Study on Electrohydrodynamic Capillary Instability with Heat and Mass Transfer

D. K. Tiwari, Mukesh Kumar Awasthi, and G. S. Agrawal

Abstract—The effect of an axial electric field on the capillary instability of a cylindrical interface in the presence of heat and mass transfer has been investigated using viscous potential flow theory. In viscous potential flow, the viscous term in Navier-Stokes equation vanishes as vorticity is zero but viscosity is not zero. Viscosity enters through normal stress balance in the viscous potential flow theory and tangential stresses are not considered. A dispersion relation that accounts for the growth of axisymmetric waves is derived and stability is discussed theoretically as well as numerically. Stability criterion is given by critical value of applied electric field as well as critical wave number. Various graphs have been drawn to show the effect of various physical parameters such as electric field, heat transfer capillary number, conductivity ratio, permittivity ratio on the stability of the system. It has been observed that the axial electric field and heat and mass transfer both have stabilizing effect on the stability of the system.

Keywords—Capillary instability, Viscous potential flow, Heat and mass transfer, Axial electric field.

I. INTRODUCTION

Capillary instability arises when a fluid cylinder in an infinite fluid collapses under the action of capillary forces due to surface tension [16], [19]. The capillary instability occurs in various situations such as film boiling, Breaking of liquid jet and in many Chemical and Metallurgical processes. The study of heat and mass transfer across the interface is very important in many situations such as boiling heat transfer in chemical engineering and in geophysical problems. The general formulation of the interfacial flow problem of two inviscid incompressible fluids with heat and mass transfer for Rayleigh-Taylor and Kelvin-Helmholtz instabilities in plane geometry was established by Hsieh [7], [8]. Hsieh [8] found that when the vapour layer is hotter than the liquid layer, the effect of heat and mass transfer tends to inhibit the growth of instability. Nayak and Chakraborty [5] established the formulation of Kelvin-Helmholtz instability of the cylindrical interface between the liquid and vapour phases with heat and mass transfer.

Viscous potential flow theory has played an important role in studying various stability problems. In viscous potential flow, we consider irrotational flow, so the viscous term i.e. $\mu \nabla^2 \mathbf{u}$ in the Navier-Stokes equation is identically zero when the vorticity is zero but the viscous stresses are not zero, where $\mu$ denotes viscosity and $\mathbf{u}$ denotes velocity of fluid flow. Tangential stresses are not considered in viscous potential theory and viscosity enters through normal stress balance. Stability of the system is discussed theoretically as well as numerically. Stability criterion is given by critical value of applied electric field as well as critical wave number. Various graphs have been drawn to show the effect of various physical parameters such as electric field, heat transfer capillary number, conductivity ratio, permittivity ratio on the stability of the system. It has been observed that the axial electric field and heat and mass transfer both have stabilizing effect on the stability of the system.

