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Abstract—The paper describes an approach for defining of k-best night vision devices based on multi-criteria mixed-integer optimization modeling. The parameters of night vision devices are considered as criteria that have to be optimized. Using different user preferences for the relative importance between parameters different choice of k-best devices can be defined. An ideal device with all of its parameters at their optimum is used to determine how far the particular device from the ideal one is. A procedure for evaluation of deviation between ideal solution and k-best solutions is presented. The applicability of the proposed approach is numerically illustrated using real night vision devices data. The proposed approach contributes to quality of decisions about choice of night vision devices by making the decision making process more certain, rational and efficient.

Keywords—K-best devices, mixed-integer model, multi-criteria problem, night vision devices.

I. INTRODUCTION

The night vision devices (NVDs) offer significant benefits for night time performed tasks over unaided vision. They allow viewing in night time during numerous applications as military, security, rescue actions, navigation, hidden-object detection, wildlife observation, hunting, tourism, entertainment, etc. [1]-[4]. As a result of technological developments there exist a constantly growing number of different NVDs types and models with different parameters values. The existing wide variety of NVDs puts the problem of proper selection of most appropriate device conforming to given user requirements.

The performance evaluation and optimal selection of engineering systems have multi-level and multi-factor features. This defines essential difficulties that can be approached by multiple criteria decision-making methods [5]. Most decision making problems deals with multiple objectives which cannot be optimized simultaneously due to the inherent incommensurability and conflict between these objectives. Making a trade-offs between these objectives becomes a major subject to get the best compromise solution. A variety of methodologies for solving multiple objective decision-making problems have been proposed [6]-[15]. There are no better or worse techniques, but some techniques better suit to particular decision problems than others do [6]. The advantage of these methods is that they can account for different impacts. The most popular multiple criteria decision-making methods include such models as scoring models [7], analytic hierarchy process (AHP) [8], analytic network process (ANP) [9], ELECTRE [10], PROMETHEE [11] utility models [12], TOPSIS [13], and axiomatic design [14]. The preference structure of PROMETHEE is based on pairwise comparisons. In this case the deviation between the evaluations of two alternatives on a particular criterion is considered. The AHP/ANP is fundamentally a process of laying out a structure of all the essential factors that influence the outcome of a decision. Numerical pairwise comparison judgments are then elicited to express people’s understanding of the importance or likely influence of these elements on the final outcome [15]. The ELECTRE is a comprehensive evaluation approach that tries to rank a number of alternatives, each one of which is described in terms of a number of criteria. The main idea of ELECTRE approach is usage of proper utilization called “outranking relations” [10]. A variant of the ELECTRE approach is the TOPSIS method. It is based on the concept that the chosen alternative should have the shortest geometric distance from the positive ideal solution and the longest geometric distance from the negative ideal solution.

In contrast to these methods where pairwise comparisons and deviation between the evaluations of two alternatives on a particular criterion is considered, the proposed approach defines k-best devices as a solution of a single multi-criteria optimization task. The basic idea is to reduce a given set of alternatives to k-best accordingly the user point of view and taking into account given relations and restrictions. The proposed optimization model allows definition of k-best solution instead determination of a single Pareto-optimal solution.

The aim of current paper is to propose an approach to assist the user by selection of k-best devices in accordance to the importance of NVDs performance parameters. The obtained subset of k-best devices can be a basis for more rational and efficient decision-making. A procedure for evaluation of deviation between ideal solution and each of k-best devices is presented.

II. PROBLEM DESCRIPTION

When choosing a NVD the user acts as a decision-maker (DM) and should consider all the relevant costs and benefits of the options for the set of devices to choose from. The preferred device should be that which comes close to the decision maker’s objectives, which may often conflict. The performance of the NVDs depends on many parameters where
the most essential are:

- **working range** \((R)\) of NVD depends on ambient light illumination, atmospheric transmittance, contrast between target and background, target area, diameter of the inlet pupil, objective focal length, objective transmittance, image intensifier tube (IIT) luminous sensitivity, IIT limiting resolution, IIT photocathode limiting light flow and signal-to-noise ratio;
- **field of view** (FOV) is parameter defining the amount of visual information provided via the device. In principle, the larger the FOV is the more information is available;
- **objective focus range** (FR) define the minimum focusing range of near objects;
- **battery life** (BL) determine the operational time duration of devices accordingly used battery types and capacity and the current of image intensifier tube;
- **weight** – currently the most of NVDs are portable devices and the weight is an important parameter that should be minimized.
- **price** – a parameter that depends on used NVDs modules that is always worth to consider when making some choice decision.

