Application New Approach with Two Networks Slow and Fast on the Asynchronous Machine

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Abstract—In this paper, we propose a new modular approach called neuroglial consisting of two neural networks slow and fast which emulates a biological reality recently discovered. The implementation is based on complex multi-time scale systems; validation is performed on the model of the asynchronous machine. We applied the geometric approach based on the Gerschgorin circles for the decoupling of fast and slow variables, and the method of singular perturbations for the development of reductions models.

This new architecture allows for smaller networks with less complexity and better performance in terms of mean square error and convergence than the single network model.

Keywords—Gerschgorin’s Circles, Neuroglial Network, Multi time scales systems, Singular perturbation method.

I. INTRODUCTION

Since the artificial neural networks try to emulate the brain, researchers have continued to focus their attention on the importance of neurons in the nervous system. However in recent decades, the importance of glial system was observed and is believed that the glial system is involved in the nervous system processes information in a much more supported than before [1] and how these glial cells also communicate, forming a separate parallel network to the neural network. Also information is treated in two time scales.

For this we developed a close architecture of biological reality called Neuroglial two networks of different speeds. The Development of this architecture is to better organize using the powerful concept of modularity.

II. SINGULAR PERTURBED METHOD

Singularly perturbed systems analyzed by this technique must have a special form called standard:

Suppose that the model \( \dot{X} = AX + BU \) distribution

\[
\begin{bmatrix}
\dot{x} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
A_1 & A_2 \\
A_21 & A_22
\end{bmatrix}
\begin{bmatrix}
x \\
z
\end{bmatrix} +
\begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} U
\]

\[y = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix}\]

with: \( x(t_0) = x_0 \), 
\( z(t_0) = z_0 \).

Which \( A_{21}, A_{22}, B_2 \) are very large in modulus compared to those \( A_{11}, A_{12}, B_1 \). So normalization can be done by introducing a parameter \( \varepsilon \), with: \( A'_{21} = \varepsilon A_{21}, A'_{22} = \varepsilon A_{22}, B'_{2} = \varepsilon B_{2} \) and the model appears in a form singularly perturbed:

\[
\begin{bmatrix}
\dot{x} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
A_1 & A_2 \\
A_21 & A_22
\end{bmatrix}
\begin{bmatrix}
x \\
z
\end{bmatrix} +
\begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} U
\]

The introduction of a small parameter \( \varepsilon \) is considered as a parasite. Slow reduced model is obtained by considering \( \varepsilon = 0 \).

\[
\begin{bmatrix}
\dot{x}_s \\
\dot{y}_s \\
\dot{z}_s
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
x_s \\
z_s
\end{bmatrix} +
\begin{bmatrix}
B_{11} \\
B_{12}
\end{bmatrix} U_s
\]

which \( x_s, z_s, u_s, y_s \) represent the slow components of variables \( x, z, u, y \).

\[
A_{11} = A_{21}, A_{22}^{-1} A_{12} \\
B_{11} = B_1, A_{22}^{-1} B_2 \\
C_{1} = C_{11}, A_{22}^{-1} A_{21} \\
D_{1} = -C_{22}^{-1} A_{21}
\]

With:

\[
A_{21} = A_{11} - A_{12} A_{22}^{-1} A_{21} \\
B_{2} = B_1 - A_{12} A_{22}^{-1} B_2 \\
C_{2} = C_{1} - C_{2} A_{22}^{-1} A_{21} \\
D_{2} = -C_{22}^{-1} A_{21}
\]

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\[ z_i(t_0) = -A_{kk}^{-1}A_k^i x_i(t_0) \]

And fast model is obtained by introducing:

\[ z_f = z - z_s \]

\[ \frac{dz_f}{d\tau} = A_{kk} z_f(\tau) + B_{i\mu} y_f(\tau) \]

\[ y_f = C_{ij} z_f(\tau) \]

\[ z_f(t_0) = z_0 + A_{kk}^{-1}A_{kj} x_k \]

III. GEOMETRICAL IDENTIFICATION OF DYNAMIC

To determine the dynamic elements that are fast (or slow), we use Gerschgorin’s circles technique.

After transformations affecting the state matrix, the latter comes in diagonally dominant form and whose eigenvalues correspond to the different time scales. The necessary conditions and sufficient are set for this technique to be applied correctly.

A. Theorem 1

Each eigenvalue of the square matrix \( A \) is located in at least one circle \( C_i \), centered in \( a_{ii} \) and having radii: \( R_i = R_{ii} \) or \( R_i = R_{ij} \) [3], [4]:

\[ R_{ii} = p_i \quad \text{avec} \quad p_i = \sum_{j \neq i} |a_{ij}|, \quad i = 1, 2, \ldots, n \]

\[ R_{ij} = q_i \quad \text{avec} \quad q_i = \sum_{j \neq i} |a_{ij}|, \quad j = 1, 2, \ldots, n \]

B. Theorem 2

If we have two sets \( I \) and \( K \), with \( I \cap K = \emptyset \) and \( I \cup K = \{1, \ldots, n\} \), with: \( \forall (i, j) \in I \times K \), circles \( C_i(a_{ii}, R_i) \) and \( C_k(a_{kk}, R_k) \) check:

\[ |a_{ii} - a_{kk}| \geq (R_i + R_k) \quad \forall i \in I \quad \text{and} \quad \forall k \in K \]

Then the matrix \( A \) has two separate sets of eigenvalues. If \( |a_{ii}| \) is greater than expected \( |a_{kk}| \), \( x_i \) variables, \( i \in I \) are so fast and \( x_k \) variables, \( k \in K \) are slow [3], [4].

