Limitation Imposed by Polarization-Dependent Loss on a Fiber Optic Communication System

Farhan Hussain and M.S. Islam

Abstract—Analytically the effect of polarization dependent loss on a high speed fiber optic communication link has been investigated. PDL and the signal’s incoming state of polarization (SOP) have a significant co-relation between them and their various combinations produces different effects on the system behavior which has been inspected. Pauli’s spin operator and PDL parameters are combined together to observe the attenuation effect induced by PDL in a link containing multiple PDL elements. It is found that in the presence of PDL the Q-factor and BER at the receiver undergoes fluctuation causing the system to be unstable and results show that it is mainly due to optical–signal-to-parallel-noise ratio (OSNR par) that these parameters fluctuate. Generally the Q-factor, BER deteriorates as the value of average PDL in the link increases except for depolarized light for which the system parameters improves when PDL increases.

Keywords—Bit Error Rate (BER), Optical-signal-to-noise ratio (OSNR), Polarization-dependent loss (PDL), State of polarization (SOP).

I. INTRODUCTION

As the network bandwidth is increased, either by increasing modulation rates or by multiplexing channels in wavelength or time domain, performance begins to suffer as a result of the detrimental effects produced by polarization-dependent loss. So PDL has become an important parameter in the design of today’s long-haul fiber-optic communication systems. PDL mainly occurs in optical components, such as isolators and couplers, whose insertion loss is dependent on the polarization states of input signals. When many PDL elements are linked together by optical fibers, the end-to-end transmission properties in terms of total PDL and insertion loss become statistical quantities [1]. In practice, spontaneous emission noise is generated by optical amplifiers along the link which then becomes partially polarized and fluctuates due to PDL concatenation. The fluctuation of noise, signal power and the global attenuation due to global PDL [2] all affect the quality of the transmitted signal. In previous papers the effect on signal concerned with attenuation due to PDL were analyzed by using simulations. Here the same attenuation pattern in a link containing multiple PDL elements is obtained by using Pauli’s spin operators and PDL parameters. Similarly many facts were drawn from the behavior of the system parameters of an optical communication link with PDL in the previous papers [3],[4]. Here some new findings regarding OSNR, Q-factor, BER and their relation with average PDL of the link has been acquired.

II. THEORETICAL ANALYSIS

A. Derivation of Attenuation Equations

In the process of relating the Pauli spin operators to PDL parameters [5] a relationship has been derived describing the transmission co-efficient which is given below,

\[ T_p = e^{-\alpha} \left[ \frac{e^\alpha + e^{-\alpha}}{2} + (\hat{\alpha}, \hat{\mathcal{S}}) \frac{e^\alpha - e^{-\alpha}}{2} \right] \]  

\( T_p \) is the loss co-efficient,\( \hat{\alpha} \) is an unit vector in Stokes space and \( \mathcal{S} \) points in the direction of maximum transmission. The transmission depends not only on the loss co-efficient of PDL \( \alpha \) but the relative orientation of the PDL \( \hat{\alpha} \) to the incoming state of polarization \( \hat{\mathcal{S}} \). From (1) it is found that for a particular value of \( \alpha \), \( T_p \) is maximum when \( (\hat{\alpha}, \hat{\mathcal{S}}) = 1 \) and \( T_p \) is minimum when \( (\hat{\alpha}, \hat{\mathcal{S}}) = -1 \). So the following is acquired,

\[ T_p(\text{max}) = e^{-\alpha} \left[ \frac{e^\alpha + e^{-\alpha}}{2} + (\hat{\alpha}, \hat{\mathcal{S}})_{\text{max}} \frac{e^\alpha - e^{-\alpha}}{2} \right] \]

\[ T_p(\text{min}) = e^{-\alpha} \left[ \frac{e^\alpha + e^{-\alpha}}{2} + (\hat{\alpha}, \hat{\mathcal{S}})_{\text{min}} \frac{e^\alpha - e^{-\alpha}}{2} \right] \]

So by simplifying the above equations the following is obtained,

\[ T_p(\text{max}) = 1 \]

\[ T_p(\text{min}) = e^{-2\alpha} \]  

When the input is completely depolarized, the transmission is averaged over all polarization states. In case of completely depolarized situation, \( (\hat{\alpha}, \hat{\mathcal{S}}) = 0 \) i.e. orientation of the PDL \( \hat{\alpha} \) is perpendicular to the incoming state of polarization \( \hat{\mathcal{S}} \).