The most essential NVDs parameter taken into account when choosing a particular night vision device is its working range. A distinguish feature of working range is its dependency not only on the device modules parameters, but also on the external surveillance conditions as ambient light illumination, atmospheric transmittance, contrast between target and background, and not at last on the target area. In practice, it is unlikely that some device will perform best against all objectives and parameters to be clearly preferred. Each one will demonstrate different advantages and disadvantages. Describing the balance between objectives, and identifying the preferred option is a complex problem. The choice is usually done intuitively based on the decision-maker experience. The choice of a NVD adjusted to the user requirements is an example of complex combinatorial problem characterized by the presence of many conflicting preferences (criteria) about the NVDs parameters values. For example, choosing of the NVD using the latest technological solutions reflects on higher prices to pay. It is reasonable to look for the “user best” device among the offered NVDs, i.e. whose parameters values are best accordingly to the user point of view.

There are considerable advantages in making an explicit decision-aiding framework ensuring that all concerns are identified and addressed and the reasons behind a particular choice are made clear. The advantages of such a structured approach are particularly apparent where there are many alternative devices with numerous different parameters values. Moreover, often user is interesting in more than one alternative to make his final selection. To define k-best alternatives conforming to the given user preferences toward NVDs parameters a proper mathematical model is developed.

### III. Multi-Criteria Model Formulation of k-Best NVDs Problem

The purpose of multi-criteria problem is to support users in exploring solutions that correspond best to their preferences, i.e. multi-criteria approach fits to the situations in which users are not able to define a single goal function. On the other hand, mixed-integer optimization provides a powerful framework for mathematically modeling of many optimization problems that involve discrete and continuous variables. Therefore, the NVDs performance could be modeled as multi-criteria mixed-integer optimization problem for determining of k-best selection of devices taking into account NVDs parameters (working range, field of view, battery life, focus range, weight and price) and external surveillance conditions as follows:

\[
\text{maximize } \{R, \text{FOV}, \text{BL}\} \\
\text{minimize } \{\text{FR, Weight, Price}\}
\]

subject to

\[
R = \sum_{i=1}^{n} x_i \sqrt[0.07E_{\text{F}i}C_{\text{S}i}D_{\text{m}i}f_{\text{ob}i}f_{\text{ob}i}S_{\text{F}i}(\Phi_{\text{min}}M_{\text{F}i})}
\]

\[
\text{FOV} = \sum_{i=1}^{n} x_i \text{FOV}_{x_i}
\]

\[
\text{BL} = \sum_{i=1}^{n} x_i \text{BL}_{x_i}
\]

\[
\text{FR} = \sum_{i=1}^{n} x_i \text{FR}_{x_i}
\]

\[
\text{Weight} = \sum_{i=1}^{n} x_i \text{Weight}_{x_i}
\]

\[
\text{Price} = \sum_{i=1}^{n} x_i \text{Price}_{x_i}
\]

\[
\sum_{i=1}^{n} x_i = k, \quad x_i \in [0, 1]
\]

\[
1 < k < n
\]

where \(R\) is the NVD working range [16], \(E\) – ambient light illumination in \(lx\), \(\tau_{\text{r}}\) – atmospheric transmittance, \(C\) – contrast, \(A_{\text{r}}\) – reduced target area in \(m^2\) [16], \(D_{\text{m}}\) – diameter of the inlet pupil in \(m\), \(f_{\text{ob}}\) – objective focal length in \(mm\), \(\tau_{\text{ob}}\) – objective transmittance, \(S\) – IIT luminous sensitivity in \(A/\text{Im}\), \(\delta\) – IIT limiting resolution in \(lp/mm\), \(\Phi_{\text{min}}\) – IIT photocathode limiting light flow in \(Im\), \(M\) – IIT signal-to-noise ratio; FOV – field of view, FR – objective focus range, BL – battery life (operational time duration of NVD), weight and price of NVD,
where \( x_i \) are binary integer variables corresponding to each device, \( k \) is integer decision variable determining the number of k-best solutions, and \( n \) is the number of devices to choose from.