C. Change the Size of Radius

Let \( S_k = \text{diag}(1, \ldots, 1, a_{kk}, 1, \ldots, 1) \), \( k = 1, 2, \ldots, n \)

Change’s base \( X^* = S_k X \) shows a new state matrix [4].

Radii \( R_i \) and \( R_k \) become respectively \( R_i a_{ii} \) and \( R_k / a_{kk} \).

If the operation is repeated several times, the transformation is obtained:

\[ \begin{cases} X = S \hat{X} \\ A = S \hat{A} S^{-1} \end{cases} \]

with: \( S = \prod_k S_k \)

If there are two disjoint circles, the permutation matrix is found:

\[ \begin{cases} X = P \hat{X} \\ A = P \hat{A} P^{-1} \end{cases} \]

D. Move Centers of Circles

Sometimes it is necessary to improve the separation of dynamic with introducing movement of circles. This movement is characterized by the following transformation [4]:

\[ T_i = I + B_i \]

Only the elements of the line \( i \) and the column \( j \) change, the centers of circles \( i \) and \( j \) are displaced from \( a_{ii} \) and \( a_{jj} \) to \( a_{ii} + B_i a_{ii} \) and \( a_{jj} + B_j a_{jj} \) respectively.

Choice \( B_i \) can be made so that:

\[ X_f = a_{ii} + B_i (a_{ii} - a_{ii}) - B_i a_{ii} = 0 \]

If several circles intersect, the terms \( B_i (l = 1, 2, \ldots) \) are calculated by the same method, the final transformation is:

\[ \begin{cases} X = T \hat{X} \\ A = T \hat{A} T^{-1} \end{cases} \]

If two groups of circles are disjoint, the permutation matrix is found:

\[ \begin{cases} X = P \hat{X} \\ A = P \hat{A} P^{-1} \end{cases} \]

IV. MODULARITY

The design of the new neuroglial approach requires executing the four following steps:

A. The Decomposition

Two vertical decompositions are executed. A decomposition is performed on the input space and another decomposition is performed on the variables of the model [5].

B. The Organization of the Architecture

A parallel decomposition is applied in our approach:

1. Cooperation

There are two ways to establish a cooperative relationship between the modules, AND and OR decomposition [6]. AND decomposition is used when solving a task involves resolving all sub-tasks and an OR decomposition when solving a task is to solve a sub task.

C. Nature of Learning

Independent learning is used [7]:

The learning of a module is performed independently when the other modules involved and do not affect during the
learning phase. The modules are connected only to the use phase / restitution.

D. Communication between Modules

The modules are connected only to the use phase / restitution. We applied the technique that minimizes the mean square error of the overall output [8]. We presented approaches and references on which we rely to provide a modular architecture with two networks.

E. Modular Neuronal Architecture with two Networks

\[ A = \begin{bmatrix} -\frac{1}{T_{sp}} I_2 & B \frac{R(-\theta)}{T_{sp}} \\ \frac{1-\sigma}{T_{sp}} B \frac{R(\theta)}{T_{sp}} & -\frac{1}{T_{sp}} I_2 \end{bmatrix} \]

with \( T_{sp} = \sigma T_c \), \( T_{sp} = \sigma T_c \). We applied the algorithm based on the Gershgorin’s circle technique to the electromagnetic state matrix, we obtained:

1. The circles are doubled and centered in \( C_{1,2} = -\frac{1}{T_{sp}} \) and \( C_{3,4} = -\frac{1}{T_{sp}} \), their radii respectively:

\[ R_{1,2} = \frac{B_2}{T_{sp}} (\cos(\theta) + \sin(\theta)) \text{ and} \]

\[ R_{3,4} = \frac{1-\sigma}{B_2 T_{sp}} (\cos(\theta) + \sin(\theta)) \]

2. For all values of \( R_{1,2} \) and \( R_{3,4} \), circles intersect.

Considering the fundamental electromagnetic model that is represented by the state matrix \( A \), we transform it by the matrix \( P_1 \) defined by:

\[ \phi_1 = P_1 \phi_{saf} \]

\[ P_1 = \begin{bmatrix} I_2 & 0 \\ 0 & B, R(-\theta) \end{bmatrix} \]

With \( A_1 = P_1 A P_1^{-1} \)

Or:

\[ A_1 = \begin{bmatrix} -\frac{1}{T_{sp}} I_2 & \frac{1}{T_{sp}} I_2 \\ \frac{1-\sigma}{T_{sp}} I_2 & -\frac{1}{T_{sp}} I_2 - \omega J_2 \end{bmatrix} \]

