Probabilistic Bhattacharyya Based Active Contour Model in Structure Tensor Space

Hiren Mewada, Suprava Patnaik

Abstract—Object identification and segmentation application requires extraction of object in foreground from the background. In this paper the Bhattacharya distance based probabilistic approach is utilized with an active contour model (ACM) to segment an object from the background. In the proposed approach, the Bhattacharya histogram is calculated on non-linear structure tensor space. Based on the histogram, new formulation of active contour model is proposed to segment images. The results are tested on both color and gray images from the Berkeley image database. The experimental results show that the proposed model is applicable to both color and gray images as well as both texture images and natural images. Again in comparing to the Bhattacharya based ACM in ICA space, the proposed model is able to segment multiple object too.

Keywords—Active Contour, Bhattacharyya Histogram, Structure tensor, Image segmentation.

I. INTRODUCTION

Region based active contour model (RAC) is widely used method in image segmentation. The RAC model gives continuous boundary of the desired object without utilizing any edge based information. These methods are not sensitive to the noise as they depend on regionally based information. One of the successful RAC model was proposed by T Chan and Vese [1] also known as CV model, where region information is incorporated with boundary information to separate the foreground object from the background. The CV model makes an assumption that the images are homogeneous. Due to intensity homogeneity, the foreground and background images can be separated using intensity mean/variance. Based on CV model, some of the popular models are localized region based ACM [2], Local binary fitting model [3] which is able to segment medical images, and hybrid ACM [4] which utilizes both gradient information and statistical intensity mean information.

However some natural images may not have homogeneous intensities (i.e. Grass, trees etc. are highly non-homogeneous in intensities) and therefore this CV model is not able to segment such images. Solutions to segment such images are the utilization of the histogram to evaluate the ACM energy function or use of the structure tensor which smooth out these non-homogeneous intensities. Histogram based approach to

drive the ACM has been utilized in many literatures like Aubert et al. [5], Kim et al. [6] and Michailovich et al. [7] utilized the histogram of an image and thus able to segment such images. In these models, the distance is calculated between the histogram inside the contour region and outside the contour region. The popular distances utilized to compare the histogram are the statistics (Euclidean distance), Kullback-Leibler divergence and the Bhattacharya distance [8]. This paper utilizes the Bhattacharya distance for the histogram comparison with the proposed model.

Structure tensor computed from the spatial derivatives of an image is a tool for estimating local orientation and widely utilized for structure estimation [9], [10]. Therefore Structure tensor is utilized with the ACM to segment the texture images. However this structure tensor cannot handle the non-homogeneous intensities and failed to segment such images too. To tackle this limitation, this paper proposed use of histogram calculation in structure tensor space. The rest of the paper is organized as follows: Section II is the background related to the proposed approach. It describes the Bhattacharya distance based ACM and highlights its limitation. Structure tensor based ACM and its drawback is also introduced in ACM. Section III describes the proposed model and the corresponding formulation of active contour. Experimental results are discussed in Section IV. Finally conclusions and future work are presented in Section V.

II. BACKGROUND

A. Bhattacharya Distance Based ACM

As suggested, a number of distance measurements have been proposed. From the comparative study done in [11], it is observed that Bhattacharyya distance is the most effective in texture discrimination in compared to Euclidean, Kullback-Leibler distance, Fisher et al proposed distance. Therefore this paper also utilizes the Bhattacharyya distance based approach for AC based image segmentation model. One of the ACM based on the Bhattacharyya distance was proposed by O. Michilovich in 2007 [7]. In this paper, the author utilized the geodesic ACM to minimize the energy function. A contour is embedded in the image domain and probability densities were estimated inside the contour given by Pin and outside the contour given by Pout. Distance between these two probability densities is accessed by the Bhattacharyya coefficient defined as:

$$B(\phi(x)) = \int \sqrt{P_{in}(z|\phi(x))P_{out}(z|\phi(x))}dz$$

(1)