The k-best solutions/alternatives are modeled by means of the decision variables \( x_i \). The relation (9) of the decision variables is generalization of the classical optimization problem of finding a single solution. It contains as a special case the single-choice case for \( k = 1 \).

In relative ratio method for the multiple attributes decision making problems, a compromise solution/alternative is chosen based on the concept that the chosen alternative should be as close to the ideal solution as possible [17]. The selection process is based on evaluation of the alternatives with respect to the set of relevant criteria.

The problem of evaluation of alternatives in terms of their distance to the ideal solution can be seen as a "second-order" problem of finding a single solution. It contains a special case the single-choice case for \( k = 1 \).

To illustrate the applicability of the proposed approach real parameters data for ten night vision goggles are used as input data shown in Table I.

### IV. NUMERICAL RESULTS AND DISCUSSION

To illustrate the applicability of the proposed approach real parameters data for ten night vision goggles are used as input data shown in Table I.

### TABLE I

<table>
<thead>
<tr>
<th>No</th>
<th>NVDs type</th>
<th>( \alpha )</th>
<th>( f_{\text{lm}} ) mm</th>
<th>( s_{\text{a}} )</th>
<th>( M )</th>
<th>( S_i ) A/im</th>
<th>FOV degree</th>
<th>Battery life, hours</th>
<th>Min. focus range, cm</th>
<th>Weight, gr</th>
<th>Price, S</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Night Optics D-2MV, Gen 1+</td>
<td>40</td>
<td>26</td>
<td>0.78</td>
<td>12</td>
<td>0.00024</td>
<td>40</td>
<td>40</td>
<td>25</td>
<td>482</td>
<td>650</td>
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<tr>
<td>2</td>
<td>Regal 3250, Gen 1+</td>
<td>30</td>
<td>35</td>
<td>0.78</td>
<td>12</td>
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<td>30</td>
<td>30</td>
<td>25</td>
<td>430</td>
<td>699</td>
</tr>
<tr>
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<td>32-40</td>
<td>35</td>
<td>0.78</td>
<td>16</td>
<td>0.00031</td>
<td>30</td>
<td>10-20</td>
<td>100</td>
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</tr>
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<td>ATN Cougar CGT1, Gen 2+</td>
<td>45-54</td>
<td>35</td>
<td>0.78</td>
<td>15</td>
<td>0.00035</td>
<td>30</td>
<td>10-20</td>
<td>100</td>
<td>500</td>
<td>3696</td>
</tr>
<tr>
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<td>35</td>
<td>0.78</td>
<td>20</td>
<td>0.00087</td>
<td>30</td>
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<td>100</td>
<td>500</td>
<td>4884</td>
</tr>
<tr>
<td>6</td>
<td>ATN Night Cougar-4, Gen 4</td>
<td>64-72</td>
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<td>0.80</td>
<td>25</td>
<td>0.00115</td>
<td>30</td>
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<td>100</td>
<td>500</td>
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<td>7</td>
<td>ATN PS23-2, Gen 2</td>
<td>36-45</td>
<td>24</td>
<td>0.80</td>
<td>13</td>
<td>0.00070</td>
<td>40</td>
<td>60</td>
<td>25</td>
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<td>3550</td>
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<tr>
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<td>24</td>
<td>0.80</td>
<td>17</td>
<td>0.00110</td>
<td>40</td>
<td>60</td>
<td>25</td>
<td>700</td>
<td>4195</td>
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<td>9</td>
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<td>25</td>
<td>700</td>
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<tr>
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<td>24</td>
<td>0.80</td>
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<td>0.00190</td>
<td>40</td>
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</table>

Note: The values in Table I are taken from Internet resources [18]-[27]

To solve the formulated multi-criteria problem (1)-(9), the following normalization [28], [29] scheme is used:

\[
P_i^* = \frac{P_i - P_{\text{min}}}{P_{\text{max}} - P_{\text{min}}} \quad \text{for maximizing criteria} \quad (10)
\]

\[
N_i^* = \frac{N_{i,\text{max}} - N_i}{N_{i,\text{max}} - N_{i,\text{min}}} \quad \text{for minimizing criteria} \quad (11)
\]

Distinguish feature of this normalization scheme is providing the values for parameters between 0 and 1 based on the maximal and minimal objective values of each parameter. This normalization not only transforms data to have comparable values but also transforms the problem to a maximizing problem.