The new circles are still doubled and centered in \( C_{1,2} = -\frac{1}{T_{sp}} \) and \( C_{3,4} = -\frac{1}{T_{sp}} \), and their radii are respectively: \( R_{1,2} = \frac{1}{T_{sp}} \) and \( R_{3,4} = \frac{1-\sigma}{T_{sp}} + \omega \)

We apply a new transformation to separate the circles and reduce their radii by using the matrix \( P_2 \) defined by:

\[ \phi_2 = P_2 \phi_1, P_2 = \begin{bmatrix} I_2 & 0 \\ -I_2 & I_2 \end{bmatrix} \]

Fig. 2 shows that the circles are separated and centered in \( C_{1,2} = 0 \) and \( C_{3,4} = \frac{1+\alpha}{T_{sp}} \), and their radii are respectively

\[ R_{1,2} = \frac{1}{T_{sp}} \text{ and } R_{3,4} = \frac{\alpha \sigma}{T_{sp}} + 2\omega \]
In this application, the model learning is a function of two inputs: sinusoidal voltages $v_{\alpha}$ and $v_{\beta}$ output is the couple $C_{em}$.

The decomposition of the input variables: The input variables are decomposed to the slow variables and fast variables. This decomposition is performed using the wavelet.

In order to have an optimal error, a better convergence and reduced neural configuration, a comparative study of performance is made between the model of single neural network and modular model with two networks slow and fast networks.

To evaluate the results, the mean square error MSE has been used:

$$\text{MSE} = \frac{1}{m} \sum_{i=1}^{m} (y_{d} - y_{e})^2$$

$y_{d}$: The desired output
$y_{e}$: The estimated output

VI. RESULT AND DISCUSSION

A. Learning Single Network

The minimum architecture allows convergence of the network is composed of several layers: an input layer with ten neuron, a hidden layer itself contains ten neurons and output layer that contains one neuron. Presenting 400 examples to the network, we obtain the convergence after 5000 iterations for the couple, thus giving the result shown in.

Thus obtained after 5000 iterations a minimum squared error 0.000174207. By increasing the number of iterations in order to achieve the parameters of a neural network, one can give better performance. We note, according to the results of Fig. 3 that the convergence is slow, because of the size of the MLP network.

B. Modular Learning

To overcome the problems of complexity and convergence, both for better performance with reduced size, the solution presented is the use of the new modular architecture we will use two smaller networks, a fast network and a slow network. Several steps are used to simulate these models.

1. Learning Slow Network

Architecture with two layers: an input layer with 4 neurons and an output layer with a single neuron. The algorithm has converged thus giving the following result.

Fig. 4 Evolution of the MSE slow network with 400 examples and after 100 iterations
Fig. 4 shows the high convergence rate of learning. The minimum value, which is in the range of 7.23E-32, occurs just after (100 iterations).

2. Learning Fast Network

With the same architecture as the slow network, gave the following result.

![Fig. 5 Evolution of the MSE fast network with 400 examples and after 100 iterations](image)

On note that the error stabilizes at a value of: 1.082E-27. Some 100 iterations are sufficient to learn the fastest model. The algorithm converged quickly and perfectly.

C. Combination and Comparison

\[ C_p = C_	ext{ss} + C_	ext{ef} \]

The estimated slow couple will be associated with the estimated fast couple to form the estimated global couple of the machine. Thus the estimated global error is the sum of estimation errors of the slow model and those of the fast model.

The error is in the order of 27-1.08932E.

These results demonstrate the great benefits offered by the modular model with respect to the single network model. This concerning one hand the speed of convergence and the mean square error and on the other hand, the complexity is greatly reduced (size reduced architecture). The performance of the global model depends on the performance of the slow reduced model and that the fast reduced model.

VII. CONCLUSION

The equations of asynchronous machines are nonlinear and strongly coupled. However, these models can be seen on several time scales (electrical, electromagnetic and mechanical dynamics, except for the thermal time scale).

The asynchronous machine is a strongly coupled non-linear model and has different dynamics. Modeling, identification, analysis of such systems networks of single neurons in a single time scale can be difficult. This complexity makes learning more difficult, slow convergence.

To reduce complexity and to facilitate the analysis and development of control strategies of these machines, we propose to use the new approach neuroglial with two networks, decoupling the system into two subsystems the fast and other slow using singular perturbations technique.

To conclude the modular model neuroglial offers better performance in terms of convergence speed, computational complexity and mean square error. This demonstrates that such an architecture is best suited to complex systems with two time scales.

Finally, we are still at the stage of trial and error to get a better configuration of a neural network can optimally solve a specific problem. It is hoped that in the near future developments in this field of research will better understand the complexity of the human brain and by that very fact permit us to discover new possibilities of using artificial neural networks.

In the future, several lines of research can be pursued, including:

- Modeling and control systems multi-time scales using the approach
- Modeling analysis and control in the field of robotics which have different dynamics.

REFERENCES