D. K. Tiwari is with the Department of Mathematics, Indian Institute of Technology, Roorkee, India.
Mukesh Kumar Awasthi is with the Department of Mathematics, University of Petroleum and Energy Studies, Dehradun, India (corresponding author to provide phone:+91-1332-285157; e-mail:mukeshiniitr.kumar@gmail.com).
G. S. Agrawal is with the Institute of Computer Application, Maglayatan University, Allahabad, India.
with the interface admitting heat and mass transfer. It was observed that uniform electric field has stabilizing effect. It was also found that the instability criterion is independent of heat and mass transfer coefficient. Elhefnawy and Moatimid [2] have studied the effect of an axial electric field on the stability of cylindrical flows in the presence of mass and heat transfer and absence of gravity. They observed that the electric field has strong stabilizing influence for all short and long wavelengths. Elcoot [1] has studied the nonlinear analysis of capillary instability of viscous fluids in the presence of axial electric field. Recently, Asthana and Agrawal [18] have studied the viscous potential flow analysis of electrohydrodynamic Kelvin-Helmholtz instability at the plane interface and concluded that the tangential electric field has stabilizing effect on the critical value of relative velocity while relative velocity has destabilizing effect on the critical value of electric field. Awasthi et al. [14] have studied the effect of irrotational shearing stresses on the capillary instability in the presence of heat and mass transfer and found that irrotational shearing stresses stabilize the system. Awasthi and Agrawal [13] has studied the viscous contribution to the pressure for the potential flow analysis of capillary instability with axial electric field and observed that the axial electric field has stabilizing effect on the stability of the system. Awasthi and Asthana [15] have studied the effect of various physical parameters such as electric field, heat transfer capillary number on the stability of the system. Various neutral curves have been drawn to show the effect of various physical parameters such as electric field and heat and mass transfer on growth rates is as the critical wave number is obtained. The effect of the ratio of permittivity of fluids on stability of the system is also studied. The effect of the electrical conductivities and permittivities, respectively, which have not been considered earlier. The effect of gravity and free surface charges at the interface is neglected. A dispersion relation is derived and stability is discussed theoretically as well as numerically. A critical value of the electric field as well as the critical wave number is obtained. The effect of the electric field and heat and mass transfer on growth rates is studied. The effect of ratio of electrical conductivities and ratio of permittivity of fluids on stability of the system is also studied and shown graphically. Various neutral curves have been drawn to show the effect of various physical parameters such as electric field, heat transfer capillary number on the stability of the system.

### II. PROBLEM FORMULATION

A system of two incompressible and viscous fluids, separated by a cylindrical interface, is considered in an annular configuration as shown in Fig. 1. The undisturbed cylindrical interface is taken at radius $R$. In the formulation the subscripts 1 and 2 denote variables associated with the fluid inside and outside the interface, respectively. In the undisturbed state, viscous fluid of thickness $h_1$, density $\rho_1$, viscosity $\mu_1$, electrical conductivity $\sigma_1$ and permittivity $\varepsilon_1$ occupies the inner region $R_1 < r < R$ and viscous fluid of thickness $h_2$, density $\rho_2$, viscosity $\mu_2$, electrical conductivity $\sigma_2$ and permittivity $\varepsilon_2$ occupies the outer region $R < r < R_2$. The bounding surfaces $r = R_1$ and $r = R_2$ are considered to be rigid. The temperatures at $r = R_1, r = R$ and $r = R_2$ are $T_1, T_0$ and $T_2$, respectively. Both the fluids are assumed to be incompressible and irrotational. A cylindrical system of coordinates $(r, \theta, z)$ is assumed so that in the equilibrium state $z$-axis is the axis of symmetry of the system. Small axisymmetric disturbances are superimposed on the basic rest state. After disturbance, the interface is given by

$$F(r, z, t) = r - R - \eta(z, t) = 0$$ (1)

where $\eta$ is the perturbation in the radius of the interface from the equilibrium value $R$, and for which the outward unit normal vector is given by

$$\mathbf{n} = \frac{\nabla F}{|\nabla F|} = \left\{ 1 + \left( \frac{\partial \eta}{\partial z} \right)^2 \right\}^{-1/2} \left[ e_r - \frac{\partial \eta}{\partial z} e_z \right]$$ (2)

where $e_r$ and $e_z$ are unit vectors along the $r$ and $z$ directions, respectively.

The velocity is expressed as the gradient of the potential function and the potential functions satisfy the Laplace equation as a consequence of the incompressibility constraint, i.e.

$$\nabla^2 \phi_j = 0 \quad \text{for} \ (j = 1, 2)$$ (3)

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$. The two fluids are subjected to an external electric field $E_0$, acting along $z$-axis i.e. $E = E_0 \mathbf{e}_z$. It is assumed that the quasi-static approximation is valid for the problem, hence the electric field can be derived from electric scalar potential function $\psi(r, z, t)$ such that

$$E_j = E_0 \mathbf{e}_z - \nabla \psi_j, \quad (j = 1, 2)$$ (4)

Gauss’s law requires that the electric potentials also satisfy Laplace’s equation i.e.

