Non-Sensitive Solutions in Multi-Objective Optimization of a Solar Photovoltaic/Thermal (PV/T) Air Collector

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Abstract—In this paper, an attempt has been made to obtain non-sensitive solutions in the multi-objective optimization of a photovoltaic/thermal (PV/T) air collector. The selected objective functions are overall energy efficiency and exergy efficiency. Improved thermal, electrical and exergy models are used to calculate the thermal and electrical parameters, overall energy efficiency, exergy components and exergy efficiency of a typical PV/T air collector. A computer simulation program is also developed. The results of numerical simulation are in good agreement with the experimental measurements noted in the previous literature. Finally, multi-objective optimization has been carried out under given climatic, operating and design parameters. The optimized ranges of inlet air velocity, duct depth and the objective functions in optimal Pareto front have been obtained. Furthermore, non-sensitive solutions from energy or exergy point of view in the results of multi-objective optimization have been shown.

Keywords—Solar photovoltaic thermal (PV/T) air collector, Overall energy efficiency, Exergy efficiency, Multi-objective optimization, Sensitivity analysis.

I. INTRODUCTION

RENEWABLE energies are going to be a main substitute for fossil fuels in the coming years for their clean and renewable nature. Solar energy is one of the most significant renewable energy sources that world needs. The major applications of solar energy can be classified into two categories: solar thermal system, which converts solar energy to thermal energy, and photovoltaic (PV) system, which converts solar energy to electrical energy. Usually, these systems are used separately. In the solar thermal system, external electrical energy is required to circulate the working fluid through the system. On the other hand, in the PV system, the electrical efficiency of the system decreases rapidly as the PV module temperature increases. Therefore, in order to achieve higher electrical efficiency, the PV module should be cooled by removing the heat in some way. In order to eliminate an external electrical source and to cool the PV module, the PV module should be combined with the solar air/water heater collector. This type of system is called solar photovoltaic thermal (PV/T) collector. The PV/T collector produces thermal and electrical energy simultaneously. Besides the higher overall energy performance, the advantage of the PV/T collector system lies in the reduction of the demands on physical space and the equipment cost through the use of common frames and brackets as compared to the separated PV and solar thermal systems placed side-by-side.

The energy payback time (EPBT) of a PV/T air collector lies between 10 and 15 years depending on insulation and the performance of it. If the performance of a PV/T air collector can be increased, the energy payback time can be reduced. Therefore, the optimized performance evaluation of a PV/T air collector is important. The performance of a PV/T air collector can be evaluated in terms of energy analysis or exergy analysis.

The energy analysis has some deficiencies [1,2]. Fundamentally, the energy concept is not sensitive with respect to the assumed direction of the process, e.g. energy analysis does not object if heat is considered to be transferred spontaneously in the direction of increasing temperature. It also does not distinguish the quality of energy, e.g., 1 W of heat equals 1 W of work or electricity. Energy analyses on their own incompletely interpret some processes [1,2], e.g., environmental air, when isothermally compressed, maintains its energy (e.g. enthalpy) equal to zero, whereas the exergy of the compressed air is greater than zero. On the other hand, optimum operating mode of a system regarding to exergy point of view is quasi-equilibrium mode. In practical cases, it is useless or mostly impossible to have quasi-equilibrium mode. Since the processes in this mode occur so slowly.

In this paper, in order to take into account energy and exergy perspectives simultaneously, a multi-objective optimization is carried out.

A significant amount of theoretical as well as experimental studies on the energy or exergy performance evaluation of PV/T collector systems has been carried out in the last 35 years. Wolf [3] as early as in the 1970s have presented the main concept of PV/T collector with the use of either water or air as the coolant.

Fujisawa and Tani [4] have compared the annual
performance of a flat-plate solar water-heating collector, a PV module, a single-glazed PV/T collector with mono-crystalline silicon solar cells, and an unglazed one. The energetic evaluation of the measured data showed that the single-glazed PV/T collector is the best. In terms of exergy analysis, unglazed PV/T collector gives the best performance.

Saitoh et al. [5] have compared the energy and exergy efficiency of a brine-cooled PV/T collector with a PV panel and a solar collector in Hokkaido (in northern Japan) and given similar equations as taken from Fujisawa and Tani [4].

