Abstract—This paper presents the application of an enhanced Particle Swarm Optimization (EPSO) combined with Gaussian Mutation (GM) for solving the Dynamic Economic Dispatch (DED) problem considering the operating constraints of generators. The EPSO consists of the standard PSO and a modified heuristic search approaches. Namely, the ability of the traditional PSO is enhanced by applying the modified heuristic search approach to prevent the solutions from violating the constraints. In addition, Gaussian Mutation is aimed at increasing the diversity of global search, whilst it also prevents being trapped in suboptimal points during search. To illustrate its efficiency and effectiveness, the developed EPSO-GM approach is tested on the 3-unit and 10-unit 24-hour systems considering valve-point effect. From the experimental results, it can be concluded that the proposed EPSO-GM provides, the accurate solution, the efficiency, and the feature of robust computation compared with other algorithms under consideration.

Keywords—Particle Swarm Optimization (PSO), Gaussian Mutation (GM), Dynamic Economic Dispatch (DED).

I. INTRODUCTION

Dynamic Economic Dispatch (DED) schedules the generating outputs of all on-line units over a time horizon by taking the dynamic constraints of generators into account, whereas the traditional Static Economic Dispatch (SED) allocates the outputs of all committed generating units by considering the static behavior of them. It can be therefore concluded that the DED problem is an extension of the SED problem in which the ramp rate limits of the generators are taken into consideration. That makes the DED problem more difficult [1-3]. Regarding the DED problem, there were a number of traditional methods that have been applied to handle this problem such as: Dynamic Programming [4], Linear Programming [5], Lagrangian Relaxation [6], etc. However, there were some attempts to find the new methodology for dealing with this difficulty.

In recent years, evolutionary computation techniques have been developed and proposed so as to solve a wide range of power system problems including DED problem such as Genetic Algorithm (GA) [7], Simulated Annealing (SA) [8], Evolutionary Programming (EP) [2], Particle Swarm Optimization (PSO) [3], etc.

In comparison with the classical methods, characteristics of evolutionary computation techniques that make them more attractive over the traditional ones are as follows:

- They are more likely to find a global solution, while the traditional methods may become trapped in a local optimum;
- There is no mathematical limitation of the problem formulation, while the classical techniques may require approximations or specific cost function forms;
- Their calculation is based on random processes; therefore, they can generate many feasible solutions. This is in contrast to the conventional approaches that may yield only one solution [9].

Compared to other evolutionary computation techniques, PSO can solve the problems quickly with high quality solutions and stable convergence characteristics, whereas it is easily implemented. However, PSO can sometimes suffer from the lack of the diversity amongst the particles, which can lead to a stagnation stage [10]. Therefore, although PSO has been a subject of an extensive research, there is a number of issues that need to be addressed in order to exploit the full potential of PSO in solving complex power system problems [11].

This paper is organized as follows: section II presents DED problem formulation and section III provides an overview of PSO. A brief introduction to Gaussian Mutation is also provided in section IV. Then, section V illustrates the details of the EPSO-GM implementation for solving the DED problem. Section VI shows the simulation results of the EPSO-GM method and the comparison with other approaches. Finally, some concluding remarks are made in Section VII.

II. DED PROBLEM FORMULATION

Dynamic Economic Dispatch (DED) problem is to determine the optimum scheduling of generation at a certain period of time that minimizes the total production cost while satisfying equality and inequality constraints, i.e. power balance, operating limits, and ramp rate constraints, respectively. In general, the mathematical model of the DED problem is as follows [2]:
Minimize : \[ TC = \sum_{i=1}^{T} \sum_{k=1}^{N} F_i(P_i) \] (1)

Subject to:

a) Power balance constraint

\[ \sum_{i=1}^{N} P_i = P_{Di} \] (2)

b) Operating limit constraints

\[ P_{i_{\text{min}}} \leq P_i \leq P_{i_{\text{max}}} \] (3)

c) Ramp rate constraints

\[ -DR_i \leq P_{i_{t-1}} - P_{i_{t}} \leq UR_i \] (4)

From the different characteristics of cost function; therefore, they can be categorized as smooth and non-smooth cost functions as presented in [12-14]. For the sake of simplicity, the cost function of the Economic Dispatch problem (smooth cost function) is generally a single quadratic function. The generator’s fuel cost function can be represented by [15]:

\[ F_i(P_i) = a_i P_i^2 + b_i P_i + c_i. \] (5)

In some large generators, their cost functions are also nonlinear, due to the effect of valve-point loading [13]. Taking the valve point loading into account will increase multiple local minimum points in the cost function and make the problem more difficult [16]. The fuel cost function with valve-point loading can be expressed as [17]:

\[ F_i(P_i) = a_i P_i^2 + b_i P_i + c_i + \left| e_i \times \sin(f_i \times (P_{i_{\text{min}}} - P_i)) \right|. \] (6)

Fig. 1 illustrates an example of smooth cost function and non-smooth cost function with valve-point loading.

