World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:7, No:8, 2013

Mathematical Rescheduling Models for Railway Services

Zuraida Alwadood, Adibah Shuib, and Norlida Abd Hamid

***Abstract*—**This paper presents the review of past studies concerning mathematical models for rescheduling passenger railway services, as part of delay management in the occurrence of railway disruption. Many past mathematical models highlighted were aimed at minimizing the service delays experienced by passengers during service disruptions. Integer programming (IP) and mixed-integer programming (MIP) models are critically discussed, focusing on the model approach, decision variables, sets and parameters. Some of them have been tested on real-life data of railway companies worldwide, while a few have been validated on fictive data. Based on selected literatures on train rescheduling, this paper is able to assist researchers in the model formulation by providing comprehensive analyses towards the model building. These analyses would be able to help in the development of new approaches in rescheduling strategies or perhaps to enhance the existing rescheduling models and make them more powerful or more applicable with shorter computing time.

***Keywords*—**Mathematical modelling, Mixed-integer programming, Railway rescheduling, Service delays.

I. INTRODUCTION

IN rail transportation system, train punctuality is a very important attribute. When people choose to commute by train, they expect to arrive at their destination at the scheduled time. However, when disruptions occur, the control managers need to reshuffle train orders, make unplanned stops, re-route and even delay or cancel scheduled services.

In practice, train rescheduling activities do not start from scratch. Rather, the new provisional timetable is obtained based on the existing schedule, of which the information is gathered from the time-space diagram. Rescheduling process may involve adjusting the train schedules, adding or cancelling a service trip, changing the platform, relocating the stops or any other necessary modifications. As the process is very complex and time consuming if it is to be done manually by human, therefore, mathematical models offer a resolution technique which saves time in the process of searching for the optimal solution.

The purpose of the work presented in this paper is to address the supporting elements in mathematical optimization models that have been applied in rescheduling railway services when disturbances arise. Generally, the mathematical programming models intend to minimize the consequences of the service disruptions, particularly in keeping the delays at the optimum level. In this paper, a cross-analysis on the common parameters and variables that are used in the model formulation will be presented. The significant contribution of the review is to provide comprehensive analyses that can provide the basis for the development of new models in the research area.

This paper first describes the model approach used in the selected literatures. It is followed by the cross analysis which compares the criteria relevant in the formulation of integer programming (IP) and mixed-integer programming (MIP) models, such as the decision variables, sets and parameters. The complexities of the models are then briefly described, before the final remark and future direction of the study are presented.

II. MATHEMATICAL MODEL FORMULATION

A mathematical programming model is widely formulated for the railway rescheduling problem with the aim to minimize service delay. Mathematical programming model is usually selected to be the tool for the rail traffic analysis because its use has been facilitated by major advances in methods of modeling objectives and real system constraints into structured formulation. It offers unique advantages over other quantitative methods because it can address the highly combinatorial problem and highly interlinked nature of the rail traffic system.

Among the various types of quantitative models used in rescheduling railway services, a study done by Alwadood et al. [1] has shown that IP and MIP are widely used in formulating the optimization problem. They are technically chosen because the models are able to accommodate the linearity of the objective functions and constraints. The following section summarizes and compares the criteria that are relevant in the formulation of the mathematical models which are used in selected literatures of train rescheduling problem. Among the published results are the works of Narayanaswami and Rangaraj [2], Caimi *et al*. [3], Acuna-Agost [4], Stanojevic *et al.* [5], Murali [6], Afonso [7], Zhou and Zhong [8], Tornquist and Persson [9] and Tornquist and Persson [10].

Zuraida Alwadood is currently pursuing doctoral degree program in mathematics at the Universiti Teknologi Mara, Malaysia (e-mail: zuraida794@salam.uitm.edu.my).

Adibah Shuib is an Associate Professor at Mathematics Studies Center, Faculty of Computer and Mathematical Sciences, Universiti Teknologi Mara, Malaysia (e-mail: adibah253@salam.uitm.edu.my).

Norlida Abd. Hamid is an Associate Professor at Department of Transport, Logistics and Operation Management, Faculty of Business Management, Universiti Teknologi Mara, Malaysia (e-mail: norlida054@salam.uitm.edu.my).