$$\nabla^2 \psi_j = 0, \quad (j = 1, 2)$$ (5)
The boundary conditions at the rigid cylindrical surfaces \( r = R_1 \) and \( r = R_2 \) are given by

\[
\frac{\partial \theta_j}{\partial r} = 0 \quad \text{at} \quad r = R_j \quad (j = 1, 2) \tag{6}
\]

\[
\frac{\partial \psi_j}{\partial z} = 0 \quad \text{at} \quad r = R_j \quad (j = 1, 2) \tag{7}
\]

It is assumed that phase-change takes place locally in such a way that the net phase-change rate at the interface is equal to zero. The interfacial condition, which expresses the conservation of mass across the interface, is given by (Hsieh [8])

\[
\| \rho \left( \frac{\partial F}{\partial t} + \nabla \cdot \nabla F \right) \| = 0 \quad \text{at} \quad r = R + \eta \tag{8}
\]

where \( \| X \| \) represents the difference in a quantity across the interface, it is defined as \( \| X \| = X_1 - X_2 \).

The tangential component of the electric field must be continuous across the interface i.e.

\[
\| E_t \| = 0 \tag{9}
\]

where \( E_t = (\mathbf{n} \times \mathbf{E}) \) is the tangential component of the electric field.

There is discontinuity in the normal current across the interface; charge accumulation within a material element is balanced by conduction from bulk fluid on either side of the surface. The boundary condition, corresponding to normal component of the electric field, at the interface is given by

\[
\| E_n \| = 0 \tag{10}
\]

where \( E_n = (\mathbf{n} \cdot \mathbf{E}) \) is the normal component of the electric field.

The interfacial condition for energy transfer proposed by Hsieh [8] can be expressed as

\[
L \rho_1 \left( \frac{\partial F}{\partial t} + \nabla \cdot \nabla F \right) = S(\eta) \quad \text{at} \quad r = R + \eta \tag{11}
\]

where \( L \) is the latent heat released during phase transformation and \( S(\eta) \) denotes the net heat flux from the interface. In deriving equation (11), Hsieh [8] assumed that the amount of latent heat released depends mainly on the instantaneous position of the interface.

In the equilibrium state, the heat fluxes in positive radial-direction in the fluid phases 1 and 2 are \(-K_1(T_1 - T_0)/R \ln (R_1/R)\) and \(-K_2(T_0 - T_2)/R \ln (R_2/R_1)\) respectively where \( K_1 \) and \( K_2 \) denote the heat conductivities of the two fluids. The net heat flux \( S(\eta) \) is expressed as (Nayak and Chakraborty [5])

\[
S(\eta) = \frac{K_2(T_0 - T_2)}{(R + \eta) \ln R_2 - \ln (R + \eta)} - \frac{K_1(T_1 - T_0)}{(R + \eta) \ln (R + \eta) - \ln R_1} \tag{12}
\]

Expanding \( S(\eta) \) about \( \eta = 0 \) as

\[
S(\eta) = S(0) + \eta S'(0) + \frac{1}{2} \eta^2 S''(0) + \frac{1}{6} \eta^3 S'''(0) + \ldots \tag{13}
\]

Since \( S(0) = 0 \), from equation (12) we get

\[
\frac{K_2(T_0 - T_2)}{R \ln (R_2/R)} = \frac{K_1(T_1 - T_0)}{R \ln (R/R_1)} = G \tag{14}
\]

Hence in the equilibrium state, heat fluxes across the interfaces are equal. The interfacial condition for conservation of momentum is given by:

\[
\rho_1 \left( \nabla \cdot \nabla F \right) = \rho_2 \left( \nabla \cdot \nabla F \right) + \left( p_2 - p_1 + 2 \mu \frac{\nabla \cdot \nabla \psi}{\nabla \cdot \nabla F} \right) + \frac{1}{2} \left( \epsilon \left( E_0^2 - E_2^2 \right) + T \nabla \cdot \mathbf{n} \right) |\nabla F|^2 \tag{15}
\]

where \( p \) represents the pressure and \( T \) denotes the surface tension. Surface tension has been assumed to be a constant, neglecting its dependence on temperature. Pressure can be obtained using Bernoulli’s equation.