In addition, some generators can be operated with multiple fuels [13, 14]. Therefore, changes of fuel type will be responsible for changes in the cost function from a single quadratic function to a piecewise quadratic function [18, 19]. The generator’s fuel cost function can be defined as follows [14]:

\[\begin{align*}
F_i(P_i) &= a_{i1} P_i^2 + b_{i1} P_i + c_{i1}, \text{ (fuel 1), if } P_{i_{\text{min}}} \leq P_i \leq P_{i_{\text{max}}} \\
&= a_{i2} P_i^2 + b_{i2} P_i + c_{i2}, \text{ (fuel 2), if } P_{i_{1}} < P_i \leq P_{i_{2}} \\
&\vdots \\
&= a_{ik} P_i^2 + b_{ik} P_i + c_{ik}, \text{ (fuel } k), \text{ if } P_{i_{k-1}} < P_i \leq P_{i_{k_{\text{max}}}}.
\end{align*}\] (7)

Fig. 2 illustrates an example of non-smooth cost functions with multiple fuels.

III. OUTLINE OF THE PARTICLE SWARM OPTIMIZATION

In 1995, Kennedy and Eberhart [20] initially introduced a modern heuristic technique called Particle Swarm Optimization (PSO) for solving nonlinear and non-continuous optimization problems [21]. It is rather similar to other evolutionary computation techniques (i.e. Genetic Algorithm (GA)) in that PSO also utilizes the principle of a random initialized population and the concept of evaluation and modification of a population to discover the global solution. However, PSO does not utilizes the mutation and crossover operators during the modification step, since it can update itself [22, 23]. The basic principle of PSO is that it initializes a population of particles with the randomness of both positions and velocities. Subsequently, each particle adjusts its velocity dynamically corresponding to its flying experiences and its colleagues [21, 24]. There are three main components that affect the changing of the velocity i.e. inertial, cognitive, and
social components. For the inertial component, it represents the particle’s behavior for moving in the previous direction, while the cognitive component represents the memory of the particle for attracting to its previous best position (pbest). Concerning the social component, it represents the memory of the particle for attracting its previous best position among the group (gbest) [25]. Correspondingly, each particle can be adjusted or updated its new position according to its modified velocity. The updated velocity \((v_{x}^{(t+1)})\) and position \((x_{x}^{(t+1)})\) of each particle can therefore be expressed by [26-29]:

\[
v_{x}^{(t+1)} = k \times [w \times v_{x}^{(t)} + c_1 \times \text{rand}_1 \times (p\text{best}_{x}^{(t)} - x_{x}^{(t)}) + c_2 \times \text{rand}_2 \times (gbest_{x}^{(t)} - x_{x}^{(t)})],
\]

\[
x_{x}^{(t+1)} = x_{x}^{(t)} + v_{x}^{(t+1)}.
\]

Constriction factor \((k)\) is expressed by:

\[
k = \frac{2}{2 - \phi - \sqrt{\phi^2 - 4\phi}}, \quad \phi = c_1 + c_2, \quad \phi > 4,
\]

where

\(v_{x}^{(t)}\): velocity of \(i^{th}\) particle at iteration \(t\) in \(d\)-dimensional space,

\(x_{x}^{(t)}\): current position of \(i^{th}\) particle at iteration \(t\) in \(d\)-dimensional space,

\(w\): inertia weight factor,

\(t\): number of iterations,

\(k\): constriction factor,

\(c_1, c_2\): acceleration constant.

IV. GAUSSIAN MUTATION

The proposed EPSO-GM technique utilizes a mutation operator, called Gaussian mutation (GM) that is generally applied to Genetic algorithm (GA). It is aimed at coping with the loss of diversity in global search by incorporating Gaussian mutation into the traditional PSO as presented in [10, 27, 30]. Applying Gaussian mutation improves the PSO searching ability by mutating some selected particles. The procedures of the implementation in this section can therefore be expressed in details as follows:

**Step 1**: Determine the mutation probability \((P_m)\) by:

\[
P_m = \frac{R_m}{m}
\]

where \(R_m\) and \(m\) are mutation rate and the number of particles, respectively. As reported in [27], \(R_m\) is set to 1 at the first iteration and linearly decreases to 0 at the final iteration.

