Unambiguous Signal Acquisition Based On Recombination of Sub-Correlations of BOC Signals

Hongdeuk Kim, Youngpo Lee, and Seokho Yoon

Abstract—Due to side-peaks of autocorrelation function, the binary offset carrier (BOC) signal acquisition suffers from an ambiguity when one of the side-peaks is acquired. In this paper, we first analyze that the BOC autocorrelation is made up of the sum of sub-correlations, and then, remove the side-peaks causing the ambiguity by recombining the sub-correlations. The proposed scheme is shown to remove the side-peaks completely. From numerical results, it is confirmed that the proposed scheme outperforms the conventional schemes in terms of the receiver operating characteristic and mean acquisition time.

Keywords—Binary offset carrier (BOC), acquisition, ambiguity problem, side-peak.

I. INTRODUCTION

Recently, new global navigation satellite systems (GNSSs) such as Galileo and global positioning system (GPS) modernization are being developed to satisfy the increasing demand for GNSS-based services such as location-based service (LBS) and emergency rescue service (ERS) and complement the existing GNSSs such as GPS [1]-[3]. Currently, new GNSSs are designed to use the same frequency band of the existing GNSSs: for example, the E1 and E5 bands of Galileo are overlapped with the L1 and L5 bands of GPS, respectively [1], [4]. Thus, if a Galileo signal is modulated by a conventional scheme such as phase shift keying (PSK) used in GPS, it would suffer from co-channel interference.

To overcome these problems, binary offset carrier (BOC) modulation has been proposed, where a high degree of spectral separation between the BOC-modulated signals and the others is achieved by shifting the signal energy from the band center [5]. The BOC signal is generated through the product of a spreading pseudo random noise (PRN) code and a sine-phased or cosine-phased square wave sub-carrier, and denoted by BOCsin(\(kn,n\)) or BOCcos(\(kn,n\)) depending on which of the sine-phased and cosine-phased sub-carriers are used, where \(k\) and \(n\) are the ratios of the PRN code chip period to the sub-carrier period and the PRN code chip rate to 1.023 MHz, respectively [4], [6]. For larger values of \(k\), more separated spectrums are obtained, reducing the co-channel interference more effectively. However, the BOC signal has multiple side-peaks on both sides of the main-peak of its autocorrelation function, causing an ambiguity problem in the BOC signal acquisition process. Moreover, the number of side-peaks increases as the value of \(k\) becomes larger. Several unambiguous acquisition schemes [7]-[11] have been proposed in order to tackle the problem. In [7]-[9], sideband filtering was used to deal with the ambiguity problem in the BOC signal acquisition; however, these schemes destroy the sharpness of the main-peak of the BOC autocorrelation function, degrading the BOC signal tracking performance severely. In [10], an interesting unambiguous acquisition scheme that maintains the sharp main-peak of the BOC autocorrelation function was proposed combining the correlation between the BOC and PRN signals with the BOC autocorrelation; however, this scheme is applicable to only BOCsin(\(kn,n\)) signals. In [11], a generalized unambiguous acquisition scheme including the scheme in [10] as a special case was proposed. This scheme is applicable to generic BOCsin(\(kn,n\)) signals; however, its extension to generic BOCcos(\(kn,n\)) signals is not straightforward.

In this paper, a novel unambiguous acquisition scheme applicable to both BOCsin(\(kn,n\)) and BOCcos(\(kn,n\)) signals is proposed based on a recombination of the sub-correlations making up the BOC autocorrelation, which is found to remove the side-peaks of the BOC autocorrelation completely, while keeping the sharp shape of the main-peak, and also, to offer a performance improvement over the scheme in [11] in terms of the correlation function, receiver operating characteristic (ROC) curves (which is the probability of detection \(P_D\) as a function of the probability of false alarm \(P_{F_A}\)), and mean acquisition time (MAT) (which is the time that elapses prior to acquisition on the average and the ultimate performance metric of interest for signal acquisition).