Sahin et al. [6] have carried out the exergy analysis of a PV array based on chemical potential components. They have also obtained exergy components and PV array exergy efficiency. Finally, they have compared energy, electrical, exergy efficiencies under given climatic and operating conditions.

Joshi and Tiwari [7] have carried out the energy and exergy analysis of a PV/T parallel-plate air collector for the cold climate region of India (in Srinagar). They have reported the instantaneous energy and exergy efficiency of a PV/T air collector varies between 55–65% and 12–15%, respectively.

Nayak and Tiwari [8] have presented the performance of a PV integrated greenhouse system for New Delhi climatic condition and reported that the exergy efficiency of the system is 4%.

Joshi et al. [9] have compared the thermal performance of a glass-to-tedlar PV/T air collector and a glass-to-glass PV/T air collector. Their results have been shown a glass-to-glass PV/T air collector have a better thermal performance than a glass-to-tedlar PV/T air collector.

Dubey et al. [10] have evaluated the energetic and exergetic performance of a PV/T air collector with air duct above the absorber plate and the one with air duct below the absorber plate. They have investigated the effect of design and operating parameters and four weather conditions on the performance of above-mentioned PV/T air collectors for five different cities of India and found that the latter one gives better results in terms of thermal energy, electrical energy and exergy gain.

Sarhaddi et al. [11,12] have investigated and optimized the exergetic performance evaluation of a PV array. They have shown that the exergy efficiency of PV array can be improved if the heat can be removed from the PV array surface.

Sarhaddi et al. [13] have obtained a new equation for the exergy efficiency of a PV array based on exergy destruction components.

Sarhaddi et al. [14] have investigated the thermal and electrical performance of a PV/T air collector using an improved thermal and electrical model.

Sarhaddi et al. [15] have carried out exergetic performance assessment of a PV/T air collector using a modified exergy model.

Sarhaddi et al. [16] have carried out the exergetic optimization of a PV/T air collector and obtained the optimal operating mode of a PV/T air collector from the exergy point of view.

In the previous studies [3–16], the multi-objective optimization of solar PV/T collector systems has not been carried out.

In this paper, the multi-objective optimization of a PV/T air collector will be carried out. The selected objective functions are overall energy efficiency and exergy efficiency. A detailed energy and exergy analysis will be fulfilled to calculate the thermal and electrical parameters, overall energy efficiency, exergy components and exergy efficiency of a typical PV/T air collector. The thermal and electrical parameters of a PV/T air collector include solar cell temperature, back surface temperature, outlet air temperature, open-circuit voltage, short-circuit current, maximum power point voltage, maximum power point current, etc. An improved thermal and electrical model will be used to estimate the thermal and electrical parameters of the PV/T air collector. Furthermore, a modified equation will be employed for calculating the exergy efficiency of the PV/T air collector. A computer simulation program will be developed to predict the thermal and electrical parameters of a PV/T air collector. Finally, multi-objective optimization will be carried out; also, non-sensitive solutions from energy or exergy point of view in the results of multi-objective optimization will be obtained.

II. ENERGY ANALYSIS

The proof of governing equations on PV/T air collector energy analysis is not included in order to have a brief note. More details of governing equations derivation is found in Refs. [14–16].

A. Thermal Analysis

Fig. 1 shows the equivalent thermal resistant circuit of a PV/T air collector [14–16].

Writing the energy balance equation for each component of a
PV/T air collector gives the thermal parameters and thermal efficiency of it as follows [14–16]:

\[ T_{\text{cell}} = \left\{ (\alpha_{\text{cell}}) G + U_{\text{amb}} + U_{L} T_{\text{bs}} \right\} / (U_{T} + U_{Y}) \]  

(1)

\[ T_{\text{bs}} = [h_{\text{bs}} (\alpha_{\text{cell}}) G + U_{\text{amb}} + h_{T} T_{\text{amb}}] / (U_{T} + h_{T}) \]  

