On One Mathematical Model for Filtration of Weakly Compressible Chemical Compound in the Porous Heterogeneous 3D Medium. Part I: Model Construction with the Aid of the Ollendorff Approach

Sharif E. Guseynov, Jekaterina V. Aleksejeva, Janis S. Rimshans

Abstract—A filtering problem of almost incompressible liquid chemical compound in the porous inhomogeneous 3D domain is studied. In this work general approaches to the solution of two-dimensional filtering problems in anisotropic, inhomogeneous and multilayered medium are developed, and on the basis of the obtained results mathematical models are constructed (according to Ollendorff method) for studying the certain engineering and technical problem (for instance, see [3]-[7]), as well as when designing and constructing them; in the fight against the problem of pollution and salification of agricultural areas by ground waters (for instance, see [8], and in other dynamic processes, described by 2D elliptic equations. Solution of such problems requires elaboration of filtering process theory in those models of porous medium, which are most adequate to the natural conditions.

II. FORMULATION OF THE DIRECT LINEAR FILTERING PROBLEM IN THE ANISOTROPIC POROUS ENVIRONMENT

Suppose that our studied inhomogeneous porous domain is an anisotropic structure having a periodic volume (not compulsory with a constant period), and the main element of the structure is a rectangular prism. The coefficient of permeability written as a product

\[ k(x, y, z) = K(\alpha, \beta, \gamma) = K^{(1)}(\alpha) K^{(2)}(\beta) K^{(3)}(\gamma), \]

where \( \alpha = \alpha(x, y, z), \beta = \beta(x, y, z), \gamma = \gamma(x, y, z) \) are auxiliary functions of arguments, which, firstly, define the geometry of the periodic structure of the porous environment, and periods by \( \alpha, \beta, \gamma \) are dimensions of periodic structure elements (rectangular prisms), which form a porous region; and secondly, satisfies these conditions:

1. \( \langle \nabla \alpha, \nabla \beta \rangle = \langle \nabla \alpha, \nabla \gamma \rangle = \langle \nabla \beta, \nabla \gamma \rangle = 0, \)

2. \[
\begin{align*}
\max \left\{ & \frac{\alpha(x, y, z) + T_{\text{per.}}(\alpha(x, y, z))}{\alpha(x, y, z)} + y(\alpha, \beta, \gamma) + z(\alpha, \beta, \gamma) \right\} d\alpha,
\beta(x, y, z) + T_{\text{per.}}(\beta(x, y, z)) \left[ \frac{\partial}{\partial \beta(x, y, z)} x(\alpha, \beta, \gamma) \right] d\beta,
\gamma(x, y, z) + T_{\text{per.}}(\gamma(x, y, z)) \left[ \frac{\partial}{\partial \gamma(x, y, z)} x(\alpha, \beta, \gamma) \right] d\gamma \right\} \ll L.
\end{align*}
\]
where \( T_{\text{per.}}(\omega(x,y,z)) \) is a period of an argument function \( \omega = \{\alpha; \beta; \gamma\} \), and if along one of directions of \( \omega = \{\alpha; \beta; \gamma\} \) the coefficient of permeability \( k(x,y,z) \) is constant, then the period \( T_{\text{per.}}(\omega(x,y,z)) \) can be equal to \( T_{\text{per.}}(\omega) \), \( \tilde{\omega} = \{\alpha; \beta; \gamma\} / \omega \); \( L \) is a descriptive size of a filtering domain; \( \omega(\alpha, \beta, \gamma) = \sigma^{-1}(x,y,z) \), \( \omega(x,y,z) = \sigma(\alpha, \beta, \gamma) \).