II. ANALYSIS OF BOC AUTOCORRELATION FUNCTION

The BOC signal \(b(t)\) can be expressed as

\[
b(t) = \sqrt{P} \sum_{i=-\infty}^{\infty} c_i p_{T_c}(t - i T_c) d_{[i/T_c]}(t) s(t),
\]

where \(P\) is the signal power, \(c_i \in \{-1,1\}\) is the \(i\)th chip of a PRN code with a period of \(T_c\), \(T_c\) is the PRN code period, \(p_{T_c}(t)\) is the PRN code waveform defined as a unit rectangular pulse over \([0,T_c]\), \(d_{[i/T_c]}(t)\) is the navigation data, where \(d_{x}(t)\) is the \(x\)th navigation data and \([x]\) is the largest integer not larger than \(x\), and

\[
s(t) = \begin{cases} 
\sum_{u=0}^{2k-1} (-1)^u p_{T_c}(t - i T_c - u T_s), & \text{for } \text{BOCsin}(kn,n), \\
\sum_{u=0}^{4k-1} (-1)^u \left[\frac{1}{2}\right] p_{T_c}(t - i T_c - u T_s - \frac{T_s}{2}), & \text{for } \text{BOCcos}(kn,n)
\end{cases}
\]

is the square wave sub-carrier, where \(T_s\) is the sub-carrier pulse duration of \(T_c/2k = 1/(2kn \times 1.023\text{ MHz})\), \(p_{T_c}(t)\) is the signal power, \(c_i \in \{-1,1\}\) is the \(i\)th chip of a PRN code with a period of \(T_c\), \(T_c\) is the PRN code period, \(p_{T_c}(t)\) is the PRN code waveform defined as a unit rectangular pulse over \([0,T_c]\), \(d_{[i/T_c]}(t)\) is the navigation data, where \(d_{x}(t)\) is the \(x\)th navigation data and \([x]\) is the largest integer not larger than \(x\), and

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s(t) = \begin{cases} 
\sum_{u=0}^{2k-1} (-1)^u p_{T_c}(t - i T_c - u T_s), & \text{for } \text{BOCsin}(kn,n), \\
\sum_{u=0}^{4k-1} (-1)^u \left[\frac{1}{2}\right] p_{T_c}(t - i T_c - u T_s - \frac{T_s}{2}), & \text{for } \text{BOCcos}(kn,n)
\end{cases}
\]

is the square wave sub-carrier, where \(T_s\) is the sub-carrier pulse duration of \(T_c/2k = 1/(2kn \times 1.023\text{ MHz})\), \(p_{T_c}(t)\) is

\[
s(t) = \begin{cases} 
\sum_{u=0}^{2k-1} (-1)^u p_{T_c}(t - i T_c - u T_s), & \text{for } \text{BOCsin}(kn,n), \\
\sum_{u=0}^{4k-1} (-1)^u \left[\frac{1}{2}\right] p_{T_c}(t - i T_c - u T_s - \frac{T_s}{2}), & \text{for } \text{BOCcos}(kn,n)
\end{cases}
\]

is the square wave sub-carrier, where \(T_s\) is the sub-carrier pulse duration of \(T_c/2k = 1/(2kn \times 1.023\text{ MHz})\), \(p_{T_c}(t)\) is
the unit rectangular sub-carrier pulse waveform over \([0, T_s]\), and \([x]\) is the smallest integer not less than \(x\). In this paper, focusing on the problem of ambiguity due to the side-peaks, we assume that there is a pilot channel for acquisition \([12]\) so that no data modulation is present during acquisition (i.e., \(d_{i[T_d, T_f]}(t) = 1\) for all \(i\)), and do not consider the effect of the secondary code. Then, considering that the PRN code period \(T\) is generally much larger than the PRN code chip period \(T_c\) and the out-of-phase autocorrelation of a PRN code is designed to be as low as possible for easy signal acquisition, we can obtain the correlation (normalized to the signal power) between the received and locally generated BOC signals as \([13]\)

