
Adil Salam, Nadia Bhuiyan, and Gerard J. Gouw

Abstract—The Canadian aerospace industry faces many challenges. One of them is the difficulty in estimating costs. In particular, the design effort required in a project impacts resource requirements and lead-time, and consequently the final cost. This paper presents the findings of a case study conducted for recognized global leader in the design and manufacturing of aircraft engines. The study models parametric cost estimation relationships to estimate the design effort of integrated blade-rotor low-pressure compressor fans. Several effort drivers are selected to model the relationship. Comparative analyses of three types of models are conducted. The model with the best accuracy and significance in design estimation is retained.

Keywords—Effort estimation, design, aerospace.

I. INTRODUCTION

The Canadian Aerospace industries employed 81,050 Canadians in 2010[1], and according to [2], 55% of the jobs in 2007 were in the province of Quebec. The aerospace industry is an important element of the Canadian economy: in 2010, it generated $21 billion dollars of revenue, and has exported over $15 billion dollars [1].

Estimating the cost is a key element of many engineering and managerial decisions [3]. As target costing focuses on the product and its characteristics, [4], those characteristics will be the basis of estimating the cost. Therefore, in order to develop an accurate cost model, the design efforts, i.e. the characteristics of the product that will influence the cost [5], have to be defined. In a regression based model they are used to develop the final target cost model, or the cost estimating relationship (CER). The target cost of the product can also be broken down into design and manufacturing cost. In order to understand the cost of design of a product, the time, in person hours, also known as design effort, is required. If it can be established within a reasonable degree of accuracy, the ability to schedule, forecast, conducting trade-offs, amongst others, will become much easier, and doing so correctly is critical [5]-[8].

The focus of this paper is on the development and comparison of parametric CERs to estimate design effort for an aerospace component. The methodology can be adapted to other domains.

II. METHODOLOGY

Parametric cost estimation is a technique that can be used to develop an estimate based on the statistical relationship of the input variables [9], [10]. In the context of projects, it determines estimates for parameters (e.g. cost or design effort) using historical data and/or other variables. It can be used to determine the feasibility of a project, to determine a budget, and to compare projects (products), among others [11]. The input variables, which are the design efforts, will be used to formulate the cost model or the cost estimating relation (CER). The parametric CERs are commonly utilized to estimate the cost during the design phase of a product, when only few, yet key design parameters or input variables (in this case, design efforts) are known. The generic formula is as follows:

\[ \text{CER} (y) = f(x) \]  

The CER is a function of its input variable(s) \( x \). Simple CERs depend on a single design effort driver, whereas the complex CERs depend on multiple design efforts [10]. By identifying the design efforts, parametric models can be developed.

In this paper, two models based on linear regression are selected to formulate the CER. The other parametric CER studied in this paper is that of a complex non-linear model (CNLM), which is also a regression based model.

A. CER Based Upon Linear Regression

The simple CER based on a linear regression (LR) model can be denoted as following.

\[ \hat{y} = \beta_0 + \beta_1 X_1 \]  

where \( \hat{y} \), Predicted design effort; \( \beta_0 \), Intercept; \( \beta_1 \), Slope; \( X_1 \), Design effort driver.

As can be seen from (2), there is only one independent variable, hence only one design effort driver. However in many cases, several design efforts are selected, thus the CER will be complex. A more complex form of the CER using regression can be denoted by the multiple linear regression model (MLRM). The MLRM function is as follows:

\[ \hat{y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k \]  

where \( \hat{y} \), Predicted design effort dependent on \( k \) predictor values; \( \beta_i \), Regression coefficients; \( X_i \), \( k^{th} \) independent variable.
The regression coefficients are deduced by the method of least squares, [12].

Another complex CER will be developed based upon a standard non-linear regression model (NLR model). The purpose of developing another parametric CER is to use it as a comparison mechanism to that based upon the MLRM, in terms of accuracy of prediction. In statistics, the NLR is a type of regression that utilizes data modeled in the form of non-linear combinations [13]. Below is the formula for the NLM used in this study:

\[ \hat{y} = \beta_0 x_1^{\beta_1} x_2^{\beta_2} x_3^{\beta_3} \]  

(4)

where \( \hat{y} \), Predicted design effort; \( x_m \), m\(^{th}\) independent variable; \( \beta_m \), Regression coefficient.