\[ T_p(\text{depol}) = e^{-\alpha} \left[ \frac{e^\alpha + e^{-\alpha}}{2} \right] \]

\[ T_{\text{depol}} = T_p(\text{depol}) = \frac{1+e^{-2\alpha}}{2} \]  

The global PDL or the cumulative PDL over a concatenation is also known as normalized loss co-efficient. The relation between normalized co-efficient and the transmission extrema is,

\[ \Gamma = \frac{T_{\text{max}} - T_{\text{min}}}{T_{\text{max}} + T_{\text{min}}} \]

\[ \Gamma = \frac{1-e^{-2\alpha}}{1+e^{-2\alpha}} \]  

In Fig.1 a signal is transmitted through an optical fiber having n PDL elements including the optical fiber. The PDL
elements possess PDL loss co-efficient of $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n$. As the signal passes through all the $n$ PDL elements the mean attenuation according to [2] is,  

$$M(T_1, T_2, \ldots, T_n) = T_{\text{depol}1} T_{\text{depol}2} \cdots T_{\text{depol}n}$$  

(6)

If it is considered that the PDL loss co-efficient of all the PDL elements are the same as it has been assumed in [2] and is equal to $\alpha$, then,  

$$M(T_1, \ldots, T_n) = \frac{(1+e^{-2\alpha})}{2} \cdot \frac{(1+e^{-2\alpha})}{2} \cdots \frac{(1+e^{-2\alpha})}{2}$$  

(7)

As the signal passes through all the $n$ PDL elements the maximum transmission co-efficient i.e. minimum attenuation is obtained by [2] which is,  

$$T_{\text{max}}(1,2,\ldots,n) = T_{\text{max}1} T_{\text{max}2} \cdots T_{\text{maxn}}$$  

(8)

Similarly as the signal passes through all the $n$ PDL elements the minimum transmission co-efficient i.e. maximum attenuation is obtained by [2] which is,  

$$T_{\text{min}}(1,2,\ldots,n) = T_{\text{min}1} T_{\text{min}2} \cdots T_{\text{minn}}$$  

(9)

Again if it is considered that the PDL loss co-efficient of all the PDL elements are the same and is equal to $\alpha$ then,  

$$T_{\text{min}}(1,2,\ldots,n) = e^{-2\alpha} . e^{-2\alpha} \cdots e^{-2\alpha}$$  

$$T_{\text{min}}(1,2,\ldots,n) = e^{-2\alpha n}$$  

(10)

Co-efficient of transmission intensity, amplitude is taken into account as transmission co-efficient.

**B. Derivation of OSNR, Q-factor and BER Equations**

A polarized signal with an arbitrary polarization state and two noise modes passing through a PDL element as in Fig.2 can be described as given below derived from both [4,6]  

$$\left( \begin{array}{c} E_{\text{out}}^x \\ E_{\text{out}}^y \\ \end{array} \right) = \sqrt{1-\alpha_{\text{pdl}}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \left( \begin{array}{c} E_{\text{in}}^x \\ E_{\text{in}}^y \\ \end{array} \right)$$  

(11)

Using the above matrix equation and the input signals along with the noise fields equations given in [3] the following equations are obtained,  

$$S_{\text{out}}(t) = \sqrt{P_{\text{in}}(t)} \left( \sqrt{1-\alpha_{\text{pdl}}} \cos \theta + \sqrt{1+\alpha_{\text{pdl}}} \sin \theta \right)$$  

$$n_{\text{out}}^\text{par}(t) = \frac{N_{\text{in}}^\text{par}(t)}{\sqrt{1+\alpha_{\text{pdl}}}}$$  

$$n_{\text{out}}^\text{ort}(t) = \frac{N_{\text{in}}^\text{ort}(t)}{\sqrt{1+\alpha_{\text{pdl}}}}$$  

(12a)

(12b)

(12c)

Where $S_{\text{out}}(t)$, $n_{\text{out}}^\text{par}(t)$, $n_{\text{out}}^\text{ort}(t)$ are the output signal field, output noise field polarized parallel to the signal, output noise field polarized orthogonally to the signal, $\theta$ indicates the polarization state of the signal which is uniformly distributed within the range [-1,1].