3.

\[
\begin{align*}
&\min \left\{ \frac{\alpha(x,y,z) + T_{\text{per.}}(\alpha(x,y,z))}{\alpha(x,y,z)} \left| \frac{\partial}{\partial x(x,y,z)} x(\alpha, \beta, \gamma) \right| \tilde{t}_1 \right. \\
+ &y(\alpha, \beta, \gamma) \tilde{t}_2 + z(\alpha, \beta, \gamma) \tilde{t}_3 \bigg| d\alpha, \\
&\beta(x,y,z) + T_{\text{per.}}(\beta(x,y,z)) \left| \frac{\partial}{\partial y(x,y,z)} x(\alpha, \beta, \gamma) \right| \tilde{t}_1 \\
+ &y(\alpha, \beta, \gamma) \tilde{t}_2 + z(\alpha, \beta, \gamma) \tilde{t}_3 \bigg| d\beta, \\
&\gamma(x,y,z) + T_{\text{per.}}(\gamma(x,y,z)) \left| \frac{\partial}{\partial z(x,y,z)} x(\alpha, \beta, \gamma) \right| \tilde{t}_1 \\
+ &y(\alpha, \beta, \gamma) \tilde{t}_2 + z(\alpha, \beta, \gamma) \tilde{t}_3 \bigg| d\gamma \\
\geq & \max \{\Delta_1, \Delta_2, \Delta_3\} .
\end{align*}
\]

where \( \Delta_i \) \( (i = 1, 3) \) is a distance between the arc end of the \( \omega = \{\alpha; \beta; \gamma\} \) coordinate line and the end of the corresponding \( i \)-th \( (i = 1, 3) \) edge of the structure element (rectangular prism), i.e. if each point \( O(x,y,z) \) of the structure element (rectangular prism) is considered as a local reference point and, if the ends of three edges \( O A_i \) \( (i = 1, 3) \) of the rectangular prism are marked by points \( A_i(x,y,z) \) \( (i = 1, 3) \), and the ends of three arcs \( \{i = 1, 3\} \) of the corresponding curvilinear prism are marked by points \( A_i(\alpha, \beta, \gamma) \) \( (i = 1, 3) \), then the value of \( \Delta_i \) \( (i = 1, 3) \) is determined as \( \Delta_i \equiv |A_i - A_i| \) \( (i = 1, 3) \).

The condition 1 means that in the porous region surfaces of the level \( \alpha(x,y,z) = \alpha_0 \equiv \text{const} \), \( \beta(x,y,z) = \beta_0 \equiv \text{const} \) and \( \gamma(x,y,z) = \gamma_0 \equiv \text{const} \) create a system of triorthogonal surfaces; the condition 2 means that in the whole filtering domain the arc length

\[
\begin{align*}
\omega(x,y,z) + T_{\text{per.}}(\omega(x,y,z)) \\
\int \frac{\partial}{\partial \omega(x,y,z)} x(\alpha, \beta, \gamma) \tilde{t}_1 \\
+ y(\alpha, \beta, \gamma) \tilde{t}_2 + z(\alpha, \beta, \gamma) \tilde{t}_3 \bigg| d\omega
\end{align*}
\]

of the \( \omega = \{\alpha; \beta; \gamma\} \) coordinate line, which corresponds to the period \( T_{\text{per.}}(\omega) \), \( \omega = \{\alpha; \beta; \gamma\} \), is infinitesimal in comparison to its descriptive size \( L \); the condition 3 means that in the curvilinear prism, limited by surfaces \( \alpha = \alpha_0 \equiv \text{const} \) and \( \alpha = \alpha_0 + T_{\text{per.}}(\alpha(x,y,z)) \); \( \beta = \beta_0 \equiv \text{const} \) and \( \beta = \alpha_0 + T_{\text{per.}}(\alpha(x,y,z)) \); \( \gamma = \alpha_0 \equiv \text{const} \) and \( \gamma = \gamma_0 + T_{\text{per.}}(\gamma(x,y,z)) \), are lengths with the length

\[
\begin{align*}
\omega(x,y,z) + T_{\text{per.}}(\omega(x,y,z)) \\
\int \frac{\partial}{\partial \omega(x,y,z)} x(\alpha, \beta, \gamma) \tilde{t}_1 \\
y(\alpha, \beta, \gamma) \tilde{t}_2 + z(\alpha, \beta, \gamma) \tilde{t}_3 \bigg| d\omega,
\end{align*}
\]

are slightly deviated from the corresponding ends of the tangent lines (to be more exact – line segments).