A. Model Approach

Over the last decades, a wide range of mathematical model formulations have focused on the train rescheduling problem in managing train service delays. There are several model approaches for rescheduling has been proposed. A recent MIP model which is established by Narayanaswami and Rangaraj [2] include disruption and conflicts-resolving constraints in the model itself. The novelty of the method ensures that only disrupted trains will be rescheduled, leaving alone the unaffected trains. This is done by partitioning train sets into conflicting and non-conflicting trains by means of linear constraints. To solve the model, the approach of a traveling salesman problem (TSP) was applied. As it is reduced to a TSP in a polynomial time, then the model is NP-Complete. Since the model used a small size fictive data, it should be extended to a larger scale of real data so that the validity of the model can be proven.

A model predictive control approach which has been proposed by Caimi *et al*. [3] attempts to reschedule trains by a discrete-time control. In this approach, a set of alternative blocking-stairways is used as the basis in each rescheduling step. They are then followed by several planning steps which are linked to each other by different temporal scopes. The concept of bi-level multi-objective formulation sees that three criteria were considered separately in the first level. They are then aggregated into one objective function as a weighted sum. This method is very suitable when it comes to optimizing a multi-criteria objective because the choice of weights depends absolutely on the dispatcher or the experts, based on the importance of the criteria valued.

Tornquist and Persson [10] formulate their combinatorial problem of real-time disturbances in railway traffic rescheduling with the objective of minimizing the total service delay. An iterative two-level process is used to solve the problem. The order of meeting and overtaking of trains on the track section is done in the upper level process using simulated annealing and tabu search while the lower level process determines the start and end time for each train. Local reordering trains are used to obtain good quality solutions.

They extended their work later using goal programming in the attempt to satisfy two different objective functions [9]. The first goal of minimizing the total service delay is for the use of current practice while the second objective of minimizing the total service cost is valued for future planning. This type of multi objective optimization is categorized as a multi-criteria decision analysis (MCDA) which is normally used to handle more than one conflicting objective measures. They discovered a relationship between certain disturbance characteristics and the ability to find good solutions within short time.

Preserving the goal of minimizing the total service cost, the mathematical model was later extended by Acuna Agost [4] who did some modifications on some constraints to form groups of blocks, known as a section. As a section consists of more than a single block, therefore the constraints need to be amended to accommodate spacing area between blocks. Due to the large number of variables and constraints, a constraint programming (CP) model was used as an alternative to the former MIP method. A CP engine performs a set of logical inferences to reduce the available options for the remaining variables' domains. Thus, the technique allows the problem to have less variables and constraints compared to the MIP. It is also able to formulate the large instances of railway rescheduling problem. As a result of these advantages, unlike MIP, it requires less memory.

Murali [6] presents an IP-based railway capacity management model with the objective of minimizing the sum of total travel time and delay for trains, utilizing rail track capacity through efficient routing and scheduling. The objective function of the IP is expressed as the weighted sum of each train, assuming all trains are of equal importance. As the routing and scheduling was done at a medium to large-scale rail network, it involves thousands of binary variables and constraints. Thus, a technique known as *Aggregation* is used to combine the portion of the network under consideration into a single node, so that the number of nodes and arcs in the general network could be possibly reduced. This effort has led to the development of train movement plans in complex network based on the possibility of real-time rerouting trains to alternative tracks during service disruptions.

A real-time optimization using a weighted sum is also put forward by Afonso [7] who introduced a capacity conflicts in the problem when two trains meet or pass at a segment track. The mathematical model is aimed to minimize the total weighted tardiness in order to improve quality in the train rescheduling process of which it is mostly done manually by human operator. A heuristic approach was presented to find a feasible solution in short computing time while search methods were aimed for optimal or near optimal solution.

Zhou and Zhong [8] formulated a train timetabling problem as a resource-constrained project scheduling with the aim to minimize the total travel time. The scheduling problem is modeled as a generalized resource-constrained project with minimum time lags, which refer to the time units between two consecutive tasks in a train route. The limited resources are referred to the segment and station headway capacities. The feasible solution for the large-scale instances of the integer programming model with computational time and space constraints are obtained by using a branch-and-bound algorithm.