(2)

\[ T_{\text{f,out}} = T_{\text{amb}} + h_{p} h_{p2} (\alpha_{\text{cell}}) G / (U_{L} L) \times \left[ 1 - \exp \left( -WU_{L} L / (mC_{p}) \right) \right] + T_{\text{fin}} \exp \left( -WU_{L} L / (mC_{p}) \right) \]  

(3)

\[ \dot{Q}_{u} = \eta C_{p} (T_{\text{f,out}} - T_{\text{fin}}) = \left( nC_{p} / U_{L} \right) [h_{p} h_{p2} (\alpha_{\text{cell}}) G - U_{L} (T_{\text{fin}} - T_{\text{amb}})] \times \left[ 1 - \exp \left( -WU_{L} L / (mC_{p}) \right) \right] \]  

(4)

\[ \eta_{\text{th}} = \eta_{u} / (WLG) = \left( mC_{p} / UW_{L} \right) \times \left[ h_{p} h_{p2} (\alpha_{\text{cell}}) G - U_{L} (T_{\text{fin}} - T_{\text{amb}}) / G \right] \times \left[ 1 - \exp \left( -WU_{L} L / (mC_{p}) \right) \right] \]  

(5)

Where \( T_{\text{cell}}, T_{\text{bs}}, T_{\text{f,out}}, T_{\text{fin}}, \eta_{u}, G, m, C_{p}, L, W \) and \( \eta_{\text{th}} \) are solar cell temperature, back surface temperature, ambient temperature, outlet air temperature, the rate of useful thermal energy, solar radiation intensity, mass flow rate of flowing air, the heat capacity of flowing air, the length of airflow duct, the width of air duct and PV/T air collector thermal efficiency, respectively.

In order to increase the calculations precision of PV/T air collector thermal parameters, some corrections have been carried out on heat loss coefficients in a same manner of Refs. [14–16]. These corrections are not mentioned to have a brief note.

### B. Electrical Analysis

Five-parameter photovoltaic for the current–voltage (I–V) characteristic curve of a PV module is defined as follows [14–16]:

\[ I = I_{sc} - I_{lo} \exp (V + IR_{sh}) / (a) - 1 - (V + IR_{sh}) / R_{sh} \]  

(6)

Where, \( I \) and \( V \) represent current and voltage at load, \( a, I_{sc}, I_{lo} \) and \( R_{sh} \) are ideality factor, light current, diode reverse saturation current, series resistance and shunt resistance, respectively. The calculation relations of five parameters \( a, I_{sc}, I_{lo}, R_{sh} \) at reference conditions \( (T_{\text{cell,ref}} = 25^\circ \text{C}, G_{\text{ref}} = 1000 \text{W/m}^{2}) \) or at other climatic and operating conditions \( (G_{\text{sun}}, T_{\text{cell,ref}}) \) have been mentioned in Refs. [11–16]. These relations are not included to have a brief note. The electrical efficiency of a PV module can be defined as the ratio of actual electrical output power to input the rate of solar energy incident on the PV surface as follows [11–16]:

\[ \eta_{\text{el}} = [W_{\text{mp}} / W_{\text{fmp}}] / \left( N W_{\text{mp}} L_{1} L_{2} \right) \]  

(7)

Where \( V_{\text{mp}}, I_{\text{mp}}, N_{W}, N_{m}, Power_{\text{fan}}, Power_{\text{fan}} \) and \( S \) are maximum power point voltage, maximum power point current, the number of strings, the number of modules in series per string, the electrical power consumed by fans and the rate of solar energy incident on the PV surface, respectively. The overall energy efficiency of a PV/T air collector can be calculated by adding the thermal efficiency (Eq. (5)) and thermal efficiency equivalent of electrical efficiency as follows [14–16]:

\[ \eta_{\text{en,ov}} = \eta_{\text{th}} + \eta_{\text{el,th}} = \eta_{\text{th}} + (0.36) \]  

(8)

In the above equation, the coefficient 0.36 is the conversion factor of the thermal power plant [14–16].