It is assumed that the principal axes of the PDL element are aligned with the x and y axes and the lossy axis of the PDL element is the x-axis. PDL(dB) = 10log10($\frac{1+\alpha_{\text{pdl}}}{1-\alpha_{\text{pdl}}}$), where $\alpha_{\text{pdl}}$ is one half of the normalized attenuation difference between the two principal axes of PDL. Using the above field equations to get the signal and noise power after the PDL element as in [3] from which the following equations are obtained,  

$$G_s = 1 - \alpha_{\text{pdl}} \cos \theta$$  

$$G_N^\text{ort} = \frac{1 - \alpha_{\text{pdl}} \cos \theta}{1 - \alpha_{\text{pdl}} \cos \theta}$$  

(13)

(14)

Where $G_s, G_N^\text{ort}$ are the gains of output signal power and output orthogonal noise power as the signal passes through the PDL element,  

$$G_N^\text{par} = \frac{1 - 2\alpha_{\text{pdl}} \cos \theta + a_{\text{pdl}}^2 \cos^2 \theta + \sqrt{1-\alpha_{\text{pdl}}} \sin^2 \theta}{1 - \alpha_{\text{pdl}} \cos \theta}$$  

$$G_N^\text{total} = \frac{1 - 2\alpha_{\text{pdl}} \cos \theta + a_{\text{pdl}}^2 \cos^2 \theta + \sqrt{1-\alpha_{\text{pdl}}} \sin^2 \theta}{1 - \alpha_{\text{pdl}} \cos \theta}$$  

(15)

Here $G_N^\text{par}$ is the gain of the output parallel noise power and $G_N^\text{total}$ is assumed as the ratio of input orthogonal noise power to that of input parallel noise power. Now from the above gain equations the following equations are acquired,  

$$G_{\text{OSNR}} = \frac{G_s}{G_N^\text{par}}$$  

$$G_{\text{OSNR}} = \frac{1 - 2\alpha_{\text{pdl}} \cos \theta + a_{\text{pdl}}^2 \cos^2 \theta}{1 - \alpha_{\text{pdl}} \cos \theta}$$  

(16)

Where $G_{\text{OSNR}}$ represents the gain of the orthogonal optical signal to noise ratio as the signal passes through the PDL element. Similarly the followings are obtained,  

$$G_{\text{OSNR}} = \frac{G_s}{G_N^\text{ort}}$$  

$$G_{\text{OSNR}} = \frac{1 - 2\alpha_{\text{pdl}} \cos \theta + a_{\text{pdl}}^2 \cos^2 \theta}{1 - \alpha_{\text{pdl}} \cos \theta}$$  

(17)

$$G_{\text{OSNR}} = \frac{G_s}{G_N^\text{total}}$$  

(18)
Where $G_{\text{par}}^{\text{OSNR}}$ and $G_{\text{Total}}^{\text{OSNR}}$ represents the gain of the parallel optical signal to noise ratio and gain of the total optical signal to noise ratio respectively. The gain of total OSNR is defined as the ratio of total signal gain to that of total noise gain. Considering the contribution of both signal-spontaneous and spontaneous-spontaneous beating noises, the Q-factor at the receiver when the signal passes through the PDL element can be expressed [4] as in (19), shown at the bottom of the page where $B_s$ and $B_o$ are the 3-dB bandwidths of the electrical and optical filters at the receiver, $B_{\text{OSNR}}$ is the noise bandwidth for OSNR calculation, $r$ is the ratio of peak power to average power.

$$Q = \frac{4r}{\sqrt{\frac{1}{OSNR_{\text{par}}^{\text{OSNR}}} + \frac{1}{OSNR_{\text{ort}}^{\text{OSNR}}}} + \frac{B_s (2R_o - R_e) B_o^{2} OSNR}{B_{\text{OSNR}}^{2}}} + \frac{1}{\sqrt{\frac{1}{OSNR_{\text{par}}^{\text{OSNR}}} + \frac{1}{OSNR_{\text{ort}}^{\text{OSNR}}}}} + \frac{B_s (2R_o - R_e)}{B_{\text{OSNR}}}$$

The BER value indicates the quality of the communication link. PDL which is a stochastic phenomenon has a significant influence on Q-factor which consequently affects the BER.