Simplifying conditions (2)-(4) allow considering an elementary curvilinear prism as elementary rectangular prism with edges \( O A_i \) \( (i = 1, 3) \), with lengths equal to

\[
\begin{align*}
L(\omega) &\equiv \int \frac{\partial}{\partial \omega(x,y,z)} x(\alpha, \beta, \gamma) \tilde{t}_1 \\
y(\alpha, \beta, \gamma) \tilde{t}_2 + z(\alpha, \beta, \gamma) \tilde{t}_3 \bigg| d\omega, \quad \omega = \{\alpha; \beta; \gamma\} .
\end{align*}
\]

Let us denote this elementary rectangular prism as an elementary approximately-averaged prism.

III. CONSTRUCTION OF A SIMPLIFIED MODEL ON THE BASIS OF OLLENDORFF METHOD (SEE [2], [9], [10])

It is necessary to construct a simplified model, which describes the main filtering properties in anisotropic environments with a coefficient of permeability found by the formula (1), and where the functional coefficients in filtering equations are continuous with respect to space variable functions, which are not compulsory periodic. The general fluid filtering model in inhomogeneous environments with a periodically changing permeability contains filtering equations, where functional coefficients are fast oscillating functions and, in general, are piecewise continuous functions. So, finding of analytical solution for the corresponding initial and boundary problem gets more sophisticated (see fundamental monograph [1] and also [7], [8]). Since in relation to all three homogeneous flows, which are perpendicular to the surfaces of the level \( \alpha(x,y,z) = \alpha_0 \equiv \text{const} \), \( \beta(x,y,z) = \beta_0 \equiv \text{const} \) and \( \gamma(x,y,z) = \gamma_0 \equiv \text{const} \), the porous medium with the permeability coefficient (1) has various filtering properties, then the approximating porous medium with curvilinear layers in the model has to be replaced with a "fictive" anisotropic environment, which has to have fully identical filtering properties in relation to all three above mentioned flows (generally speaking, not anymore one-dimensional). Depending on such filtering flow approximation level – at a structure element level (i.e. within the limits of an elementary approximately-averaged rectangular prism) or at a filtering level in general - it is possible to talk about approximation methods, namely, about Ollendorff local homogeneously-anisotropic approximation method (obviously, at first [2], see also [1], [9], [10]) or about Leibenzon integral homogeneously-anisotropic approximation method (presumably, at first [3], see also [1], [4]-[6], [9]-[11]). Notice that local and integral homogeneously-anisotropic approximation methods set various permeability values in anisotropic models along \( \omega = \{\alpha; \beta; \gamma\} \) coordinate
lines (see [1], [6]). Taking into account the assumptions (3)-(5), we will apply Ollellendorf method for our problem to construct a filtering model in porous curvilinear layer environment with permeability, which changes periodically in space and the coefficient of which is set by the formula (1). For this purpose let us compare all three one-dimensional flows, which are perpendicular to the surfaces of the level \( \alpha(x,y,z) = \alpha_0 \equiv const, \beta(x,y,z) = \beta_0 \equiv const, \gamma(x,y,z) = \gamma_0 \equiv const \) in an elementary approximately-averaged rectangular prism (a single structure element), which is filled by a liquid (inhomogeneous environment) with permeability coefficients in the form (1), and in the same elementary approximately-averaged rectangular prism, which is filled by a homogeneous environment with rectilinear anisotropy. Because of the condition (3) it is possible to consider that within the boundaries of an elementary structure unit (elementary approximately-averaged rectangular prism) density \( \rho(\alpha, \beta, \gamma; t) \), dynamic viscosity \( \mu(\alpha,\beta,\gamma; t) \) and other physical characteristics of the liquid are constant, and, moreover, filtering process within the boundaries of this elementary structure unit is done at one moment, i.e. all physical characteristics within the boundaries of an elementary approximately-averaged rectangular prism do not depend on time (more precise mathematical interpretation of this statement/assumption by using Dirac’s delta function is stated in the work [12]; see also [13],[14]). So, in the local Cartesian coordinate system \( \alpha \times \beta \times \gamma \) (locality is understood this way: for each elementary structure unit its own Cartesian coordinate system is introduced) we can write such widely known liquid filtering equations (see, for example, [1], [12]):

\[
\frac{\partial}{\partial \alpha} \left\{ K(\alpha,\beta,\gamma) \frac{\partial p(\alpha,\beta,\gamma)}{\partial \alpha} \right\} + \frac{\partial}{\partial \beta} \left\{ K(\alpha,\beta,\gamma) \frac{\partial p(\alpha,\beta,\gamma)}{\partial \beta} \right\} + \frac{\partial}{\partial \gamma} \left\{ K(\alpha,\beta,\gamma) \frac{\partial p(\alpha,\beta,\gamma)}{\partial \gamma} \right\} = 0,
\]