### III. Linearized Equations

It has been observed that the asymmetric disturbances are always stable for capillary instability. A long cylinder of liquid is unstable to the axisymmetric disturbances with wavelengths greater than \( 2\pi R \), where \( R \) is the radius of the cylinder. Hence, we considered only axisymmetric disturbances in this analysis. Now, axisymmetric disturbances are imposed on the equations (8), (9), (10), (11) and (15) and retaining the linear terms we can get the following equations.

\[
\| \rho \left( \frac{\partial \phi}{\partial t} - \frac{\partial \eta}{\partial t} \right) \| = 0 \tag{16}
\]

\[
\| \frac{\partial \psi}{\partial z} \| = 0 \tag{17}
\]

\[
\| \sigma \left( \frac{\partial \psi}{\partial r} + E_0 \frac{\partial \eta}{\partial z} \right) \| = 0 \tag{18}
\]

\[
\left[ \rho_1 \left( \frac{\partial \phi_j}{\partial t} - \frac{\partial \eta}{\partial t} \right) \right] = \alpha \eta \tag{19}
\]

\[
\| \rho \frac{\partial \phi_j}{\partial t} + E_0 \frac{\partial \eta}{\partial z} + 2 \mu \frac{\partial^2 \psi}{\partial r^2} \| = T \left( \frac{\eta}{R^2} + \frac{\partial^2 \eta}{\partial z^2} \right) \tag{20}
\]

where equation (19) is obtained using equations (12)-(14) with equation (11) and

\[
\alpha = \frac{G}{LR \ln (R/R_1) \ln (R_2/R)} \tag{21}
\]

Now the normal mode technique is used to find the solution of the governing equations. Let

\[
\eta = A e^{i(k z - \omega t)} + c.c. \tag{21}
\]

where \( A \) represents the amplitude of the surface wave, \( k \) denotes the real wave number, \( \omega \) is the growth rate and \( c.c. \) refers the complex conjugate of the preceding term. On solving equations (3) and (5) with the help of boundary conditions we get

\[
\phi_j = \left( -\frac{\alpha}{\rho_j} + i \omega \right) \left( \frac{I_0(k R) K_0(k R_j) - I_0(k R_j) K_0(k R)}{D_j(k)} \right) A e^{i(k z - \omega t)} + c.c., \quad (j = 1, 2) \tag{22}
\]
In equation (26) putting \( \psi_k \) reduces to
\[
\psi_1 = \frac{i_k (\sigma_2 - \sigma_1) E_0 g_2(k)}{\sigma_1 g_2(k) G_1(k) - \sigma_2 g_1(k) G_2(k)}
\]
\[\left[ I_0(k R) K_0(k R_1) - I_0(k R_1) K_0(k R) \right] \text{Ai}'(k z - \omega t) + \text{c.c.} \tag{23}
\]
\[
\psi_2 = \frac{i_k (\sigma_2 - \sigma_1) E_0 g_1(k)}{\sigma_2 g_2(k) G_1(k) - \sigma_1 g_1(k) G_2(k)}
\]
\[\left[ I_0(k R) K_0(k R_2) - I_0(k R_2) K_0(k R) \right] \text{Ai}'(k z - \omega t) + \text{c.c.} \tag{24}
\]
where
\[
D_1(k) = I_0^2(k R) K_0^2(k R) - I_0^2(k R) K_0^2(k R_1),
\]
\[
g_1(k) = I_0(k R) K_0(k R) - I_0(k R_1) K_0(k R_1),
\]
\[
G_1(k) = I_0^2(k R) K_0(k R) - I_0^2(k R_1) K_0^2(k R_1), \ (j = 1, 2)
\]
and symbols \( I_0 \) and \( K_0 \) are modified Bessel functions of first and second kind respectively and \( \rho \) on modified Bessel functions denotes the differentiation with respect to \( r \) when \( r = R, R_1 \) or \( R_2 \).