Since (4) is represented in a non-linear form, it must be transformed into a linear model to conduct the regression analysis. The linear equation generated is shown below:

\[ \ln \hat{y} = \ln(\beta_0) + \beta_1 \ln(x_1) + \beta_2 \ln(x_2) + \beta_3 \ln(x_3) \]  

(5)

Thereafter, as the equation is in the standard linear regression form, the least squares method will be utilized to calculate the regression coefficients. As the models described in this section are based upon linear regression, it is important that the linearity and normal assumptions are met.

1. Linearity Assumption

In order to determine if the function is linear for a given case of the MLRM, scatter plots or residual plots can be made. In the case of the scatter plot, the standardized residuals could be plotted against the non-standardized predicted value. For a linear function, the scatter plot should not have any curvilinear patterns.

Another graphical manner to statistically prove the validity of the MLRM is to create statistical process control (SPC) charts. In the context of this research, the predicted values against the actual values of design effort will be plotted. The errors or residuals will be the deviation from the line \( \ln(\hat{y}) = \ln(\text{Actual}) + \varepsilon \). The expected error, assuming a normal distribution, is zero, i.e. \( \varepsilon \sim N(0, \sigma^2) \).

2. Normality Assumption

The second assumption that must be validated is that the error values follow a normal distribution. The test to determine error normality requires the coefficient of correlation, \( r \). The value of \( r \) is calculated from the following equation.

\[ r = \pm \sqrt{R^2} \]  

(8)

where \( R^2 \), Coefficient of determination. The value of \( R^2 \) is calculated from the following equation:

\[ R^2 = \frac{SSR}{SSTO} \]  

(9)

where SSR, Regression sum of squares; SSTO, Total sum of squares.

This will also involve a hypothesis test in which the critical values for the correlation coefficient, \( r \), described by [15], are to be compared against the resulting correlation coefficient, \( r \), from the generated regression model. The \( H_0 \) assumes that the error has a normal behavior. The \( H_1 \) assumes that the error does not have a normal behavior. The outcome of the test is as follows:

\[ \begin{align*}
    &\text{if } r \geq r_1(1-\alpha) \quad \text{conclude } H_0 \\
    &\text{if } r < r_1(1-\alpha) \quad \text{conclude } H_1
\end{align*} \]

(10)

The value of \( \alpha \), also known as the Type 1 error, is set and is the probability to reject \( H_0 \) given \( H_0 \) is in fact true [16]. These values will be used to fulfill the normality assumption. The next model studied is the complex non-linear model.

3. Analysis of Variance

ANOVA for an MLRM will help to determine which cost drivers have a statistical significance [12]. The ANOVA will indicate, with the use of the F-test for linear regression, if the CER only contains statistically significant cost drivers [17]. According to [12], the \( p \)-value, one of the values generated in the ANOVA, will indicate the statistical significance of a given parameter, which in this case, the cost drivers. The following set of equations will be utilized to determine if the selected input variables are statistically significant.

\[ \begin{align*}
    &\text{If } p_i \geq 1 - \text{Confidence Level} \\
    &\text{then Factor is statistically significant} \quad (11)
\end{align*} \]

\[ \begin{align*}
    &\text{If } p_i < 1 - \text{Confidence Level} \\
    &\text{then Factor is not statistically significant} \quad (12)
\end{align*} \]

For the purpose of this study, the selected confidence level, which would relate to the reliability of the estimate, is set at 75%. It should be noted that the subscript, \( i \) for the term, \( p_i \), represents the \( i^{th} \) cost drivers, as several cost are considered in the complex CER.

After creating the regression model and computing the ANOVA, if there is a single value of \( p_i > 0.25 \), the regression...
analysis will have to be repeated by eliminating the selected cost driver with the greatest value of \( p_i \). This procedure is repeated until all the values of \( p_i \geq 0.25 \). The resultant equation will be utilized to predict the target cost solely upon statistically significant cost drivers at a specified confidence level, 75% in this case.

### B. CER Based Upon a Complex Non-Linear Model

The CNLM used in this study has the following notation.

\[
\hat{y} = a_0x_1^{b_0} + a_1x_2^{b_1} + a_2x_3^{b_2} + c_1x_1^{d_1}x_2^{d_2} + c_2x_2^{d_1}x_3^{d_2} + c_3x_1^{d_2}x_3^{d_3} + c_4x_1^{d_1}x_2^{d_2}x_3^{d_3} + c_5x_1^{d_1}x_2^{d_2}x_3^{d_3} + e_0x_1^{f_1}x_2^{f_2}x_3^{f_3} \]  

(13)