(6)

where \( p(\alpha,\beta,\gamma) \) is pressure, and

\[
\vec{v} = \frac{K(\alpha,\beta,\gamma)}{\mu(\alpha,\beta,\gamma)} \cdot \nabla p,
\]

(7)

is the Darcy’s law, where \( \vec{v} = \vec{v}(\alpha,\beta,\gamma) \) defines a filtering velocity field within the elementary approximately-averaged rectangular prism.

Then for a one-dimensional filtering flow, which flows along the coordinate line within the elementary approximately-averaged rectangular prism, from (6) we can write

\[
\frac{\partial}{\partial \alpha} \left\{ K^{(1)}(\alpha) \cdot \frac{\partial p(\alpha,\beta,\gamma)}{\partial \alpha} \right\} = 0,
\]

and after integration it gives

\[
p(\alpha,\beta,\gamma) = \left[ K^{(1)}(\alpha) \frac{\partial p(\alpha,\beta,\gamma)}{\partial \alpha} \right]_{\alpha=0} + \int_{0}^{\alpha} \frac{d\alpha}{K^{(1)}(\alpha)} + p(\alpha,\beta,\gamma)|_{\alpha=0}.
\]

Assuming that a function \( p(\alpha,\beta,\gamma)|_{\alpha=L(\alpha)} \) is defined, from

the last formula we can find that

\[
\left[ K^{(1)}(\alpha) \frac{\partial p(\alpha,\beta,\gamma)}{\partial \alpha} \right]_{\alpha=0} = \frac{p(\alpha,\beta,\gamma)|_{\alpha=L(\alpha)} + p(\alpha,\beta,\gamma)|_{\alpha=0}}{L(\alpha)} \int_{0}^{\frac{d\alpha}{K^{(1)}(\alpha)}} K^{(1)}(\alpha) L(\alpha) \cdot d\alpha.
\]

Thus, we have got

\[
p(\alpha,\beta,\gamma) = \frac{1}{L(\alpha)} \int_{0}^{\frac{d\alpha}{K^{(1)}(\alpha)}} \left\{ p(\alpha,\beta,\gamma)|_{\alpha=L(\alpha)} + \frac{L(\alpha)}{K^{(1)}(\alpha)} \right\} \frac{d\alpha}{K^{(1)}(\alpha)}
\]

(8)

where \( L(\alpha) = |OA_1| \) is determined by the formula (5) and denotes the length of the elementary approximately-averaged rectangular prism edge \( OA_1 \) (we remind the according to the above mentioned construction/assumption the edge \( OA_1 \) is located at \( \alpha \)-coordinate line; the edge \( OA_2 \) is located at \( \beta \)-coordinate line; the edge \( OA_3 \) is located at \( \gamma \)-coordinate line); \( K^{(1)}(\cdot) \) is a function from (1).

The full liquid filtering flux, which flows through the side surface of the elementary approximately-averaged rectangular prism, which is perpendicular to \( \alpha \)-coordinate line can be calculated by the formula

\[
Q_\alpha \equiv \frac{d\gamma}{\mu(\alpha,\beta,\gamma)} \int_{0}^{\frac{d\alpha}{K^{(1)}(\alpha)}} L(\beta) \left( \beta_1 \right) d\beta_1 \frac{K^{(1)}(\gamma(\beta_1))}{\mu(\alpha,\beta,\gamma)} d\gamma.
\]