IV. DISPERSION RELATION

Substituting the values of \( \eta, \phi_1, \phi_2, \psi_1 \) and \( \psi_2 \) in equation (20) we get the dispersion relation
\[
D(\omega, k) = a_0 \omega^2 + i \omega_0 - a_2 = 0 \tag{25}
\]
where
\[
a_0 = \frac{\alpha_1 M_1(k)}{D_1(k)} - \frac{\rho_2 M_2(k)}{D_2(k)}
\]
\[
a_1 = \alpha \left( \frac{M_1(k)}{D_1(k)} - \frac{M_2(k)}{D_2(k)} + \frac{2 \mu_1 N_1(k)}{D_2(k)} - \frac{2 \mu_2 N_2(k)}{D_2(k)} \right)
\]
\[
a_2 = \frac{2 \mu_2 N_2(k)}{D_2(k)} - \frac{2 \mu_1 N_1(k)}{D_2(k)} - \frac{\omega^2 c^2 g_2(k)}{k^2} + \frac{T}{k^2} (k^2 R^2 - 1)
\]
\[
= \frac{k^2 E^2 g_1(k) g_2(k)(\varepsilon_1 - \varepsilon_2)(\sigma_1 - \sigma_2)}{\sigma g_2(k) G_2(k) - \sigma g_2(k) G_2(k)}
\tag{27}
\]
Neural curves are obtained by putting \( \omega_0(k) \). Equation (26) reduces to \( a_2 = 0 \), which in turn implies that
\[
2 \mu_2 N_2(k) - 2 \mu_1 N_1(k) = \frac{T}{k^2} (k^2 R^2 - 1)
\]
\[
= \frac{k^2 E^2 g_1(k) g_2(k)(\varepsilon_1 - \varepsilon_2)(\sigma_1 - \sigma_2)}{\sigma g_2(k) G_2(k) - \sigma g_2(k) G_2(k)}
\tag{27}
\]
VI. RESULTS AND DISCUSSIONS

Following parametric values have been considered for the system of interest containing vapour in the inner region and liquid in the outer region.
\[
\rho_1 = 0.0012 g m^{-3}, \rho_2 = 1.0 g m^{-3}, \mu_1 = 0.0018 \text{ poise},
\]
\[
\mu_2 = 0.01 \text{ poise}, T = 60.0 \text{ dynes/cm}.
\]
The diameters of the inner and outer cylinders are taken as 1 cm and 2 cm, respectively. The conductivity ratio \( \sigma \) and permittivity ratio \( \varepsilon \) are taken as 0.2 and 0.01, respectively for numerical calculations, otherwise mentioned. At the interface, phase change is taking place. Neutral curves divide the plane into a stable region denoted by S (above the curve) and an unstable region denoted by U (below the curve). In the following the effect of various physical parameters on the onset of instability is interpreted through various Figures and Tables.

In Fig. 2, the neutral curves for the critical wave number \( k_c \) versus vapor fraction \( h \) have been drawn for various values of heat transfer capillary number \( Ca \) when there is
no electric field i.e. $E = 0$. It has been observed that as $Ca$ increases, the stable region (upper region) grows. Therefore, $Ca$ has a stabilizing effect on the stability of the system. The effect of heat and mass transfer on the stability of the system can be explained in terms of local evaporation and condensation at the interface. At a perturbed interface, crests are warmer because they are closer to the hotter boundary on the vapour side, thus local evaporation takes place, whereas troughs are cooler and thus condensation takes place. The liquid is protruding to a hotter region and the evaporation will diminish the growth of disturbance waves. Fig. 3 shows the neutral curves for the critical wave number $k_c$ versus vapor fraction $h$ for various values of heat transfer capillary number $Ca$ when electric field intensity $E = 5$. The heat and mass transfer phenomenon has stabilizing effect on the stability of the system even in the presence of electric field and this effect is enhanced in the presence of an electric field.

For a fixed value of vapour thickness $h$, on increasing $Ca$, the critical wave number $k_c$ decreases and finally vanishes at threshold $Ca$.