The widely used approach for solving multi-objective optimization problems is to transform a multiple objective (vector) problem into single-objective (scalar) problems. Among decision methods, weighted-sum aggregation of preferences is by far the most common, as it is a direct specification of importance weights. The weighted sum method transforms multiple objectives into an aggregated scalar objective function by multiplying each objective function by a weighting coefficient and summing up all contributors to look for the Pareto solution [30]. The transformed scalar optimization problem determining the k-best solution is defined as follows:

\[
\begin{align*}
\text{maximize} & \ h(x) = \sum_{i=1}^{6} w_i R_i(x) + w_j F_{\text{ov}}(x) + w_k B_{\text{f}}(x) + \cdots + w_{6} P_{\text{Price}}(x) \\
\text{subject to} & \quad \sum_{i=1}^{6} x_i = k, x_i \in [0, 1] \\
\sum_{i=1}^{6} w_i = 1, 0 \leq w_i \leq 1
\end{align*}
\]

where \( w_i, i = (1, 2, \ldots, 6) \) are weighting coefficients for each of the normalized objective functions.

The proposed model defines k-best Pareto optimal solutions considering the importance of each criteria expressed by DM preferences. The applicability of the proposed approach is demonstrated by four different sets of weighting coefficients reflecting four different DM preferences about importance of criteria shown in Table II.
First set of weighting coefficients (DM-1) expresses equivalent importance toward all criteria – detection range, field of view, battery life, focus range, price and weight. The second set (DM-2) simulate the DM preferences emphasizing on working range, weight and price; the corresponding set for DM-3 reflect the preferences on working range, field of view, weight and price and DM-4 expresses DM preferences about working range, focus range, weight and price. The solutions of task (12) – (14) for different sets of weighting coefficients considering all 10 devices of Table I for k=3 and k=5 best devices selections are illustrated in Fig. 1.

These groups of devices satisfy DM preferences expressed by defined weighted coefficients sets in Table III. These k-best selections of devices could be the base from which the user can make final choice decision. From the formal point of view, every Pareto-optimal solution is equally acceptable as the solution to the multi-objective optimization problem. In practice, only one solution has to be chosen as final decision and this is realized by involvement of decision maker. A procedure is proposed for helping the DM in taking of his final decision, for defining how far each of these k-best devices is from the “ideal” device:

Step 1: Definition of an “ideal” device with “ideal” parameters i.e. device whose parameters values have their optimal (maximal/minimal) values. Having in mind the normalization scheme, the objective function value of (12) – (14) for the “ideal” device is equal to 1.

Step 2: Calculation of the objective function value for each of the selected k-best devices.

Step 3: Subtract calculated value of objective function for each k-best device from objective function value of “ideal” device and determine in percentage the relative distance of devices from “ideal” one.

The results of execution of the described procedure for each of selected k-best devices are shown in Tables III-VI.

Imposing the DM-1 preferences, where all NVDs parameters are considered as of equal importance, the results show that the device #1 has a minimal deviation from the ideal solution followed by devices #8, #7 and #2. In case of DM-2 preferences minimal deviation from the ideal solution also has device #1 followed by devices #9, #10, #8 and #7. For DM-3 preferences the order of devices is #1, #8, #7, #9 and #2 and for DM-4 set of weightings the devices are ranked as #2, #1, #5, #4 and #3.

The relative distances for determined k-best devices for different sets of weighting coefficients (different DM preferences) comparing the alternatives in terms of their rank acceptability are shown on Fig. 2.
alternatives as a result of single run of the optimization task.

Four different scenarios for DM preferences are numerically tested to demonstrate the applicability of the proposed approach: 1) equivalent importance toward all defined device criteria; 2) emphasizing on working range, weight and price; 3) interesting in working range, field of view, weight and price 4) emphasizing on working range, focus range, weight and price.

The problem of comparing the alternatives in terms of their rank is considered as a “second-order” decision problem. After determining the k-best solutions, they are arranged using the relative estimation between “ideal” device and devices from particular k-best selection. A proper procedure is proposed for this goal.

The described approach for k-best choice by multi-criteria mixed-integer optimization modeling is suitable for other real-life problems to support users in exploring solutions that correspond best to their preferences. Using such an approach would contribute to improving the quality of decisions by making the decision-making process more comprehensible, efficient and rational.

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