\[
R^k_{\sin}(\tau) = \frac{1}{PT} \int_0^T (b(t - \tau) + w(t))b(t)dt \\
\simeq \sum_{u=0}^{2k-1} \frac{1}{T_c} \sum_{v=0}^{2k-1} (-1)^{u+v} \Lambda_T(\tau - (u - v)T_s) + w^u_{\sin} (3)
\]

for BOC_{sin}(kn, n) and

\[
R^k_{\cos}(\tau) \simeq \sum_{u=0}^{4k-1} \frac{1}{T_c} \sum_{v=0}^{4k-1} (-1)^{\frac{T}{2} + \frac{T}{2}} \Lambda_T(\tau - (u - v)T_s) + w^u_{\cos} (4)
\]

for BOC_{cos}(kn, n), where \(\tau\) is the phase difference between the received and locally generated BOC signals, \(w(t)\) is the additive white Gaussian noise (AWGN) process with mean zero and one-sided noise power spectral density \(N_0\), \(w^u_{\sin} = \frac{1}{\sqrt{PT} \int_0^T \sum_{v=-\infty}^{\infty} (-1)^{v} c_{pt}(t - iT_c - uT_s)w(t)dt, \quad w^u_{\cos} = \frac{1}{\sqrt{PT} \int_0^T \sum_{v=-\infty}^{\infty} (-1)^{\frac{T}{2} + \frac{T}{2}} c_{pt}(t - iT_c - uT_s)w(t)dt, \quad \Lambda_T(\tau) = \begin{cases} x - |\tau|, & |\tau| \leq x, \\ 0, & \text{otherwise} \end{cases} \)

is the triangular function of height \(x\) and area \(x^2\). Denoting the terms \(\frac{1}{T_c} \sum_{v=0}^{2k-1} (-1)^{u+v} \Lambda_T(\tau - (u - v)T_s) + w^u_{\sin}\) and \(\frac{1}{T_c} \sum_{v=0}^{4k-1} (-1)^{\frac{T}{2} + \frac{T}{2}} \Lambda_T(\tau - (u - v)T_s) + w^u_{\cos}\) in (3) and (4) by \(R^k_{\sin, u}(\tau)\) and \(R^k_{\cos, u}(\tau)\), respectively, we can re-write \(R^k_{\sin}(\tau)\) and \(R^k_{\cos}(\tau)\) as

\[
R^k_{\sin, u}(\tau) = \frac{1}{T_c} \sum_{u=0}^{2k-1} (-1)^{u+v} \Lambda_T(\tau - (u - v)T_s) + w^u_{\sin}
\]

\[
= \sum_{l=0}^{L-1} \frac{1}{PT} \int_{(2k+l+1)T}^{(2k+l+1)T} b(t - \tau)b(t) + w(t)b(t)\ dt, \quad R^k_{\cos, u}(\tau) = \frac{1}{T_c} \sum_{u=0}^{4k-1} (-1)^{\frac{T}{2} + \frac{T}{2}} \Lambda_T(\tau - (u - v)T_s) + w^u_{\cos}
\]

\[
= \sum_{l=0}^{L-1} \frac{1}{PT} \int_{(2k+l+1)T}^{(2k+l+1)T} r(t)b(t)dt, \quad (6)
\]

and similarly,

\[
R^k_{\cos, u}(\tau) = \sum_{l=0}^{L-1} \frac{1}{PT} \int_{(2k+l+1)T}^{(2k+l+1)T} r(t)b(t)\ dt, \quad (7)
\]

where \(r(t) = b(t - \tau) + w(t)\) and \(L\) is a correlation length and would be generally limited to be equal to or less than the PRN code period (normalized to \(T_s\)) due to some constraints such as the frequency error, data modulation, and secondary code. From (6) and (7), we can see that \(R^k_{\sin, u}(\tau)\) and \(R^k_{\cos, u}(\tau)\) are the sub-correlations making up the correlations (3) and (4), respectively, and which are shown for \(k = 1\) and \(2\) in the absence of noise in Figure 1. From the figure, we can see that the main-peaks of the sub-correlations are coherently combined through the summation of the sub-correlations, thus forming the sharp main-peak of the BOC autocorrelation, and on the other hand, the sub-peaks of the sub-correlations are irregularly spread around the main-peaks, and thus, the summation of the sub-correlations results in the multiple side-peaks of the BOC autocorrelation. Another important observation is that the number of the side-peaks increases as \(k\) increases. From this observation, it is expected that the acquisition performance is degraded as \(k\) increases. In the next section, we propose a novel unambiguous acquisition scheme, removing the side-peaks completely through a recombination of the sub-correlations.

