a + b)^2 + d^2}} \quad (8)$$

The resolving of (7) becomes easier and is given, as in [3], by:

$$M_0 = \mu_0 N_1 N_2 \sqrt{ab} S(k_0) \quad (9)$$

In the general case with combined misalignments, the integral of mutual inductance coefficient in (7) can be done analytically only in approximate form or in a numerical way. In the following results, a numerical integration is adopted in order to obtain various curves of normalised mutual inductance $M / (\mu_0 a N_1 N_2)$ versus angular misalignment $\psi$ (in degree) or versus reduced lateral misalignment $\Delta / a$. The most representative figures are given in Fig. 2, in each of them, M is plotted under three longitudinal inter-coils distances: $d/a = 0.1, 0.4$ and 1.

In Fig. 2a and Fig. 2b M is plotted versus lateral misalignment alone with different radius ratio of coils, 0.5 and 1, respectively. In Fig. 2c and Fig. 2d, M is plotted versus angular misalignment alone with radius ratio of coils, 0.5 and 1, respectively. Fig. 2e to Fig. 2h are plotted in the same conditions as previous, except that simultaneous lateral and angular misalignment are considered. So in lateral misalignment, Fig. 2e becomes dissymmetrical relatively to Fig. 2a. This is not the case of Fig. 2f, relatively to Fig. 2b, which becomes more closer. In angular misalignment, Fig. 2g and Fig. 2h become dissymmetrical relatively to Fig. 2c and Fig. 2d, respectively. Higher values of M are noted in Fig. 2g relatively to Fig. 2c. However, lower values are noted in Fig. 2h relatively to Fig. 2d.
Fig. 2 Normalised mutual inductance versus lateral (\(\Delta/a\)) misalignment and angular (\(\psi\)) misalignment of coils for three longitudinal inter-coils distances (\(d/a\)). In lateral misalignment alone, the radius ratio of coils is 0.5 in (a) and 1 in (b). In angular misalignment alone, the radius ratio of coils is 0.5 in (c) and 1 in (d). (e) and (f) are plotted in the same conditions as (a) and (b) respectively, except that angular misalignment is fixed at \(\psi = 10^\circ\). (g) and (h) are plotted in the same conditions as (c) and (d) respectively, except that lateral misalignment is fixed at \(\Delta/a = 0.5\).

V. POWER ATTENUATION BY MISALIGNMENT COILS

By using Laplace operator \(S\), the voltage \(V_2\) in the receiving coil loaded by the implant \(R\), is the sum of the induced voltage \(M S I_1\) and the voltage at the self induction \(L_2\) caused by the induced current \(I_2\). \(V_2\) can also be calculated from the voltage \(R I_2\) at the implant, as shown in Fig. 3.

\[
V_2 = M S I_1 + L_2 S I_2 = R I_2
\]

which becomes versus transmitter coil current \(I_1\):

\[
V_2 = \frac{R M S I_1}{R - L_2 S}
\]

(11)

The power attenuation \(A_M\) of RF signal due to the misalignment coils, is the ratio of received powers by the implant in the cases of aligned and not aligned coils. So, for a same current \(I_1\) in the transmitter coil and a same impedance of load \(R\) (implant) presented at the receiving coil, we obtain with (11):

\[
A_M = \frac{V_2^2 \text{ with misalignment}}{V_2^2 \text{ without misalignment}} = \left(\frac{M}{M_0}\right)^2
\]

(12)

With (7) and (9), it follows that:

\[
A_M = -\frac{b}{4 \pi^2 S(t_0)} \left[ \int_0^2 \frac{S(k)}{b_k} \left(\Delta \cos \varphi + b \cos \psi\right) d\varphi \right]^{-2}
\]

(13)

This formula gives the power attenuation due to a combined misalignment in translation and rotation of the radio frequency transfer coils.

VI. CONFRONTATION WITH THE EXPERIMENT

An experimental bank, realized at Ensem-Cran of Nancy (in France) [1], [11], allows the study of a 3 MHz radio frequency energy transfer by magnetic coupling. The transmitter and receiver coils have diameters of 6.1 cm and 1.8 cm, respectively. Measurements of transmitted power to a load of 2 K\(\Omega\), are done into a Faraday cage with variable inter coils distance and misalignment. In manipulation, a transmitted power of 10 W is used. The received power \(P_0\) in mW without misalignment of the coupled coils versus longitudinal inter-coils distance \(d\) in cm, is approximated by an analytical expression with correlation rate of 1 [1], [11]:

\[
P_0 (\text{mW}) = 1706 e^{-0.54 d}
\]

(14)

This expression is used as reference for the powers calculation under misalignment of the transfer coils. A numerical integration of (13) gives, in relation to the measured power with aligned coils (reference \(P_0\)), the attenuated power due to a misalignment versus the longitudinal distance between the coupled coils:

\[
P(d) = A_M P_0 (d)
\]

(15)

Fig. 4 Comparison of measured and calculated powers versus longitudinal inter-coils distance, under different misalignment of transfer coils. Both of 0 and 0.5 cm lateral misalignment with fixed 10° as angular misalignment, give confounded powers at measured curves. This is the same case in the calculated ones.

The measured and calculated receiving power under different states of misalignment coils (shifted axis of coils: \(\Delta\) in cm, rotation between coils: \(\psi\) in degrees), are compared in Fig. 4.
A well agreement is noted in order of magnitude and decreasing form of power attenuations between the practical and theoretical results. The curves with $(0 \text{cm}, 10^\circ)$ and $(0.5 \text{cm}, 10^\circ)$ misalignments are confounded in case of measure and also in case of calculation. In lateral misalignment alone ($\Delta=0.5\text{cm}$, $\psi=0^\circ$), calculated and measured curves are confounded.

In order to stabilize the efficiency of RF connection in presence of possible misalignment coils, it is necessary to satisfy a good tolerance in coils misalignment. So the evaluation of a sufficient power to transmit can be done analytically by (13) or graphically from mutual inductance curves as in Fig. 2.

REFERENCES