As was the case in the regression models, the terms \( x_i \) represent the design effort drivers, and remaining terms are the constants. As this equation is not in the form of a regression model, the constants will have to be determined analytically. The manner is determining the constants will be using the gradient descent algorithm (GDA). The GDA is an optimization tool to find the local minima of a function [18]. In order to determine the constants, the function to be minimized is the square error of the predicted versus the actual costs (i.e. \( \sum(y - \hat{y})^2 \)). The gradient, \( \nabla \) for each of the constants will calculate the amount the constant has to be changed (i.e. delta) in order to minimize the function, the square error. Furthermore, the value of the constant will be adjusted by multiplying it by a step rate, \( \eta \). Each of the constants will be adjusted each iteration until the specified stopping criterion (i.e. acceptable change in error) is fulfilled.

### III. CASE STUDY

Pratt and Whitney Canada (PWC) is renowned a global leader in the design and manufacture of jet engines. For this specific case, parametric CERS are developed, to estimate the required effort (in terms of person-hours) in order to design, a compressor fan. The specific fan of interest in this case is the integrated blade-rotor low-pressure compressor (IBR LPC) fan.

#### A. Compressor Fan

The design of the CF is a complex process which involves expertise in several domains. However, there are four departments, which together carry out the majority of the effort to design the CF: design, aerodynamics, structures (analytical), and drafting departments. The design department is responsible for ensuring that all components mesh well with the rest of the engine. In order to estimate the design effort, the principal factors that potentially may have significant effect on design effort estimation, known as the design effort drivers, must be identified. Upon conducting extensive interviews key experts and discussions with managers, designers, and project engineers at PWC, the following four factors were identified as the effort drivers to be used for this model: Type of design (TD); Degree of change (DC); Experience of departmental personnel (DE) and Concurrency (Con).

It should be noted that even though, concurrency was initially selected as a design effort driver, our research for this case, indicates it not to be a significant factor, and will therefore not be considered [19].

1. **Type of Design**

The effort required in designing a component will vary depending on the type of design (TD). As can easily be understood, the effort required from an initial design will not be similar to that of a redesign. Therefore, for each of the design jobs (DJs), they were assigned one of the following attributes.

1. Initial Design → 1
2. Redesign → 2

It should be noted, that the redesign can be further divided based upon its degree of change.

2. **Degree of Change**

This purpose of this factor is to attribute a value for the level of rework created from the initial design to a redesign, or from a redesign to a second redesign. If a major change or degree of change (DC) is required from the initial design, the amount of rework generated would be expected to be different (greater) than if only a minor change is required. The values attributed to the designs are shown below.

1. Initial design → 1
2. Redesign with minor modifications → 2
3. Redesign with major modifications → 3
4. Experience of Departmental Personnel

The amount of experience an individual has will also play a major role when determining the time they require in order to complete their work. This may be contrasted to the notion of the learning curve. The learning curve was first introduced by [20]. The following attributes for the degree of experience (ED) is assigned.

1. 0-2 years of experience → 1
2. 3-4 years of experience → 2
3. 5 + years of experience → 3

As the effort required for a department may be divided into several personnel, the experience level for the job will be determined as the weighted average of experience from all the individuals working on the job. The formula used to calculate the degree of experience is as follows:

\[
ED = \sum_{i=1}^{n}(\% \text{hours of } i) \times \text{(experience of } i) \]  

(14)

### IV. RESULTS AND ANALYSIS

As mentioned in the previous section, three design effort drivers were identified for the CF: type of design (TD), degree of change (DC), and degree of experience (ED). Table I presents the masked data obtained from PWC.

As can be seen from Table I, the term “masked design effort” is used, as discussed in Section II A 1 to maintain confidentiality. From this point forward, it will simply be referred to as “design effort.”

Table I: Historical Data for Various Design Jobs

<table>
<thead>
<tr>
<th>DJ</th>
<th>X1:TD</th>
<th>X2:DC</th>
<th>X3:ED</th>
<th>y: Masked Design Effort</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>259.07</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>121.39</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>288.73</td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>249.98</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
<td>2</td>
<td>2.25</td>
<td>462.65</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
<td>3</td>
<td>2.65</td>
<td>480.04</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
<td>1</td>
<td>2.44</td>
<td>734.14</td>
</tr>
</tbody>
</table>

where Xn: design effort drivers; y: masked design effort

The regression models are developed using the method of least squares. As there are a total of seven design jobs, the models are developed containing the full data set (Trial 0). Two more trials are conducted where one job is randomly removed and used to validate the results. The first sub-sample removed DJ C, and is referenced as Trial 1 from this point forward. Similarly, another sub-sample was created, namely Trial 2, which omitted DJ E. The following analysis presented in that of the linear regression based CER.

