A Bi-Objective Preventive Healthcare Facility Network Design with Incorporating Cost and Time Saving
Mehdi Seifbarghy, and Keyvan Roshan

Abstract—Main goal of preventive healthcare problems are at decreasing the likelihood and severity of potentially life-threatening illnesses by protection and early detection. The levels of establishment and staffing costs along with summatior of the travel and waiting time that clients spend are considered as objectives functions of the proposed nonlinear integer programming model. In this paper, we have proposed a bi-objective mathematical model for designing a network of preventive healthcare facilities so as to minimize aforementioned objectives, simultaneously. Moreover, each facility acts as M/M/1 queuing system. The number of facilities to be established, the location of each facility, and the level of technology for each facility to be chosen are provided as the main determinants of a healthcare facility network. Finally, to demonstrate performance of the proposed model, four multi-objective decision making techniques are presented to solve the model.

Keywords—Preventive healthcare problems, Non-linear integer programming models, Multi-objective decision making techniques

I. INTRODUCTION AND LITERATURE REVIEW

NOWADAYS, preventive healthcare problems (PHPs) have been used in substantial savings in the costs of diagnosis and therapy along with the lower capital investment [1]. The main advantage of preventive plans is saving better quality of life by decreasing the requirement for radical treatments, such as surgery or chemotherapy. Among these, the most well-known preventive services are flu shots, blood tests, mammograms, and anti-smoking advice. [2] shows that mammograms taken on a regular basis have the potential to decrease deaths from breast cancer for women between the ages of 50 and 69 by up to 40 percent. Moreover, [3] discovered that 36 percent of breast cancer patients without a mammogram received the diagnosis of late stage cancer, whereas this ratio was 20 percent for the patient group with a mammogram.

Preventive healthcare programs can be divided into three groups with regard to their objectives: (I) primary prevention aims at reducing the likelihood of diseases in people with no symptoms, e.g., immunizations of healthy children; (II) secondary prevention aims at identifying and treating people who have risk factors or are at very early stage of diseases, e.g., pap smears to detect early forms of cervical cancer; (III) tertiary prevention aims at treating symptomatic patients in an effort to decrease complications or severity of disease, e.g., sugar control in a diabetic in order to mitigate vision and nerve problems. [4] found through a survey that the convenience of access to the facility was a very important factor in a client's decision to have prostate cancer screening. The survey by [5] revealed that the perceptions of lack of access to services were related to the decrease of mammography participation.

Since many diseases can be prevented, the current healthcare systems do not make the best use of their available resources to support preventive programs. Most of these systems are based on responding to acute problems, urgent needs of patients, and pressing concerns [6]. An effective procedure to improve the efficiency of a regional healthcare system under limited resources is to increase the number of people receiving preventive services [7]. The point is accessibility of facilities is an important factor for the success of a preventive healthcare program. [8] introduced three groups of factors that influence the individuals' use of services in healthcare including structural, financial, and personal barriers. This article concentrates on structural barriers that are directly related to the number, type, and concentration, level of technology and location of healthcare facilities, as well as transportation to services and availability of providers.

Most general literature reviews by [9], [10], which focus on public facility location problems with stochastic demand and congestion in the context of fixed versus mobile servers, do not cite any articles on preventive healthcare. [11] consider waiting time as one of the attributes in a client's overall utility for alternative primary care facilities. The second key factor is the apparent link between volume and quality of preventive healthcare services. Although many design issues exist for preventive healthcare programs, our paper focuses on the configuration of a network of preventive healthcare facilities so as to minimize establishment and staffing costs and the average total time. In representing demand elasticity, the accessibility of a facility can be modeled in terms of its proximity to the potential clients [12], the total time required for receiving the service [13], or an overall utility (Parker and Srinivasan, 1976). Marianov et al. (2008) propose a facility location problem with congestion, by using a probabilistic-choice model to represent client allocation behavior. Recently, [15] proposed a multi-objective facility location problem within batch arrival queuing framework. [16] presented a facility location problem within competitive environment with considering M/M/m/k queuing system for each facility.

In this paper, a bi-objective mathematical model for designing a network of preventive healthcare facilities to
minimize establish and staffing costs and the average total time, is presented. To do that, we utilize the summations of travel and waiting time spent to receive preventive services as a proxy for accessibility of healthcare facilities. This time includes the time spent in transportation to the facility as well as the time spent at the facility while waiting and receiving services. The number of facilities to be established, the location, and the level of technology of each facility are the main determinants of the configuration of the healthcare facility network.