### III. Exergy Analysis

Exergy analysis is a technique that uses the conservation of mass and conservation of energy principles together with the second law of thermodynamics for the analysis, design and improvement of energy and other systems. The exergy efficiency of a PV/T air collector is defined as the ratio of net output (desired) exergy rate to net input exergy rate [15,16]:

\[ \eta_{\text{ex}} = \frac{m_{\text{cell}} C_{\text{p,cell}} / \Delta t (T_{\text{cell}} - T_{\text{amb}} - T_{\text{amb}} \ln(T_{\text{cell}} / T_{\text{amb}}))}{S[1 - 4(T_{\text{amb}} / T_{\text{sun}})^{3} / 3] + (V_{\text{oc}} I_{\text{sc}} - V_{\text{mp}} I_{\text{mp}}) T_{\text{cell}} / T_{\text{sun}}} + \frac{m_{\text{cell}} C_{\text{p,cell}} / \Delta t (T_{\text{f,out}} - T_{\text{fin}} - T_{\text{amb}} \ln(T_{\text{f,out}} / T_{\text{fin}}))}{S[1 - 4(T_{\text{f,out}} / T_{\text{sun}})^{3} / 3] + (V_{\text{mp}} I_{\text{mp}} - Power_{\text{fan}}) T_{\text{cell}} / T_{\text{sun}}} + \frac{m_{\text{cell}} C_{\text{p,cell}} / \Delta t (T_{\text{cell}} - T_{\text{amb}} - T_{\text{amb}} \ln(T_{\text{cell}} / T_{\text{amb}}))}{S[1 - 4(T_{\text{amb}} / T_{\text{sun}})^{3} / 3] + (T_{\text{amb}} / T_{\text{sun}})^{3} / 3]} \]  

(9)

Where \( V_{\text{oc}}, I_{\text{sc}}, R, T_{\text{sun}}, P_{\text{in}}, P_{\text{out}}, m_{\text{cell}}, C_{\text{p,cell}} \) and \( \Delta t \) are open-circuit voltage, short-circuit current, gas constant, the sun’s temperature, ambient pressure, agent fluid pressure at entrance and exit from PV/T air collector, PV module mass, the specific heat capacity of silicon solar cell and time interval respectively.

### IV. Formulation Optimization Problem

The formulation of multi-objective optimization problem is given as follows

\[ \begin{align*}
& \text{Maximize:} \quad \left( \eta_{\text{en,ov}} = \eta_{\text{th}} + \eta_{\text{el,th}} \right) \\
& \text{subject to} \quad 0.01 \leq \delta \leq 0.1 \text{m}, \\
& \quad 0.001 \leq V_{\text{in}} \leq 12 \text{m/s}, \\
& \quad T_{\text{cell}}, T_{\text{f,out}}, T_{\text{bs}}, P_{\text{fan}}, m_{\text{cell}}, C_{\text{p,cell}}, C_{\text{p,cell}}, I_{\text{sc}}, I_{\text{mp}} U_{1}, U_{T}, U_{Y}, h_{T}, T_{\text{fin}}, T_{\text{f,out}}, \dot{Q}_{u} \\
& \quad \eta_{\text{el,th}}, Power_{\text{fan}}, (\alpha_{\text{cell}}) G, h_{p1}, h_{p2}, \text{etc.} \geq 0 \\
& \quad \text{and other nonlinear constraints [14–16].}
\end{align*} \]

The objective functions and the constraint equations are nonlinear. Therefore, we have used MATLAB optimization toolbox to solve the optimization problem. MATLAB uses NSGA-II algorithm for multi-objective optimization [17].
V. SENSITIVITY ANALYSIS

Sensitivity analysis is the study of how the variation (uncertainty) in the output of a mathematical model can be apportioned, qualitatively or quantitatively, to different sources of variation in the input of the model [18]. Put another way, it is a technique for systematically changing parameters in a model to determine the effects of such changes. Dimensionless sensitivity analysis (logarithmic sensitivity analysis) is widely used in sciences because it allows one to compare the sensitivity of one output with respect to one parameter with the sensitivity of another output with respect to yet another parameter. If \( \eta = f(x_1, x_2, ..., x_n) \) then dimensionless sensitivity analysis with respect to parameter \( x_i \) defined as follows:

\[
\frac{\partial \ln(\eta)}{\partial \ln(x_i)} \approx \frac{\Delta \eta/\eta}{\Delta x_i/x_i}
\]  

In this paper the dimensionless sensitivity analysis is used in the results of multi-objective optimization in order to obtain non-sensitive solutions from energy or exergy point of view.