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where \( z \) is the vector of image feature, and \( \phi(x) \) is the contour defined in image domain \( \Omega \). The concept behind segmentation is to reduce the information overlap by minimizing \( B(C) \). In this model, color features are utilized and to calculation of color histogram is computationally hard. Therefore an independent component analysis (ICA) space is utilized to reduce the feature space from 3-D to 2-D. As an external force, they utilized the largest eigen value of this 2-D metric as an edge function. The overall energy minimization equation is represented as:

\[
\frac{\partial \phi}{\partial t} = \alpha \int [VH(\phi(x))]|dx + v(x) \tag{2}
\]

and

\[
v(x) = 0.5B(\phi(x))(A_i^{-1} - A_o^{-1}) + 0.5 \int K_{in}(z-J(x)) \frac{1}{A_i} \sqrt{\frac{P_{in}(z|\phi(x))}{P_{in}(z|\phi(x))}} dz - 0.5 \int K_{out}(z-J(x)) \frac{1}{A_o} \sqrt{\frac{P_{out}(z|\phi(x))}{P_{out}(z|\phi(x))}} \]

where \( H \) is Heaviside function, \( J \) is feature space extracted using fast ICA and \( K \) is a kernel density function. Finally isotropic diffusion function is utilized during the minimization of energy function which reduces the noise within the image.

**B. Limitation of the Bhattacharya Gradient Flow and Proposed Variation**

In this algorithm, ICA space is utilized which reduced the 3-D space into 2-D space. The advantage achieved is in computational complexity of histogram calculation. But to utilize this ICA space, input image must be in the 3-D / color image. Therefore it cannot be utilized to segment the grayscale image. In the Bhattacharya based ACM, the isotropic diffusion function is utilized in minimization of the energy function. However this isotropic diffusion function not only removes the noise, but also removes the desired weak edges and thus makes edges harder to identify. At the same time, it also does not allow the splitting of an active contour model. Thus multiple object segmentation is not possible with this model. This paper proposed the solution towards these above said disadvantages. With reference to the above said model, in the proposed model, two variations have been proposed:

- The Bhattacharya distance is calculated on image features obtained from non-linear structure tensor space (using anisotropic diffusion function) instead of ICA space which allows the segmentation of grayscale images too.
- An anisotropic diffusion function required for updating level set function in the original model is replaced by a gradient descent method in the proposed model which allows the splitting of the level set function. Thus proposed model is able to segment the multiple objects too.

**III. PROPOSED MODEL**

As discussed in previous section, the Bhattacharya distance based model is not applicable to gray scale image because as ICA space is utilized. The function of ICA space is to reduce the 3-D features space into 2-D space. Therefore in the proposed model, use of non-linear anisotropic diffusion function based features calculation is proposed. This non-linear anisotropic smoothes images using gradient based structure tensor. The isotropic diffusion function of image (I) is given by

\[
\frac{\partial \phi}{\partial t} = \text{div}(VI) \tag{3}
\]

where \( t \) is artificial time parameter, \( VI \) is the image gradient. Perona and Malik [12] replaced this classical isotropic diffusion function with

\[
\frac{\partial \phi}{\partial t} = \text{div}(g(\|V\|, \sigma), VI) \tag{4}
\]

where \( \|V\| \) is gradient magnitude and, \( g \) is an edge stopping function. This edge function \( g \) acts as a weight age function and gives higher weight to the small values of gradient magnitude and weight age assigned to the higher values of gradient is equal to some nearby gradient values. In the proposed model, anisotropic diffusion function calculated using (4) is utilized as an edge stopping function in comparison to the largest eigenvalues obtained from the ICA space in the original model. The advantage of this function is that it is applicable to gray scale image too.

The second advantage is that it smoothes out the texture region at the same time it also preserves the edges too. So accurate stopping of contour can be obtained in comparison to the original model. Below Fig. 1 shows the comparison of images obtained using (4) and using largest eigenvalues obtained from the ICA space.
It is clear that, eigenvalues have broken edges while anisotropic diffusion function smoothes texture region between the edges and preserve the edges too. Therefore utilization of this nonlinear anisotropic function gives better segmentation in comparison to the original model. The second problem of the model is not able to segment multiple objects. The reason is that to update the level set function, they utilized the isotropic diffusion function and the properties of isotropic diffusion function do not allow the splitting of the level set function. Therefore in the proposed model, the level set function is updated using gradient decent flow.