The presented model incorporates the differentiating features of preventive healthcare. First, the number of people who seek the services at the facility is not controlled by the policy maker, i.e., preventive healthcare is a user-choice environment in terms of the allocation of clients to facilities. Unless the services are offered at convenient locations, people are not likely to participate. That is, the demand for preventive programs at population zones reduces with the time that needs to be spent for receiving services. In the event that people have to wait for a long time to receive the services due to limited capacity, their willingness to participate in preventive programs could decrease significantly. Therefore, the level of congestion at the facilities is a crucial factor that is incorporated in our model as an M/M/1 queuing system at each facility to determine active facility, allocation process, and technology level of each facility. To solve the proposed model, four multi-objective decision making (MODM) techniques are implemented and analyzed.

The paper is organized as follows. The next section describes the problem in detail and formulates it as a nonlinear programming model. In Section 3, MODM techniques are analyzed to solve the model. At end, Section 4 gives conclusion and future research directions.

II. PROPOSED BI-OBJECTIVE MODEL

In this section, we first define the problem and then the nonlinear integer mathematical programming model is presented. In order to explain the problem, let \( G=(N,L) \) be a network with a set of nodes \( N (|N|=n) \) and a set of links \( L \). The nodes represent the neighborhoods of a city or the population zones, and the links are the main transportation arteries. The fraction of clients residing at node \( i \) is denoted by \( h_i \), \( i \in N \). In our model, the assumptions are as follow:

- Number of clients who require medical service over the entire network is Poisson distributed with a rate of \( \lambda \) per unit of time, and thus from each node \( i \) at a rate \( \lambda h_i \), \( i \in N \)
- There is a finite set of potential locations \( X \subseteq N \) for the facilities
- There is a single service team in facility located at point \( j \) that can provide an average of \( \mu_i \) services per unit of time, \( i \in X \)
- The service time is exponentially randomly distributed. Therefore, each facility is an M/M/1 queue.
- The number of technology levels for each potential location of facility denotes by \( k \in M (|M|=m) \).
- Service rate \( \mu_{jk} \) is for each potential location and each level of technology, simultaneously. For the ease of exposition, we also assume that \( \mu_{jk} = \mu_k \), \( j \in X \), although this assumption can easily be relaxed within the context of our model.

- All individuals from the same node request service from the same facility
- In the long run, the clients will gather sufficient information about the total time required to obtain preventive healthcare services at the facilities in their vicinity, although each client may visit these facilities, infrequently.

In our model, \( T_{ij} \) is the average total time that individuals from node \( i \) spend in order to receive service at facility located at point \( j \in X \). The average total time \( T_{ij} \) comprises of two components:

1. The travel time from node \( i \) to facility located at point \( j \) through the shortest path denoted by \( t_{ij} \).
2. The average waiting time clients spend at the facility with the special level of technology possibly waiting and receiving service which we denote as \( \tilde{W}_{jk} \), i.e.,

\[
T_{ij} = t_{ij} + \sum_{k=1}^{m} \tilde{W}_{jk} \gamma_{jk} \tag{1}
\]

The fraction of clients from node \( i \) who request service from facility \( j \), denoted by \( a_{ij} \), is a decreasing function of the expected travel time.

\[
a_{ij} = \begin{cases} 
\frac{A_{ij} - \gamma (t_{ij})}{\gamma} & \text{if } t_{ij} < \frac{A_{ij}}{\gamma} \\
0 & \text{otherwise}
\end{cases} \tag{2}
\]

where \( A_{ij} \) is the fraction of clients from node \( i \) who would visit facility located at point \( j \) when \( T_{ij} = 0 \), i.e., the intercept of the demand decay function, and \( \gamma \) is the slope of the demand decay function. Also, \( \lambda j \) is the rate of clients requesting service from node \( j \). Then,

\[
\lambda_j = \lambda \sum_{i=1}^{n} a_{ij}, \quad j \in X.
\]

Since the system is an M/M/1 queue,

\[
\tilde{W}_{jk} = \frac{1}{\mu_k - \lambda_j}, \quad j \in X, \quad k \in M.
\]

The objectives of our problem are to find the optimal set of locations \( j \in X \), so as to minimize establishment and staffing costs and the average total time. To formulate the problem as a mathematical program, our model is included three decision variables as following definitions:

\[
x_{ij} = \begin{cases} 
1 & \text{if clients from node } i \text{ require service from facility located at point } j \\
0 & \text{otherwise}.
\end{cases}
\]

\[
y_j = \begin{cases} 
1 & \text{if facility is located at node } j \\
0 & \text{otherwise}.
\end{cases}
\]

\[
z_{jk} = \begin{cases} 
1 & \text{if facility located at point } j \text{ use the level of technology } k \\
0 & \text{otherwise}.
\end{cases}
\]

Finally, the proposed mathematical model is presented as

\[
imin \sum_{j \in X} H_{ij} y_j + \sum_{j \in X} \sum_{k=1}^{m} C_{jk} + \frac{C'_{jk}}{\mu_k} \z_j \lambda_j \tag{5}
\]
\[
\min \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{M} \tilde{T}_{ij} x_{ij} = \sum_{i=1}^{m} \sum_{j=1}^{n} (t_{ij} + \sum_{k=1}^{M} \bar{W}_{jk} z_{jk}) x_{ij} \quad (6)
\]

S.t.