A. Linear Regression Based CER

The resulting equation for the three trials based upon the linear regression based CER are as follows:

\[
\hat{y}_{LRT, 0} = 2389.84 - 454.48 \times_{1} + 215.83 \times_{2} - 617.35 \times_{3} \tag{15}
\]

\[
\hat{y}_{LRT, 1} = 2416.37 - 443.98 \times_{1} + 203.67 \times_{2} - 626.19 \times_{3} \tag{16}
\]

\[
\hat{y}_{LRT, 2} = 2772.61 - 271.64 \times_{1} + 125.40 \times_{2} - 786.25 \times_{3} \tag{17}
\]

The normality plot showed no patterns, and the R^2 values seen below, were both above the critical values specified by [15], thus confirming the data to be normal.

Table II: R^2 for LR Based CER

<table>
<thead>
<tr>
<th>Trial</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial 0</td>
<td>0.9206</td>
</tr>
<tr>
<td>Trial 1</td>
<td>0.9195</td>
</tr>
<tr>
<td>Trial 2</td>
<td>0.9808</td>
</tr>
</tbody>
</table>

B. Non-Linear Regression Based CER

The resulting equation for the three trials based upon the non-linear regression model based CER are as follows:

\[
\ln \hat{y}_{NLR, 0} = 10.81 - 3.21 \ln X_{1} + 2.08 \ln X_{2} - 4.77 \ln X_{3} \tag{18}
\]

\[
\ln \hat{y}_{NLR, 1} = 10.88 - 3.11 \ln X_{1} + 1.98 \ln X_{2} - 4.85 \ln X_{3} \tag{19}
\]

\[
\ln \hat{y}_{NLR, 2} = 10.95 - 3.10 \ln X_{1} + 2.03 \ln X_{2} - 4.91 \ln X_{3} \tag{20}
\]

The models in the standard form presented in (4) will be the following:

\[
\hat{y}_{NLR, 0} = 49281.89 \times_{1}^{-3.21} \times_{2}^{2.08} \times_{3}^{-4.77} \tag{21}
\]

\[
\hat{y}_{NLR, 1} = 53191.85 \times_{1}^{-3.11} \times_{2}^{1.98} \times_{3}^{-4.85} \tag{22}
\]

\[
\hat{y}_{NLR, 2} = 56781.45 \times_{1}^{-3.10} \times_{2}^{2.03} \times_{3}^{-4.91} \tag{23}
\]

The analysis of this model is complete; the next CER developed is that of complex non-linear model.

Table IV: R^2 for NLR Based CER

<table>
<thead>
<tr>
<th>Trial</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trial 0</td>
<td>0.9959</td>
</tr>
<tr>
<td>Trial 1</td>
<td>0.9978</td>
</tr>
<tr>
<td>Trial 2</td>
<td>0.9963</td>
</tr>
</tbody>
</table>

Similarly to the case of the CER based upon LR, none of the p-values from the NLR ANOVA are greater than 0.25, therefore all the selected design effort drivers have statistical significance. The resulting errors for one trial are as follows:

Table V: Errors for the NLR Based CER (Trial 0)

<table>
<thead>
<tr>
<th>DJ</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.67</td>
</tr>
<tr>
<td>B</td>
<td>1.71</td>
</tr>
<tr>
<td>C</td>
<td>4.09</td>
</tr>
<tr>
<td>D</td>
<td>4.33</td>
</tr>
<tr>
<td>E</td>
<td>1.74</td>
</tr>
<tr>
<td>F</td>
<td>4.26</td>
</tr>
<tr>
<td>G</td>
<td>4.79</td>
</tr>
</tbody>
</table>

The analysis for this model is complete; the final CER developed is that of complex non-linear model.
**C. Complex Non-Linear Based CER**

The CNLM selected to model the design effort presented in Section II B is shown below is used to model this relationship.

\[
\hat{y} = a_0x_1^{b_0} + a_1x_2^{b_1} + a_2x_3^{b_2} + c_0x_4^{d_0}x_2^{d_1} + c_1x_4^{d_2}x_3^{d_3} + c_2x_4^{d_4}x_3^{d_5} + e_0x_1^f_0x_1^f_1x_2^f_2 + \epsilon
\]

It should be noted the values of input and cost data are normalized, with the following equations, to facilitate in the convergence of the model.