By repeating this assumption, we can find full fluid filtering fluxes, which flow through other two side surfaces of the elementary approximately-averaged rectangular prism, which are perpendicular to the \( \beta \) and \( \gamma \) coordinate lines, respectively:

\[
Q_\beta \equiv \frac{d\alpha}{\mu(\alpha,\beta,\gamma)} \int_{0}^{\frac{d\alpha}{K^{(1)}(\alpha)}} L(\gamma) \left( \gamma_1 \right) \frac{K^{(1)}(\beta(\gamma_1))}{\mu(\alpha,\beta,\gamma)} d\gamma,
\]

\[
Q_\gamma \equiv \frac{d\alpha}{\mu(\alpha,\beta,\gamma)} \int_{0}^{\frac{d\alpha}{K^{(1)}(\alpha)}} L(\alpha) \left( \alpha_1 \right) d\alpha_1 \frac{K^{(1)}(\gamma(\alpha_1))}{\mu(\alpha,\beta,\gamma)} d\gamma.
\]

Moreover, if our assumption that within the elementary approximately-averaged rectangular prism the dynamic viscosity of the liquid is constant (as well as other physical characteristics of the liquid) is valid, then in the last three formulæ \( \mu(\alpha,\beta,\gamma) = \mu_{const} \equiv const \), and therefore this value can be taken out of the integral sign and in addition, in the right sides of these formulæ the multiple integrals are
reduced to repeated integrals:

\[
Q_\alpha \triangleq \frac{p(\alpha,\beta,\gamma)|_{\alpha=L(\alpha)} - p(\alpha,\beta,\gamma)|_{\alpha=0}}{\mu_{\text{const.}} \int_0^{L(\alpha)} \frac{d\alpha}{K^{(1)}(\alpha)}} \times \int_0^{L(\beta)} K^{(2)}(\beta_1) d\beta_1 \int_0^{L(\gamma)^{\gamma_1}} K^{(3)}(\gamma_1) d\gamma_1,
\]

\[Q_\beta \triangleq \frac{p(\alpha,\beta,\gamma)|_{\beta=L(\beta)} - p(\alpha,\beta,\gamma)|_{\beta=0}}{\mu_{\text{const.}} \int_0^{L(\beta)} \frac{d\beta}{K^{(2)}(\beta)}} \times \int_0^{L(\alpha)} K^{(1)}(\alpha_1) d\alpha_1 \int_0^{L(\gamma)^{\gamma_1}} K^{(3)}(\gamma_1) d\gamma_1,
\]

\[Q_\gamma \triangleq \frac{p(\alpha,\beta,\gamma)|_{\gamma=L(\gamma)} - p(\alpha,\beta,\gamma)|_{\gamma=0}}{\mu_{\text{const.}} \int_0^{L(\gamma)} \frac{d\gamma}{K^{(3)}(\gamma)}} \times \int_0^{L(\alpha)} K^{(1)}(\alpha_1) d\alpha_1 \int_0^{L(\beta)} K^{(2)}(\beta_1) d\beta_1.
\]

As it is shown in [1], [15], [16], in the local Cartesian coordinate system \(x \times \beta \times \gamma\) within small volumes (in our case it is an elementary approximately-averaged rectangular prism) it is possible to consider that rectilinear anisotropic porous environment permeability coefficients along the local Cartesian coordinate axis are constant: \(k_\alpha \equiv \text{const.}, k_\beta \equiv \text{const.}, k_\gamma \equiv \text{const.}\) Then fluid filtering equation within small volume of such rectilinear anisotropic porous environment looks like this [see [1], [15]-[17]]:

\[
\sum_{i=1}^{3} k_{i} \frac{\partial^2 p(\omega_1, \omega_2, \omega_3)}{\partial \omega_i^2} = 0; \{\omega_1; \omega_2; \omega_3\} \equiv \{\alpha; \beta; \gamma\}.
\]

Moreover, the fluid filtering velocity field is found by the tensor Darcy’s law:

\[
\vec{v}_i = -\frac{k_{i \alpha}}{\mu_{\text{const.}}} \frac{\partial p}{\partial \omega_i}; \{\omega_1; \omega_2; \omega_3\} \equiv \{\alpha; \beta; \gamma\}.
\]

For a one-dimensional filtering flux, which flows along coordinate line within the elementary approximately-averaged rectangular prism, from (12) we obtain that

\[
p(\alpha, \beta, \gamma) = p(\alpha, \beta, \gamma)|_{\alpha=0} + \left[p(\alpha, \beta, \gamma)|_{\alpha=L(\alpha)} - p(\alpha, \beta, \gamma)|_{\alpha=0}\right].
\]