Stanojevic *et al.* [5] presented an integer programming model to determine a timetable for a set of trains, subject to some operational constraints, within the track capacities. The model formulation attempts to minimize the sum of train delays for a single track linking a number of stations. In addition to compromise with the scheduled times of the trains, the model also treats the problem that involve total waiting time incurred during service disruptions.

Table I presents the summary of the relevant literatures discussed earlier, along with the description of test data used.

TABLE I
OVERVIEW ON RELATED WORK USING INTEGER AND MIXED INTEGER PROGRAMMING MODELS

Publications	**Research objectives**	**Solution approach**	**Sample data**
Narayanaswami and Rangaraj (2013)	To minimize total delay of all trains	Heuristics	Fictive data
Caimi et al. (2012)	To maximize train punctuality and reliability	Heuristics	Swiss Federal Railways
Acuna Agost (2010)	To minimize cost of delays and cost of final delays.	Local search/branching	Railway network in France and Chile
Stanojevic et al. (2010)	To minimize the sum of train delays	(not mentioned)	Serbian Railways
Murali (2010)	To minimize total travel time and delay.	Linear relaxation and routing constraints	Union Pacific-Alhambra Rail Network
Afonso (2008)	To minimize the total weighted tardiness	Heuristics/Branch-and-bound	Fictive data
Zhou & Zhong (2007)	To minimize the total travel time.	Branch-and-bound	Single-track rail line at Fujian, China
Tornquist & Persson (2007)	To minimize service delays and costs in rescheduling railway traffic.	Heuristics	Sweden railway network
Tornquist & Persson (2005)	To minimize the total delays in railway traffic due to disturbances.	Branch and bound	Sweden railway network

The mathematical solution approaches which yields the optimal or near-optimal solutions are presented explicitly next to the research objectives column of which majority are concerned with the aim to minimize the average delays and total delays. The rightmost column shows the sample of the real or fictive data being used for the models' validity experimentation.

B. Decision Variables

As all the research works aim to arrive at provisional timetables which are able to minimize service delay, then it is expected that the most important decision variables in the model formulation should be the *start time* and *end time* of the event for train. Other decision variables relate to which train that is to be used, which track the train should run on, which station the train should be leaving from or waiting at, among others.

For the decision variables involving *'yes'* or *'no'* answer, binary variables of *'1'* and *'0'* are used. The analysis of decision variables used in the mathematical programming models in the selected literatures is shown in Table II.

C. Model Sets and Parameters

Basically, the mathematical models are based on three sets namely train, block/segment and station, as shown explicitly in Table III. The set of train contains all types of train running on the rail track which may be in outbound/inbound direction, or in some works it is known as upline/downline direction. The set of block or segment are the collection of all sections of railway tracks which can only be occupied by one train in a direction, at any particular time. The set of station are the entire terminal meet points for the trains within the relevant area of study. These three sets are dominant in all the models by the seven selected literatures.

TABLE II
CROSS ANALYSIS OF DECISION VARIABLES USED IN MODELS OF RELATED WORKS

	AUTHOR(S)								
DECISION VARIABLES	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
start time of event for train	•	•	•	•	•	•	•	•	•
end time of event for train	•	•	•	•	•	•	•	•	•
event uses track *t* or not		•	•	•	•		•	•	
event on block occurs before or after an event			•				•	•	•
magnitude of delay for event for train	•	•	•					•	•
event is rescheduled to occur after an event or not	•		•					•	
an unplanned stop is added during event or not			•				•		
event uses train *h* or not				•	•				
train *h* is leaving or waiting at station *k*				•					

[1] Narayanaswami & Rangaraj, 2013; [2] Caimi et al., 2012; [3] Acuna Agost, 2010; [4] Stanojevic et al., 2010; [5]Murali, 2010, [6] Afonso, 2008; [7] Zhou & Zhong, 2007; [8] Tornquist & Persson, 2007; [9] Tornquist & Persson, 2005.

TABLE III
CROSS ANALYSIS OF SETS USED IN MODELS OF RELATED WORKS

	AUTHOR(S)								
SETS	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
train	•	•	•	•	•	•	•	•	•
block/segment		•	•	•	•	•	•	•	•
station	•	•		•		•	•	•	
events for train			•					•	•
events for block/segment			•					•	•
events in stations			•						

[1] Narayanaswami & Rangaraj, 2013; [2] Caimi et al., 2012; [3] Acuna Agost, 2010; [4] Stanojevic et al., 2010; [5]Murali, 2010, [6] Afonso, 2008; [7] Zhou & Zhong, 2007; [8] Tornquist & Persson, 2007; [9] Tornquist & Persson, 2005.