1. \[
\sum_{j \in X} x_{ij} = 1 \quad i \in N
\]

2. \[
x_{ij} \leq y_{ij} \quad i \in N \quad j \in X
\]

3. \[
\sum_{j \in X} H_{ij} y_{ij} \leq R_{\text{max}}
\]

4. \[
\sum_{k=1}^{m} z_{jk} = y_{ij} \quad j \in X
\]

5. \[
x_{ij} (t_{ij} + \sum_{k=1}^{m} \bar{W}_{jk} z_{jk}) \leq t_{ij} + \bar{W}_{jk} z_{jk} + M(1 - y_{ij})
\]

6. \[
\lambda \sum_{i=1}^{n} h_{ij} a_{ij} c_{ij} \leq \mu_{jk} z_{jk} \quad j \in X
\]

7. \[
a_{ij} x_{ij} \geq 0 \quad i \in N \quad j \in X
\]

8. \[
x_{ij}, y_{ij}, z_{jk} = 0, 1
\]

9. \[
i \in N \quad j \in X \quad k = 1, 2, ..., m
\]

Equations (5) and (6) are respectively objective functions that the first one defines to minimize establishment and staffing costs and second one is used to minimize the average total time. Constraints (7) ensure that each node is serviced by one facility. Constraints (8) guarantee that clients can require service only from open facilities. Constraints (9) specify that the maximum amount of establish costs define by \( R_{\text{max}} \). Constraints (10) define that only one technology can be used in each facility. Constraints (11), where \( M \) represents a big number, stipulate that clients choose the facility with minimum total time. Constraints (12) guarantee the stability of the queue and constraints (13) forbid negative \( a_{ij} \). Constraints (14) indicate being binary the decision variables.

### III. RESULTS

In this section, we applied four MODM techniques including single optimization method (SOM), LP-metric method (LPM), Minimax method (MIXM), \( \varepsilon \)-constraint method (ECM) to solve the model.

#### TABLE I

<table>
<thead>
<tr>
<th>MODM techniques</th>
<th>The first objective function value</th>
<th>The second objective function value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SOM</td>
<td>230019</td>
<td>5.36</td>
</tr>
<tr>
<td>SOM</td>
<td>560048</td>
<td>2.66</td>
</tr>
<tr>
<td>LPM for the norm one ((p=1))</td>
<td>230024</td>
<td>3.78</td>
</tr>
<tr>
<td>LPM for the norm two ((p=2))</td>
<td>300019</td>
<td>5.38</td>
</tr>
<tr>
<td>LPM for the norm five ((p=5))</td>
<td>250020</td>
<td>5.82</td>
</tr>
<tr>
<td>LPM for the norm thirty ((p=30))</td>
<td>260020</td>
<td>5.79</td>
</tr>
<tr>
<td>MIXM</td>
<td>300023</td>
<td>3.90</td>
</tr>
<tr>
<td>ECM</td>
<td>230019</td>
<td>5.36</td>
</tr>
<tr>
<td>ECM</td>
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<td>2.66</td>
</tr>
</tbody>
</table>

As shown Table 1, objective function values for both objectives are determined based on four MODM techniques. To clarify performance of all MODM techniques, Fig. 1 and Fig. 2 show behavior of four techniques. According to first objective function, LPM with \( p=1 \) and ECM report better outputs. For second objective one, SOM and ECM represent appropriate performance.
IV. CONCLUSION

In this paper, we have proposed a bi-objective mathematical model for designing a network of preventive healthcare facilities to determine the number of facilities to be established, the location of each facility, and the level of technology for each facility to be chosen. The objective functions are minimizing summations of travel and waiting time as well as minimizing total cost including establishment and staffing cost. Finally, to demonstrate performance of the proposed model, four MODM techniques including single optimization, Lp-metric, Minimax, and ε-constraint methods are analyzed to solve the model. As future research, the model can be formulated in fuzzy environment.

REFERENCES