The neutral curves for critical wave number $k_c$ versus vapor fraction $h$ for various values of electric field intensity $E$ at heat transfer capillary number $Ca = 0.3$ have been shown in Fig. 4. It has been observed that for a fixed value of $h$ and $Ca$, the critical wave number $k_c$ decreases on increasing electric field intensity $E$. Therefore, it is concluded that $E$ has stabilizing effect. The variation of the critical wave number for the different values of vapour fraction is illustrated in Fig. 5. As vapour thickness increases, at the crests more evaporation will take place. This additional evaporation will increase the amplitude of the disturbance waves and the system becomes destabilized as observed from Fig. 5.

In Tables I and II, maximum growth rates $(\omega_0)_m$, corresponding wave numbers $k_m$ and critical wave numbers $k_c$ as a function of heat transfer capillary number $Ca$ have been shown for different values of vapour fraction $h$ at various values of electric field intensity $E$. It is observed that the maximum growth rates decrease on increasing $Ca$ and growth rates vanish at certain $Ca$, known as threshold $Ca$. This threshold $Ca$ remains same as the electric field increases at some fixed value of $h$.

In Fig. 6, the neutral curves for the electric field intensity $E$ versus wave number $k$ have been plotted for various values of heat transfer capillary number $Ca$ when vapor thickness...
The system is unstable otherwise it is stable. It has been observed that the stable region increases on increasing the heat transfer coefficient $C\alpha$ and hence it is concluded that $C\alpha$ has stabilizing effect on the critical electric field. Variation of neutral curves for electric field for different values of kinematic viscosity ratio of two fluids $\kappa$ have been shown in Fig. 7 for $C\alpha = 0.1$ and $h = 0.01$. It is concluded that the critical wave number $k_c$ decreases with increasing $\kappa$. It is also found that for every $E$, the threshold $\kappa$ remains same for some fixed value of $C\alpha$ and $h$. As the kinematic viscosity ratio $\kappa$ decreases, the critical electric field $E_c$ increases.

### Table I

MAXIMUM GROWTH RATE $(\omega)_m$, CORRESPONDING WAVE NUMBER $k_m$, AND CRITICAL WAVE NUMBER $k_c$ FOR DIFFERENT VALUE OF $E$ AT $h = 0.01$.

<table>
<thead>
<tr>
<th>$C\alpha$</th>
<th>$(\omega)_m$</th>
<th>$k_m$</th>
<th>$k_c$</th>
<th>$(\omega)_m$</th>
<th>$k_m$</th>
<th>$k_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.5019</td>
<td>0.6707</td>
<td>0.9901</td>
<td>0.3759</td>
<td>0.4940</td>
<td>0.7145</td>
</tr>
<tr>
<td>0.02</td>
<td>0.0897</td>
<td>0.6586</td>
<td>0.9333</td>
<td>0.0477</td>
<td>0.4739</td>
<td>0.6734</td>
</tr>
<tr>
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<td>0.0355</td>
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<td>0.8729</td>
<td>0.0185</td>
<td>0.4458</td>
<td>0.6297</td>
</tr>
<tr>
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<td>0.0175</td>
<td>0.5703</td>
<td>0.8080</td>
<td>0.0091</td>
<td>0.4137</td>
<td>0.5827</td>
</tr>
<tr>
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<td>0.0091</td>
<td>0.5221</td>
<td>0.7374</td>
<td>0.0047</td>
<td>0.3775</td>
<td>0.5317</td>
</tr>
<tr>
<td>0.10</td>
<td>0.0047</td>
<td>0.4659</td>
<td>0.6592</td>
<td>0.0024</td>
<td>0.3373</td>
<td>0.4753</td>
</tr>
<tr>
<td>0.12</td>
<td>0.0022</td>
<td>0.4016</td>
<td>0.5705</td>
<td>0.0011</td>
<td>0.2892</td>
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</tr>
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<td>0.0008</td>
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<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

### Table II

MAXIMUM GROWTH RATE $(\omega)_m$, CORRESPONDING WAVE NUMBER $k_m$ AND CRITICAL WAVE NUMBER $k_c$ FOR DIFFERENT VALUE OF $E$ AT $h = 0.1$.