Thus the overall problem is solved by minimizing the Bhattacharya distance and updating the level set function as follows:

\[
\frac{\partial \phi}{\partial t} = \delta_k (\phi) \left[ \mu \text{div} \frac{\nabla \phi}{|\nabla \phi|} - v(x) \right]
\]  

(5)

where, \(v(x)\) is according to (2). But change is that feature channel \(J\) is consists of the original image and anisotropic diffusion function as defined in (4). In summary the proposed model is in the form of following pseudo code:

1. Initialize the parameter \(dt\) and \(\mu\).
2. Calculate the feature channel \(J\) using anisotropic diffusion function and gray scale version of the original image.
3. Initiate level set function \(\phi_{in}(x)\).
4. Estimate the probability densities \(P_{in}\) and \(P_{out}\).
5. For \(t>0\) until convergence

IV. EXPERIMENT RESULTS AND DISCUSSION

The experiment results are demonstrated using both grayscale images and color images. As well as the case of multiple object is also considered to show the advantage of the proposed model against original Bhattacharya based GVF model where the single level set function can split into multiple contour. Initially to prove the working of the algorithm, color images are considered. In all images, the initial contour is placed near to the desired object. All the images are obtained from the Berkeley image database [13]. As proposed model is more relevant to the Bhattacharya distance based gradient flow model, initially the proposed model is compared with this model to demonstrate the advantage of the proposed model.

From the bottom Fig. 2, it can be seen that original Bhattacharya Gradient flow model do not able segment tail part properly and contour also moves inside the tiger head region while proposed model segment the image accurately.
Similarly the Bhattacharya based gradient flow model is applied to wolf image (Fig. 3), as eigenimage do not define proper edges, the contour cannot reach to the true boundary near to ear part. While the anisotropic structure tensor in the proposed model helps contour to stop at true desired boundary.

![Fig. 3](image1.jpg) (Left) Segmentation using Bhattacharya based gradient flow (Right) Segmentation using the proposed model

The proposed model is also tested on the grayscale image. Following zebra image (Fig. 4) shows the segmented output obtained using the proposed model.

![Fig. 4](image2.jpg) Original image and its segmentation using the proposed Model

As said earlier, the proposed model is also able to segment multiple objects with initialization of single contour too in comparison to original Bhattacharya gradient flow model. Some of the images having multiple objects and corresponding segmentations are shown in Fig. 5.

![Fig. 5](image3.jpg) Multi object image segmentation (Top) Original Image (Bottom Left) Using Bhattacharya gradient flow (Bottom right) using the proposed model

Similarly Fig. 6 shows the image containing two objects and its segmentation using the proposed model.

![Fig. 6](image4.jpg) Multi object image segmentation (Left) Original Image (right) segmentation using the proposed model

The proposed model is also tested for noisy images. As suggested, the feature vector in the proposed model is generated using anisotropic diffusion function. This allows the smoothing of noise and preserves the object noise. Fig. 7 shows the segmentation obtained in noisy image using the proposed model.

![Fig. 7](image5.jpg) Noisy image segmentation using the proposed model
V. CONCLUSION AND FUTURE WORK

This paper presents texture image segmentation using the probabilistic Bhattacharya based model. In the proposed model, the feature vectors are calculated in structure tensor space. Here anisotropic diffusion function (a nonlinear structure tensor) is utilized to obtain the image features. Bhattacharya histogram is calculated on this obtained feature channels. The proposed approach is compared with the Bhattacharya gradient vector flow based Active contour model. In comparison to this model, the proposed model is applicable to grayscale images and it also allows the segmentation of multiple objects with initialization of single contour. For that gradient decent flow based level set function is utilized instead of isotropic diffusion based updation. The experimental results show the improvement achieved using the proposed model in comparison to the Bhattacharya gradient flow model. Future work includes the narrow band evolution of contour evolution. Narrowband evolution reduces the computational complexity as histogram calculation is required near to contour only instead of calculating histogram over the entire image domain.

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