VI. RESULTS AND DISCUSSION

A. Experimental Validation

The experimental results of Joshi et al. [9] for a PV/T air collector make it possible to verify the results obtained by our computer simulation. Related information about the validation process are found in Refs. [9,14–16]. In order to compare the simulated results with the experimental measurements, a root mean square percentage deviation (RMS) has been evaluated by following equation [14–16]:

\[
RMS = \sqrt{\sum_{i=1}^{n} \left( \frac{X_{\text{sim},i} - X_{\text{exp},i}}{X_{\text{sim},i}} \right)^2}\times 100
\]  

Fig. 2 shows the experimental and simulated values of overall energy efficiency, thermal efficiency and electrical efficiency during the test day.

The simulated and experimental values of exergy efficiency during the test day are shown in Fig. 3.

According to Fig. 2 and Fig. 3, it is observed that the numerical simulation results of this study are in good agreement with the experimental measurements of Joshi et al. [9]. Further, it is observed that the simulation results obtained in this paper is more precise than the one given by Joshi et al. [9].

B. Optimization and Sensitivity Analysis Results

Fig. 4 shows the values of objective functions and decision variables in optimal Pareto front.
The numerical range of the points shown in Fig. 4 is given in Table 1. The non-sensitive regions of the energy efficiency or the exergy efficiency and the non-sensitive points from the energy or exergy point of view have been also indicated in the same figure.

<table>
<thead>
<tr>
<th>Optimum range of decision variables</th>
<th>Optimum range of objective function</th>
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<tbody>
<tr>
<td>$0.016 \leq \delta \leq 0.037 \text{ m}$</td>
<td>$54.54 \leq \eta_{\text{ex}} \leq 67.68%$</td>
</tr>
<tr>
<td>$5.34 \leq V_{\text{in}} \leq 10.63 \text{ m/s}$</td>
<td>$9.26 \leq \eta_{\text{ex}} \leq 11.38%$</td>
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</table>

Unlike the conventional multi-objective optimization algorithms, the NSAG-II algorithm gives a range for each decision variables. There are no superior between exiting points in each range from the perspective of Pareto optimal conditions. Each points shown in Fig. 4, introduce a vector of decision variables, $\vec{X}(\delta, V_{\text{in}})$. In other words, each decision variable can not be selected solely; designer should choose a vector of decision variables.

It is clear from Fig. 4 that choosing appropriate values for the decision variables, namely duct depth ($\delta$) and inlet air velocity ($V_{\text{in}}$), to obtain a better value of one objective would normally cause a worse value of another objective. This subject shows Pareto optimal conditions.

Designer can choose the desired vector of decision variables among optimal Pareto solutions according his considerations such as energy or exergy application, design limitations, economic costs, etc.

According to the results of sensitivity analysis (Fig. 4), the exergy efficiency can be increase from 9.26% to ~10.5%, while the overall energy efficiency remains constant (~67.44%), approximately. On the other hand, the overall energy can be increase from 54.54% to ~62%, while the exergy efficiency has not sensible variations (~11.31%). This subject is suitable from engineering design perspective. Because, it leads to non-sensitive selection from energy or exergy point of view.

VII. CONCLUSIONS

On the basis of present study, the following conclusions have been drawn:

- The numerical simulation results of this study are in good agreement with the experimental measurements noted in the previous literature. Further, it is observed that the simulation results obtained in this paper is more precise than the one given by the previous literature.
- Unlike the conventional multi-objective optimization algorithms, the NSAG-II algorithm gives a range for each decision variables.
- The exergy efficiency can be increase from 9.26% to ~10.5%, while the overall energy efficiency remains constant (~67.44%), approximately. On the other hand, the overall energy can be increase from 54.54% to ~62%, while the exergy efficiency has not sensible variations (~11.31%). This subject is desired from engineering design perspective. Because, designer can choose the desired vector of decision variables among optimal Pareto solutions according his considerations such as energy or exergy application, design limitations, economic costs, etc.

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