III. RESULT AND DISCUSSION

A. Results Regarding Attenuation

In the process of studying the attenuation effect of PDL on high speed fiber optic communication link, some facts are acquired. The relative orientation of PDL elements are not fixed, but fluctuate in time due to the effect of connecting fibers. Also the input signal state of polarization fluctuates due to PMD. In Fig.3 the incoming state of polarization of the signal is varied from 0 to 180 degree and shows the effect on transmission co-efficient for different values of PDL. The loss co-efficient associated with PDL has been increased from 0.05 to 0.30 and it is observed that for a particular value of SOP as the PDL value increases the transmission co-efficient decays. In Fig.4 similarly the loss co-efficient is varied from 0.05 to 0.30 i.e. increasing the PDL value and shows the effect on transmission co-efficient for different values of incoming state of polarization. It is obvious that for a particular value of loss co-efficient as the angle for the input state of polarization increases the transmission co-efficient decreases. So it is found that for the loss co-efficient of 0.15 the transmission co-efficient is greater for 30 degree than for 150 degree. Similarly the transmission co-efficient is greater for 60 degree when compared to 120 degree for the same value of loss co-efficient of 0.15.

Generally as the loss co-efficient of the PDL elements in a link increase the value of global PDL over that concatenation will also increase. Fig.5 illustrates that for lower values of PDL the loss co-efficient of the PDL elements in the link and the corresponding normalized loss co-efficient are very close to each other. But as the PDL value increases the loss co-efficient of the PDL elements deviates from the corresponding normalized loss co-efficient and for higher values of PDL their values are no longer close to each other.
Now to analyze the concatenation effect due to PDL elements in fiber optic communication link a signal is transmitted through an optical fiber having 20 PDL elements as considered in [2]. All the 20 PDL elements are chosen to be identical, having the same value of $\alpha=0.100446$. In Fig. 6 shows that stage by stage as the number of PDL elements increase the mean attenuation decays and at the output of the link its value is equal to $e^{-2n\alpha}$, where $n=20$. Similarly as the number of PDL elements increase the minimum transmission co-efficient i.e. maximum attenuation curve decays and at the output of the link its value is equal to $e^{-2n\alpha}$, where $n=20$.

Theoretically the maximum transmission co-efficient value has been found to be equal to 1. So after 20 PDL elements the minimum attenuation is equal to 1 i.e. there is no decay in its graph. In [2] practically the value of minimum attenuation has been found to be equal to 0.98 for each element, so after 20 PDL elements the practical minimum attenuation curve decays somewhat from the theoretical curve obtained in Fig. 6. The curves of mean attenuation and maximum attenuation in Fig. 6 follows the pattern observed in [2].

B. Results Regarding OSNR, Q-factor and BER

The system model similar to [3] where a 10-Gb/s system consisting of 40 spans, each span with 3 PDL elements with random orientation amongst them, no nonlinearity and chromatic dispersion are considered. The transmitter generates $2^7-1$ pseudorandom bit sequence (PRBS) 10-Gb/s Gaussian-shaped signal pulses with 33% duty cycle, the receiver contains Gaussian optical filter with total bandwidth of 0.4 nm, a square-law detector, and a fifth-order Bessel electric filter with 3-dB bandwidth of 7.5 GHz is considered to analyze the effect of PDL on OSNR, Q-factor and BER. Fig. 7 shows that due to the random orientation of the PDL elements the signal’s incoming state of polarization to each PDL element throughout the link fluctuates from 0 to 180 degree and resembles the effect on OSNR$_{par}$, OSNR$_{ort}$ and OSNR$_{total}$ received at the receiver due to average PDL value of 3 dB. The curves obtained follow the gain patterns calculated in (16), (17) and (18) which are consistent with the results obtained in [3]. If the orientation of all the PDL elements is such that the signal’s incoming state of polarization to each PDL element throughout the link produces an angle of 90 degree then at the receiver maximum values of OSNR$_{par}$, OSNR$_{ort}$ and OSNR$_{total}$ would be detected. On the other hand if the angle is 0 or 180 degree then minimum values of OSNR$_{par}$ and OSNR$_{total}$ would again be minimum. The fluctuation of OSNR$_{ort}$ is much smaller than that of OSNR$_{par}$ and OSNR$_{total}$ as described in [3]. Now Fig. 7 also illustrates that the OSNR$_{total}$ is less than the minimum values.
OSNR\textsubscript{ort} and OSNR\textsubscript{par} which is obvious as it takes into account both the parallel and orthogonal noise effect.