<table>
<thead>
<tr>
<th>$E$</th>
<th>$(\omega)_m$</th>
<th>$k_m$</th>
<th>$k_c$</th>
<th>$(\omega)_m$</th>
<th>$k_m$</th>
<th>$k_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.4306</td>
<td>0.6225</td>
<td>0.9901</td>
<td>0.3759</td>
<td>0.4940</td>
<td>0.7145</td>
</tr>
<tr>
<td>0.2</td>
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<td>0.0885</td>
<td>0.2329</td>
<td>0.3274</td>
</tr>
<tr>
<td>0.4</td>
<td>0.0206</td>
<td>0.5502</td>
<td>0.7834</td>
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<td>0.2129</td>
<td>0.3021</td>
</tr>
<tr>
<td>0.6</td>
<td>0.0095</td>
<td>0.5020</td>
<td>0.7124</td>
<td>0.0014</td>
<td>0.1928</td>
<td>0.2747</td>
</tr>
<tr>
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<td>0.6335</td>
<td>0.0007</td>
<td>0.1727</td>
<td>0.2442</td>
</tr>
<tr>
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<td>0.3815</td>
<td>0.5435</td>
<td>0.0003</td>
<td>0.1486</td>
<td>0.2094</td>
</tr>
<tr>
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<td>0.3092</td>
<td>0.4534</td>
<td>0.0001</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
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</table>
Fig. 10. The neutral curve $E_c$ vs $k_c$ for different values of $Oh$ for $Ca = 0.1$ at $h = 0.01$.

Fig. 11. The growth rate $\omega_0$ vs $k$ for $E = 0$ for different values of $Ca$ at $h = 0.01$.

viscosity ratio $\kappa$ increases, the viscosity of the inside fluid increases and hence $\kappa$ has stabilizing effect on the stability of the system.

Variation of neutral curves for electric field for different values of the conductivity ratio $\sigma$ have been shown in Fig. 8 for $Ca = 0.1$ and $h = 0.1$. The stable region decreases as $\sigma$ increases and hence it is concluded that $\sigma$ has destabilizing effect on the critical electric field. The variation of critical electric field for various values of the permittivity ratio $\epsilon$ for $Ca = 0.1$ and $h = 0.1$ have been shown in Fig. 9. It is observed that $\epsilon$ has destabilizing effect on the critical value of electric field.

The evolution of the neutral curves for electric field intensity $E$ versus wave number $k$ for different values of Ohnesorge number has been shown in Fig. 10. It has been observed that Ohnesorge number has destabilizing effect on the stability of the system. Through increasing Ohnesorge number, the viscosity of the outside fluid will decrease and less resistance to the fluid flow will take place. Therefore, the flow will become unstable.

In Figs. 11 and 12, the growth rate values have been compared for the electric field intensity $E = 0$ and 5, for different values of heat transfer capillary number $Ca$ at $h = 0.01$. It is observed that on increasing $Ca$ the growth rates decrease and growth rates in the presence of an electric field decrease faster than the growth rates in the absence of an electric field. It shows that heat and mass transfer has stabilizing effect on the stability of the system and this effect is enhanced in the presence of an electric field.

VII. CONCLUSION

Viscous potential flow analysis of capillary instability with heat and mass transfer in the presence of an axial electric field has been carried out. The dispersion relation is obtained which is a quadratic equation in growth rate. The stability condition is obtained by applying Routh-Hurwitz criterion for stability. A critical value of electric field as well as critical wave number is obtained. The system is unstable when the electric field is less than the critical value of electric field, otherwise it is stable. It is observed that the heat and mass transfer has stabilizing effect on the stability of the system and this effect is enhanced in the presence of an electric field. The heat and mass transfer completely stabilizes the interface against capillary effects even in the presence of an electric field. It is also observed that the axial electric field increases the stability of the system with or without heat and mass transfer. It is found that the ratio of electric conductivity has destabilizing effect on growth rate. The same nature of result is obtained for the effect of ratio of permittivity on growth rates. The heat and mass transfer, for inviscid fluids, has no effect on the stability of the system, while it has stabilizing effect on the stability for viscous fluids.

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