However the sets of events for each of these train, block or segment and station are only used in some of the studies. These sets of events are the resource requested by a specific train, block or segment and station, respectively.

Due to the dimensions of the problem and the complex nature of the IP and MIP models, the selection of the parameters that would be taken into the model formulation needs to be closely examined. Table IV provides the analysis of the model parameters used in the selected research works. Several models share almost similar parameters but there are also models which incorporate a set of unique parameters as a result of improving an existing model or introducing a hybrid mathematical model. For instance, Acuna Agost [4] introduced the parameters of braking and accelerating time as a result of unplanned stops. In addition to this, a list of unique binary parameters has also been put forward. On the other hand, Tornquist and Perrson [10] introduced the parameters for train connection in order to handle the objective function of costs for missing connections. Having said this, to come up with a newly-developed model, it is recommended that the common listed parameters should be first included to ensure the sensibility of the model. This will then be followed by introducing fresh elements in the model formulation to offer a unique research novelty.

D.Model Complexity

Railway rescheduling is a very challenging task as it normally involves real-time schedule modification within its highly interconnected railway network. Mathematically, this is considered as a difficult, combinatorial and strongly constrained problem. The model's constraints will involve a large number of hard (operational) and soft (desirable) constraints and the complexity of problem increases with the number of decision variables and constraints. Modeling and solving such railway rescheduling problem is thus considered a highly complex task and an NP-hard problem.

In addition to this, train rescheduling model needs to be run at macro level of railway networks, so as to meet the real-world application demands.

The routing and scheduling tasks are very challenging because it normally involves large combinatorial optimization problems. In the early stage, it demands the ability to formulate the real problem into a mathematical representation, incorporating all the factors influencing the decision variables, not forgetting the constraints and uncertainties governing the problem. In later stage, it demands the ability to solve the problem and generate the feasible or optimal solution within a short time frame, using search method or exact method, whichever suits the model.

The algorithm intends to solve railway traffic conflict as fast as possible so as to assist the dispatcher in the resolution process. Solutions to conflicts may involve many combinations of stations, departure and arrival times, direction of routes and location of conflicts, especially when the disruption involves a train that interferes with other trains. These lead to a large number of feasible solutions. Therefore, depending on the chosen solution for a conflict, optimal solutions are normally unattainable in large-scale and complex instances.

Table V shows the number of sets of decision variables and constraints for the research works discussed. It is important to bear in mind that each set would contain hundreds or thousands of variables and constraints.

As the problem increases exponentially with additional nodes, it becomes an NP-hard problem. Many existing techniques that are used to solve such a large combinatorial problem would consume a huge amount of computation time and require a large memory space to produce solution with inconsistent solution accuracy. As it is very difficult to get the best solution, programs that allow time savings are often developed to find a reasonable and acceptable solution.

Alternatively, to reduce the computational complexity and implementation, in some cases, certain technique and procedures are used to simplify the program. Among these include the aggregation technique [6], incorporating priority rules into simulation framework [11], solving optimality for a specific case only [10] and decomposing the complex problem into sub problems, where branch-and-bound is one of the common chosen techniques [8].

TABLE IV
CROSS ANALYSIS OF PARAMETERS USED IN MODELS OF RELATED WORKS

	AUTHOR(S)								
PARAMETERS	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
initial start of event of train as in timetable	•	•	•	•	•	•	•	•	•
initial end of event of train as in timetable	•	•	•		•	•	•	•	•
earliest delayed start of event of train	•	•	•		•			•	•
earliest delayed end of event of train	•	•	•	•				•	•
minimum separation time if trains meet on a track	•		•			•	•	•	•
minimum separation time if one train is following the other, on a track	•		•			•	•	•	•
minimum running (waiting) time for event between stations	•		•			•	•	•	•
penalty/cost per time unit for delays		•	•					•	•
large positive constant (covers the time period)	•		•		•		•		•
time horizon/time index	•	•		•	•		•		
dwell time				•		•	•		
last event on a train			•	•					
next event on a train				•					
capacity of meet point			•	•	•	•			
direction of event of train	•	•	•				•	•	
length of each track					•			•	
length of each train								•	
train starts from a station where the start time is fixed, or not									•
there is a connecting train of event for train, or not		•							•
types of track: uni-directional or bi-directional			•						
an event is allowed to change track, or not			•						
number of parallel tracks on a block			•						
an event occurs in station, or not			•						
minimal extra time for braking			•						
minimal extra time for accelerating			•						
an event is a planned stop, or not			•						
velocity limit of block/segment					•				