So, we can calculate the full fluid filtering flux, which flows through the side surface of the elementary approximately-averaged rectangular prism, which is perpendicular to the coordinate line:

\[
Q_\alpha \triangleq -\frac{k_{\alpha}}{\mu_{\text{const.}}} \int_0^{L(\beta)} \int_0^{L(\gamma)^{\gamma_1}} \frac{\partial p(\alpha, \beta_1, \gamma_1)}{\partial \alpha} d\gamma_1 d\beta_1
\]

\[= -\frac{k_{\alpha}}{\mu_{\text{const.}}} \int_0^{L(\beta)} \int_0^{L(\gamma)^{\gamma_1}} \left[p(\alpha, \beta_1, \gamma_1)|_{\alpha=L(\alpha)} - p(\alpha, \beta_1, \gamma_1)|_{\alpha=0}\right] d\gamma_1 d\beta_1
\]

\[= -\frac{k_{\alpha}}{\mu_{\text{const.}}} \frac{L(\beta)}{L(\alpha)} \int_0^{L(\gamma)} \int_0^{L(\beta)^{\beta_1}} \left[p(\alpha, \beta_1, \gamma)|_{\alpha=L(\alpha)} - p(\alpha, \beta_1, \gamma)|_{\alpha=0}\right] d\beta_1 d\gamma_1.
\]

Absolutely likewise, we can find full fluid filtering fluxes, which flow through other two side surfaces of the elementary approximately-averaged rectangular prism, which are perpendicular to the \(\beta\)– and \(\gamma\)– coordinate lines, respectively:

\[
Q_\beta \triangleq -\frac{k_{\beta}}{\mu_{\text{const.}}} \int_0^{L(\alpha)} \int_0^{L(\gamma)^{\gamma_1}} \frac{\partial p(\alpha_1, \beta, \gamma_1)}{\partial \beta} d\gamma_1 d\alpha_1
\]

\[= -\frac{k_{\beta}}{\mu_{\text{const.}}} \int_0^{L(\alpha)} \int_0^{L(\gamma)^{\gamma_1}} \left[p(\alpha_1, \beta, \gamma_1)|_{\beta=L(\beta)} - p(\alpha_1, \beta, \gamma_1)|_{\beta=0}\right] d\gamma_1 d\alpha_1
\]

\[= -\frac{k_{\beta}}{\mu_{\text{const.}}} \frac{L(\alpha)}{L(\beta)} \int_0^{L(\gamma)} \int_0^{L(\alpha)^{\alpha_1}} \left[p(\alpha, \beta_1, \gamma)|_{\gamma=L(\gamma)} - p(\alpha, \beta_1, \gamma)|_{\gamma=0}\right] d\alpha_1 d\gamma.
\]

As it was mentioned above, our considered porous region with permeability has various filtering properties in relation to one-dimensional fluxes, which are perpendicular to the surfaces of the level \(x,y,z = \alpha_0 \equiv \text{const.}, \beta \equiv \text{const.}, \gamma \equiv \text{const.}\). Therefore, when modelling, it is necessary to substitute the porous region with curvilinear surfaces with such “fictive” anisotropic environment, in order to save identical filtering characteristics in relation to the same fluxes. So, it is necessary to find the corresponding approximating “fictive” anisotropic environment parameters, so that the filtering properties of the modeled porous anisotropic region, the periodic permeability of which is set by the formula (1), would be identical in relation to all three one-dimensional fluxes. For this purpose let us compare the each formula from formulas (9)-(11) to the corresponding formula from formulas (14)-(16):

- from (9) and (14) we obtain that in the local Cartesian coordinate system \(x \times \beta \times \gamma\) within small volumes the rectilinear permeability coefficient \(k_{\alpha}\) of the anisotropic porous environment along the local Cartesian axis has to be chosen (found, calculated) as

\[
k_{\alpha} = \frac{L(\beta) L(\gamma)}{L(\alpha)} \int_0^{L(\beta)} K^{(2)}(\beta_1) d\beta_1 \int_0^{L(\gamma)} K^{(3)}(\gamma_1) d\gamma_1.
\]