\[
x_n \rightarrow x_{(\text{original})}/10 \quad (24)
\]

\[
y \rightarrow y_{(\text{original})}/1000 \quad (25)
\]

As mentioned in Section II B the gradient descent algorithm (GDA) is used to determine the coefficients of the model. For the same three trials, using the GDA, the models converged and resulted in the following equations.

\[
\hat{y}_{\text{CNLM, T0}} = -3.94x_1^{1.18} + 1.65x_2^{0.06} + 0.72x_3^{-0.49} + 0.72x_1^{-0.06}x_2^{-0.79} - 0.31x_1^{1.12}x_3^{-0.34} - 6.09x_1^{-0.41}x_2^{+0.85} - 3.36x_1^{-0.55}x_2^{+0.19}x_3^{-2.74} \quad (26)
\]

\[
\hat{y}_{\text{CNLM, T1}} = -1.78x_1^{0.77} + 0.75x_2^{+1.2} + 0.71x_3^{-0.83} + 0.28x_1^{-0.58}x_2^{+0.87} - 1.48x_1^{+0.99}x_3^{-0.98} - 0.53x_1^{-0.96}x_2^{+1.35} + 0.30x_1^{+0.59}x_2^{+0.77}x_3^{+2.58} \quad (27)
\]

\[
\hat{y}_{\text{CNLM, T2}} = -1.42x_1^{-0.11} + 0.97x_2^{+0.76} + 0.65x_3^{-1.15} + 0.49x_1^{+0.46}x_2^{+0.70} - 1.11x_1^{+0.66}x_3^{-1.04} + 0.48x_2^{+0.41}x_3^{+2.58} + 0.65x_1^{+0.37}x_2^{+0.67}x_3^{+2.57} \quad (28)
\]

The resulting errors for one trial are as follows:

<table>
<thead>
<tr>
<th>DJ</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1.92</td>
</tr>
<tr>
<td>B</td>
<td>0.64</td>
</tr>
<tr>
<td>C</td>
<td>0.26</td>
</tr>
<tr>
<td>D</td>
<td>1.64</td>
</tr>
<tr>
<td>E</td>
<td>0.00</td>
</tr>
<tr>
<td>F</td>
<td>0.00</td>
</tr>
<tr>
<td>G</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Now that all models are developed, they can be compared.

**D. Comparative Analysis**

The following table displays one trial of the errors, their maximum, minimum, and average, for all the developed CERs.

<table>
<thead>
<tr>
<th>TABLE VII</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ERRORS FOR THE CNLM BASED CER (TRIAL 0)</strong></td>
</tr>
<tr>
<td>DJ</td>
</tr>
<tr>
<td>----</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>E</td>
</tr>
<tr>
<td>F</td>
</tr>
<tr>
<td>G</td>
</tr>
</tbody>
</table>

As can be seen from Table VIII, the CNLM is the best in terms of all the errors. However, when the other trials are examined it has different results. The reason for this is due to the fact of the modeling of the equation. As the CNLM is solved analytically based upon the maximum error, it can theoretically result in a model with zero error, which has its own implications, such as over fitting. The notion of over fitting can be seen, where the errors of the model are acceptable, yet when used to predict for the omitted program, the resulting error is high.

In trials 1 and 2, the non-linear regression based CER, performs the best in terms of prediction for the omitted program, and in terms of average error. Moreover, a statistical test (t-test), which can be found in [16], comparing the means of the errors, with a 95% confidence interval also confirms that the non-linear based CER, both outperforms the linear regression and complex based CER in terms of accuracy. A sensitivity analysis is now conducted on the NLR model to see the impact that each factor has on the estimate of design effort.

**E. Sensitivity Analysis**

Sensitivity analysis was performed on the impact that TD has on design effort. It confirmed the intuition related to the selected factors. It was deduced that the effort would be more for an initial design than of a redesign. Similarly, the degree of changes increases from minor to major, the corresponding effort will also increase. Finally, as the experience of the individual/team increases, the required effort will reduce.

V. CONCLUSION

This study uses the development of parametric based cost estimating relations to estimate the design effort. The analysis supports, the significant design effort drivers to estimate the design effort. The design effort drivers are the type of design, degree of change, and experience of departmental personnel.

The models are used to estimate the design effort for the integrated blade-rotor low-pressure compressor fan for seven design jobs at Pratt & Whitney Canada. The non-linear regression based CER performed best when compared to the other models.
REFERENCES


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