Fig.8 and Fig.9 reveals the Q-factor and BER relationship with the incoming state of polarization of the signal using (19) and (20) respectively where the OSNR patterns described in Fig.7 is assumed. In the calculation r=2 for nonreturn-to-zero (NRZ) format, the receiver having an optical filter with 3-dB bandwidth (B\textsubscript{O}) of 10 GHz, an electric filter with 3-dB bandwidth (B\textsubscript{E}) of 7.5 GHz is also assumed. According to [3] the noise bandwidth for OSNR calculation i.e. B\textsubscript{OSNR}=B\textsubscript{O} has also been considered. As the PDL value increases the Q-factor decreases and the corresponding BER increases for all values of SOP except for 90 degree. In case of 90 degree SOP i.e. for depolarized light as the average PDL value increases the Q-factor increases and the corresponding BER decreases. The fluctuation of Q-factor for different values of average PDL is approximately Gaussian-distributed, owing mainly due to the corresponding fluctuation of OSNR. As the average PDL value increases the fluctuation of Q-factor increases, creating a larger picture of Gaussian-distribution. Similarly in case of BER as the average PDL value increases the fluctuation also increases. For lower values of PDL the fluctuation is relatively small.

PDL causes a much larger fluctuation of OSNR\textsubscript{ort} than that of OSNR\textsubscript{par} but the OSNR\textsubscript{par} has a much larger impact on system performance i.e. on Q-factor and BER than OSNR\textsubscript{ort}. From Fig.7 it is observed that for SOP angle from 40 to 140 degree both OSNR\textsubscript{ort} and OSNR\textsubscript{par} are changing in the same direction i.e. both are increasing and then decreasing simultaneously. So within this specified angle both of them influence the Q-factor and the corresponding BER. But within the angle limits of 40 to 0 degree and 140 to 180 degree the OSNR\textsubscript{par} is increasing and on the other hand OSNR\textsubscript{ort} is decreasing. At one point within those angle limits the OSNR\textsubscript{par} exceeds the value of OSNR\textsubscript{ort}. So within 40-0 degree and 140-180 degree OSNR\textsubscript{par} has a larger effect on the system parameters. So it is justifying in saying that overall OSNR\textsubscript{par} has a larger influence on Q-factor and the BER. That is why the Q-factor curve’s pattern is similar to the OSNR\textsubscript{par} curve’s pattern. So in order to minimize the fluctuation of Q-factor and BER which is mainly caused by OSNR\textsubscript{par} due to effect induced by PDL it is essential to minimize the fluctuation of OSNR\textsubscript{par}. One of the parameter that controls the fluctuation of OSNR\textsubscript{par} is \( \gamma_{W} \). In the process of finding the OSNR\textsubscript{ort}, OSNR\textsubscript{par} and OSNR\textsubscript{Total} versus SOP for the system model \( \gamma_{W} = 3 \) has been assumed in the calculations and found that the values obtained are a close approximation of the values obtained in [3]. Now as the value of \( \gamma_{W} \) increases the...
fluctuation of OSNR\text{par} also increases which results in large fluctuation in system parameters that unstable the system. So it is essential to minimize the value of $\gamma$ which in turn will mitigate the fluctuation of OSNR\text{par} resulting in a stable system.

Fig. 11 Relation between BER and the increasing average PDL for different values of incoming SOP of the signal in degree.

Fig. 10 shows the relation between BER and Q-factor (expressed in dB). The curve obtained clearly follows the standard curve where Q-factor in dB is calculated by $20\log_{10} Q$. As the average PDL value increases from the transmitter side to receiver side through the link the BER increases for every value of incoming SOP (except for 90 degree) of the signal. In Fig.11 the average PDL value of the link is increased from 1 dB to 3 dB and shows the effect on BER of the system for different values of incoming SOP of the signal. This pattern of increase is similar to the pattern observed in [8]. SOP angle of 30 degree has much higher value of BER than that of 70 degree for the same value of PDL. Similarly the BER for SOP angle of 160 degree is higher than that of 120 degree for the same PDL. As stated earlier the only exception is for SOP angle of 90 degree i.e. when the light is depolarized the BER of the system decreases as the average PDL value increases.

IV. CONCLUSION

In this paper a simplified model has been introduced to analyze the input signal undergoing attenuation as it travels through the link due to increase of average PDL resulting from concatenation of the PDL elements. Also it has been discussed that OSNR\text{Total} is less than OSNR\text{par} and OSNR\text{rot} detected at the receiver. Another important fact that has also been analyzed is that in presence of PDL when the input signal is polarized the Q-factor, BER deteriorates as the average PDL value increases and the main cause for this degradation is OSNR\text{par}. So it has been justified that the PDL induces detrimental effect on the system behavior and it should be readily compensated.

REFERENCES