[1] Narayanaswami & Rangaraj, 2013; [2] Caimi et al., 2012; [3] Acuna Agost, 2010; [4] Stanojevic et al., 2010; [5]Murali, 2010, [6] Afonso, 2008; [7] Zhou & Zhong, 2007; [8] Tornquist & Persson, 2007; [9] Tornquist & Persson, 2005.

TABLE V
SETS OF DECISION VARIABLES AND CONSTRAINTS IN MODELS

	AUTHOR(S)								
	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
Sets of decision variables	12	7	7	4	N/A	5	5	7	7
Sets of constraints	13	7	34	6	10	7	10	20	13

III. CONCLUSION

This paper provides the overview of the mathematical railway rescheduling techniques used in past research, focusing on the criteria that are relevant in the formulation of the models. It can assist researchers in formulating structured models by means of the gap analysis done on the model variables and parameters. In this context, the review justifies the selection of variables and parameters that could be used in a model building. Upon developing an enhanced or hybrid mathematical model that could minimize service delays for the passenger trains, the solution approaches to the mathematical model will then be proposed to the railway operator.

The work is part of the study that is currently being conducted on train rescheduling model to cope with the railway service disruptions within Malaysia commuter rail system. The models and solution approaches will be presented in the near future.

ACKNOWLEDGMENT

The work presented in this paper is part of the Research Intensive Faculty (RIF) project funded by Research Management Institute of Universiti Teknologi MARA Malaysia. The authors gratefully acknowledge this support.

REFERENCES

[1] Z. Alwadood, *et al.*, "A review on quantitative models in railway rescheduling " *International Journal of Scientific and Engineering Research*, vol. 3, 2012, pp. 1-7.

[2] S. Narayanaswami and N. Rangaraj, "Modelling disruptions and resolving conflicts optimally in a railway schedule," *Computers and Industrial Engineering*, vol. 64, 2013, pp. 469-481.

[3] G. Caimi, *et al.*, "A model predictive control approach for discrete-time rescheduling in complex central railway station areas," *Computers and Operations Research*, vol. 39, 2012, pp. 2578-2593.

[4] R. Acuna-Agost, "Mathematical modeling and methods for rescheduling trains under disrupted operations," Dissertation PhD Thesis, Université d'Avignon et des Pays de Vaucluse, Avignon, France, 2010.

[5] P. Stanojevic, *et al.*, "Mathematical optimization for the train timetabling problem," *Mathematica Balkanica,* vol. 24, 2010, pp. 303-312.

[6] P. Murali, "Strategies for effective rail track capacity use," Dissertation Dissertation, Faculty of the USC Graduate School, University of Southern California, California, 2010.

[7] P. A. Afonso, "Railway traffic management," Dissertation MSc, Universidade Tecnica de Lisboa, 2008.

[8] X. Zhou and M. Zhong, "Single-track train timetabling with guaranteed optimality: Branch-and-bound algorithms with enhanced lower bounds," *Transportation Research Part B,* vol. 41, 2007, pp. 320-341.

[9] J. Tornquist and J. Persson, "N-tracked railway traffic rescheduling during disturbances," *Transportation Research Part B,* vol. 41, 2007, pp. 342-362.

[10] J. Tornquist and J. A. Persson, "Train traffic deviation handling using Tabu Search and Simulated Annealing," *Proceedings of the 38th Hawaii International Conference on System Sciences (HICSS38),* 2005.

[11] M. J. Dorfman and J. Medanic, "Scheduling trains on a railway network using a discrete event model of railway traffic," *Transportation Research Part B,* vol. 30, 2004, pp. 147-161.