- from (10) and (15) we obtain that the permeability of this environment has to be chosen as

\[
k_{\beta} = \frac{L(\alpha) L(\gamma)}{L(\beta)} \int_0^{L(\alpha)} K^{(1)}(\alpha_1) d\alpha_1 \int_0^{L(\gamma)} K^{(3)}(\gamma_1) d\gamma_1.
\]
from (11) and (16) we obtain that the permeability of the \( k_γ \)-th porous environment has to be chosen as

\[
k_γ = \frac{L(α) L(β)}{L(γ)} \int_0^{L(α)} K^{(1)} (α_1) dα_1 \int_0^{L(β)} K^{(2)} (β_1) dβ_1 \cdot \int_0^{L(γ)} dγ/\int_0^{K^{(3)} (γ_1)}.
\]

(19)

Let us denote that by putting the expressions (12) and (13) directly into the expressions (9)-(11) for each elementary approximately-averaged rectangular prism, we will obtain the following expressions for calculation of \( Q_α \), \( Q_β \), \( Q_γ \):

\[
Q_α = \frac{p(α,β,γ)|_{α=0} - p(α,β,γ)|_{α=0}}{p(α,β,γ)|_{α=L(α)}} \int_0^{L(α)} \frac{dα_1}{μ_α} \int_0^{L(β)} \frac{dβ_1}{μ_β} \int_0^{K^{(3)} (γ_1)} dγ,
\]

(20)

\[
Q_β = \frac{p(α,β,γ)|_{β=0} - p(α,β,γ)|_{β=0}}{p(α,β,γ)|_{β=L(β)}} \int_0^{L(α)} \frac{dα_1}{μ_α} \int_0^{L(γ)} \frac{dγ}{μ_γ} \int_0^{L(β)} \frac{dβ_1}{μ_β},
\]

(21)

\[
Q_γ = \frac{p(α,β,γ)|_{γ=0} - p(α,β,γ)|_{γ=0}}{p(α,β,γ)|_{γ=L(γ)}} \int_0^{L(α)} \frac{dα_1}{μ_α} \int_0^{L(γ)} \frac{dγ}{μ_γ} \int_0^{L(β)} \frac{dβ_1}{μ_β},
\]

(22)

Obviously, the formulas (20)-(22) substantially differ from the formulas (14)-(16).

Similarly to the assumptions taken prior to the acquisition of the formulas (17)-(19), it is necessary to compare the each of formulas (9)-(11) to the corresponding formulas (20)-(22). Such comparison gives us the following interesting result:

\[
k_α k_β k_γ = \frac{L(α)}{L(β) L(γ)} \int_0^{L(α)} K^{(3)} (γ_1) dγ_1 \int_0^{L(β)} K^{(2)} (β_1) dβ_1 \int_0^{L(γ)} K^{(1)} (α_1) dα_1.
\]

From here it is obvious that

\[
\frac{1}{L(α)} \int_0^{L(α)} K^{(1)} (α_1) dα_1 \int_0^{L(β)} K^{(2)} (β_1) dβ_1 \int_0^{L(γ)} K^{(3)} (γ_1) dγ_1.
\]

IV. CONCLUSIONS

In this work the filtering process is studied in non-homogeneous porous environment, which is anisotropic and periodic in space. Assuming that the coefficient of permeability of the porous environment is a multiplicative function, the following results are obtained:

- a mathematical model is elaborated, which describes a process of linear filtering in the anisotropic environment, the geometric form of which is a region curvilinear surfaces;
- a simplifying model is constructed on the basis of Ollendorff method (simplifying Ollendorff procedure), where in the filtering equation the functions-coefficients of the special variables are supposed to be continuous and not definitely periodic;
- for the simplifying model an analytic formula is found describing the full filtering flow of the fluid, which flows through wall surfaces of an elementary approximately averaged prism;
- for a one-dimensional filtering flow, which flows along the horizontal coordinate line within the limits of the elementary approximately averaged prism an analytical formula is found, which determines pressure by knowing the full filtering flow of the fluid;
- coefficients of permeability for the direct anisotropic porous environment are defined in the analytical form along the local lines of the Cartesian coordinate system.