### III. PROPOSED UNAMBIGUOUS ACQUISITION SCHEME

From Figure 1, we can clearly observe that \(R^k_{\sin, u}(\tau)\) and \(R^{k, 2k-1}_{\sin, u}(\tau)\) and \(R^k_{\cos, u}(\tau)\) and \(R^{k, 2k-1}_{\cos, u}(\tau)\) are symmetric with respect to \(\tau = 0\) and have only a single overlapped peak at \(\tau = 0\) for BOC_{sin}(kn, n) and BOC_{cos}(kn, n), respectively. Thus, if the two sub-correlations are summed, a main-peak with a larger magnitude (than that of the main-peak of a sub-correlation) is obtained without increasing the magnitudes.
Since the noise random variables of the side-peaks, and on the other hand, the difference between the two sub-correlations yields the side-peaks only, whose magnitudes and positions are the same as those of the side-peaks in the sum of the two sub-correlations. Thus, the difference between the two sub-correlations might be used to remove the side-peaks in the sum of the two sub-correlations, leaving only the main-peak. This observation is the key motivation of the proposed scheme.

Since the side-peaks in the sum and difference of the two sub-correlations are out-of-phase and in-phase at $\tau < 0$ and $\tau > 0$, respectively, however, we cannot remove the side-peaks in the sum of the two sub-correlations completely through the subtraction between the sum and difference of the two sub-correlations. To align the phases of the side-peaks in the sum and difference of the two sub-correlations, thus, we use the sum of the absolute values of the two sub-correlations, obtaining the side-peaks with the same slopes as those of the side-peaks in the absolute difference of the two sub-correlations. Figure 2 shows that the subtraction of the absolute difference of the two sub-correlations from the sum of the absolute values of the two sub-correlations yields an unambiguous correlation function with a single main-peak and no side-peak.

Since the unambiguous correlation function is generated with several absolute operations, it may suffer from undesired noise enhancement. To alleviate the noise enhancement while maintaining the signal part (i.e., the main-peak), we first multiply the unambiguous correlation function with each of the sub-correlations, and then, sum the product results together. Since the noise random variables $\{ w_{\sin}^{k} \}_{k=0}^{2k-1}$ and $\{ w_{\cos}^{k} \}_{k=0}^{4k-1}$ in $\{ R_{\sin}^{k-1} (\tau) \}$ are independent from each other under the AWGN-limited satellite environment, the product sum of $\{ R_{\sin}^{k-1} (\tau) \}$ will average out the noise, and moreover, the main-peak magnitude of the unambiguous correlation function can be maintained after the product sum, since all sub-correlations have an equal magnitude of $\frac{1}{2k}$ and $\frac{1}{4k}$, the inverse of the number of the sub-correlations, for $BOC_{\sin}(kn,n)$ and $BOC_{\cos}(kn,n)$, respectively.

From the above discussions, the proposed unambiguous correlation function can be expressed as

$$
P_{\sin}^{k,\text{proposed}}(\tau) = \sum_{u=0}^{2k-1} R_{\sin}^{k,u}(\tau) |R_{\sin}^{k,0}(\tau) + R_{\sin}^{k,2k-1}(\tau)| - |R_{\sin}^{k,0}(\tau) - R_{\sin}^{k,2k-1}(\tau)|$$

for $BOC_{\sin}(kn,n)$ and

$$
P_{\cos}^{k,\text{proposed}}(\tau) = \sum_{u=0}^{4k-1} R_{\cos}^{k,u}(\tau) |R_{\cos}^{k,0}(\tau) + R_{\cos}^{k,4k-1}(\tau)| - |R_{\cos}^{k,0}(\tau) - R_{\cos}^{k,4k-1}(\tau)|$$

for $BOC_{\cos}(kn,n)$.