Continuation of the current work titled "On one mathematical model for filtration of weakly compressible chemical compound in the porous heterogeneous 3D medium. Part II: Determination of the reference directions of anisotropy and permeabilities on these directions" will be prepared in the shortest terms by the authors.

Moreover, in the further research, by using the results obtained in this work, the authors are going to formulate and investigate the inverse problem of stable determination of the permeability coefficient for the porous anisotropic and anisotropic environments.

ACKNOWLEDGMENT

Present article was executed within the framework of the European Regional Development Fund’s Project No. 2010/0301/2DP/2.1.1.1.0/10/APIA/VIAA/151: “New technology and software development for biogas extraction processes optimization”, and with the pointed Project financial support.
REFERENCES


Professor Sh.E.Guseynov is the member of the Editorial Board of four International Journals; member of the Reviewers' Teams of eight International Journals; member of over ten International Societies and organizations included AMS, SIAM, EMS, ISDEDS, IAENG, etc. He was a member of the Scientific and Program Committees of more than twenty international conferences.

There is full CV of Professor Sh.E.Guseynov at www.guseynov.lv.

Jekaterina V. Aleksejeva is researcher at the Institute of Mathematical Sciences and Information Technologies, University of Liepaja, Liepaja, Latvia. Having M.S. and Ph.D. in Chemistry she was working several years at Lomonosov Moscow State University, Department of Chemistry, Chair of Natural Compounds. During 1993/1994 years was a period subject to confirmation in Max-Planck-Institut for Molecular Genetic, Berlin, Germany. In 2009 She has defended doctoral thesis in Biology and has obtained the highest scientific degree Dr.Sc.Biol. Since 2000 worked at Biomedical Research and Study Center, Riga, Latvia. Her research interests are Molecular Biology, Virology, Immunology and Epidemiology.

In 2010 Dr. J.V.Aleksejeva began the collaboration with mathematicians in the fields of mathematical modeling of complex processes in aqueous and atmospheric environment, epidemiology and biology. Jekaterina V. Aleksejeva is the author or co-author of more than 50 articles published in scientific journals and proceeding of international conferences.

Janis S. Rimshans is professor of mathematics in Numerical analysis, leading research scientist at the Institute of Mathematical Sciences and Information Technologies (in the capacity of director), Liepaja University (in the capacity of rector), Latvia.

In 1992 he has defended PhD in Mathematical Physics and Numerical Analysis, University of Latvia, Riga, Latvia. His research interests are related with Numerical Analysis, Computational Physics and Mathematical Physics with Applications Semiconductor and Device Physics, Opt-electronics, Plasma Physics, Non-linear Dynamics, Flows in Porous Media.

As author and co-author he published more than 100 scientific and technical articles including about 60 papers in refereed international journals and conference proceedings.

Sharif E. Guseynov is Professor of Mathematics and leading researcher at the Institute of Mathematical Sciences and Information Technologies, University of Liepaja, Liepaja, Latvia. He received an M.S. from Lomonosov Moscow State University in 1991, and a Ph.D. in Mathematics from Lomonosov State University in 1995. He received his Dr.Sc.Math. from University of Latvia in 2003.

His research interests are Inverse and Ill-posed Problems, Partial Differential Equations, Non-linear Equations of Mathematical Physics, Integral and Operator Equations, Mathematical Modeling, Mathematical Economics, Optimization Methods. Professor Sh.E.Guseynov is the author or co-author of about three hundred articles on inverse and ill-posed problems of mathematical physics, mathematical modeling of processes in environment and atmospheric pollution, economics, biology and epidemiology, intensive steel quenching, etc. published in scientific journals and proceedings of international conferences. He is author or co-author of three monographs: "Mathematical models of dynamics of concentration of exhaust gases in the city atmosphere", "Mathematical Modelling of Ecological Problems", "Mathematical Methods in Medical Research", M.I. GNTI, 1994, 244 p.

In Introduction to an Inverse Problems", "One-criterion Optimization Methods: Theory, Computation, Application", "Singularly Perturbed Problems for ODE and PDE", "Fundamentals of vector field and tensor analysis". He is an Editor-in-Chief of the Journal "Methods and Applications of Mathematics", ISSN 1691-614X, University of Liepaja.