Figure 3 shows the baseband receiver structure of the proposed unambiguous acquisition scheme for $BOC_{\sin}(kn,n)$ and $BOC_{\cos}(kn,n)$. The received BOC signal $r(t)$ is first multiplied with the locally generated PRN code and sub-carrier, and then, integrated and sampled every $T_s$ and $T_s/2$ seconds for $BOC_{\sin}(kn,n)$ and $BOC_{\cos}(kn,n)$, respectively. Subsequently, sub-correlation values are obtained by summing $L$ samples per sub-correlation, and then, combined according to (8) and (9) to produce a decision variable based on $R_{\sin}^{k,\text{proposed}}(\tau)$ and $R_{\cos}^{k,\text{proposed}}(\tau)$, respectively. Finally, decision variables corresponding to possible phases in the uncertainty region are collected and the process is transferred to the locking stage with the phase corresponding to the largest of the decision variables. It should be noted that the proposed scheme needs only a single correlator since each sub-correlation is sequentially obtained by sampling the single correlator output.
every $T_s (= T_s/2k)$ seconds and every $T_s/2 (= T_s/4k)$ seconds for $BOC_{\sin}(k, n)$ and $BOC_{\cos}(k, n)$ signals, respectively, as shown in Figure 3.

IV. NUMERICAL RESULTS

In this section, the proposed unambiguous acquisition scheme is compared with the unambiguous acquisition scheme in [11] called the general removing ambiguity via side-peak suppression (GRASS) in terms of the correlation function, ROC curves, MAT. In comparisons, we assume the following parameters: Galileo E1-C PRN code of $T = 4092$ chips [14], correlation length of $L = 1023$, and a search step size of $T_s$ and $T_s/2$ for the sine-phased and cosine-phased BOC signals, respectively.

Figure 4 shows the normalized correlation functions of the proposed, GRASS, and traditional BOC schemes for $BOC_{\sin}(k, n)$ and $BOC_{\cos}(k, n)$ when $k = 1$ and 2 in the absence of noise, where the GRASS correlation function is not shown for $BOC_{\cos}(k, n)$ since it is dedicated to the sine-phased BOC signals only, and the traditional BOC autocorrelation is also shown as a reference. From the figure, unlike the GRASS and traditional BOC schemes, the proposed scheme is clearly observed to remove the side-peaks completely for both $BOC_{\sin}(k, n)$ and $BOC_{\cos}(k, n)$ regardless of the value of $k$.

Figure 5 shows the ROC curves of the proposed, GRASS, and traditional BOC schemes for (a) $BOC_{\sin}(k, n)$ and (b) $BOC_{\cos}(k, n)$ when $k = 1$ and 2.

As shown in Figure 5, it is seen that the traditional BOC scheme provides the best ROC performances; however, it should be noted the probabilities of detection and false alarm of the ROC curve are associated with only the main-peak of the correlation function, i.e., the ROC curve does not reflect the effect of the side-peaks elimination. Thus, we compare the MAT performances of the three schemes as shown in Figure 6, where the penalty time and the probability of false alarm are set to $4T$ and $10^{-3}$, respectively. As shown in the figures, the proposed scheme performs better than the GRASS and traditional BOC schemes in the CNR range 20 $\sim$ 40 dB-Hz of practical interest.
V. Conclusion

In this paper, we propose a novel unambiguous acquisition scheme based on recombination of sub-correlations of BOC signals. We first have observed that the side-peaks of the BOC autocorrelation is the sum of the sub-correlations, and then, removed the side-peaks of the BOC autocorrelation completely via the recombination of the sub-correlations. Numerical results have demonstrated that the proposed scheme provides a better acquisition performance than the conventional scheme in terms of the ROC and MAT.

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References


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