**Step 2**: Generate a uniformly distributed random number \((\text{rand}_d)\) between 0 and 1 for each particle.

**Step 3**: Compare each generated random number \((\text{rand}_d)\) with \(P_m\). If \(P_m > \text{rand}_d\), then mutate the particle by following equation [27].

\[
x_{x, \text{mutate}}^{(t)} = x_{x}^{(t)} \times (1 + \text{gaussian}(\sigma))
\]

where \(x_{x}^{(t)}\) and \(x_{x, \text{mutate}}^{(t)}\) denote the current and mutated position of particle \(i\) at iteration \(t\), whilst \(\text{gaussian}(\sigma)\) is a random number drawn from a Gaussian distribution. It can be calculated from \(\sigma = 0.1 \times \text{The length of search space}\).

V. DEVELOPMENT OF THE PROPOSED EPSO-GM ALGORITHM

The basic concept of the EPSO-GM is that the Gaussian mutation (GM) is integrated into an enhanced PSO algorithm (EPSO) to increase a possibility of generating feasible solutions when applying to the DED problem. Concerning the EPSO, it consists of the standard PSO and a modified heuristic search, which is modified and developed from [12, 31, 32] for manipulating the equality and inequality constraints. The procedures of the proposed EPSO-GM method are shown in Fig. 3.
Step 3: Mutating some selected particles using Gaussian mutation operator.

Step 4: Modify the positions of the particle
Step 4.1: Set $i = 1$ and $j = 1$, where $i = 1, 2, ..., T$ and $j = 1, 2, ..., N$.
Step 4.2: Randomly select L-th generator,
Step 4.3: Calculate $P_d$ using $P_d = P_D - \sum_{j=1}^{N} P_j$.
Step 4.4: Adapt $P_d$ for its operating limit if $P_d < P_d(\text{min})$ or $P_d > P_d(\text{max})$. Otherwise, go to Step 4.8.
Step 4.5: If $j \leq \text{total number of generators (N)}$, let $j = j + 1$. Otherwise go to Step 4.8.
Step 4.6: Re-random L-th generator and re-calculate $P_d$.
Step 4.7: Adjust the value of $P_d$ if it is out of operating limit, and then return to the Step 4.5. Otherwise, go to the next step.
Step 4.8: Calculate the operating limit for the next hour considering ramp rate constraints from $P_{r,1}(\text{min}) = P_0 - DR$ and $P_{r,1}(\text{max}) = P_0 + UR_1$,
Step 4.9: If $P_{r,1}(\text{min}) < P_{r}(\text{min})$, then let $P_{r,1}(\text{min}) = P_{r}(\text{min})$ or $P_{r,1}(\text{max}) > P_{r}(\text{max})$, then let $P_{r,1}(\text{max}) = P_{r}(\text{max})$.
Step 4.10: If $i = \text{total number of hours (T)}$, then go to Step 5. Otherwise, let $i = i + 1$, and go to Step 4.2.

Step 5: Update $p_{\text{best}}$ and $g_{\text{best}}$ by evaluating and comparing the fitness value with their previous values.

Step 6: If the termination criteria are satisfied, then stop. Otherwise, return to Step 2.

VI. NUMERICAL RESULTS

In order to demonstrate and validate the effectiveness of the proposed EPSO-GM algorithm, its simulation results will be compared with the outcomes obtained from the traditional PSO (EPSO) and other algorithms by applying to two different case studies. The first case study is a traditional Static Economic Dispatch problem (SED) i.e. the standard 3-unit system considering valve-point loading. The second case study is the Dynamic Economic Dispatch problem (DED) i.e. the 10-unit 24-hour system including generator ramp rate limitation and also non-smooth cost function. The systems data can be found from [17] and [2]. The simulations are carried out using Matlab and executed on a personal computer, where in all cases; the each algorithm is run for 30 times with different initial conditions in order to diminish the random effects. The values of the simulation parameters for the EPSO and EPSO-GM method are shown in Table I.

### Table I

<table>
<thead>
<tr>
<th>Method</th>
<th>Average cost ($)</th>
<th>Best cost ($)</th>
<th>Generation schedule (MW)</th>
<th>Total Power (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA [17]</td>
<td>8234.07</td>
<td>8234.07</td>
<td>300.27</td>
<td>149.73</td>
</tr>
<tr>
<td>IEP [19]</td>
<td>8234.09</td>
<td>8234.09</td>
<td>300.24</td>
<td>149.77</td>
</tr>
<tr>
<td>EPSO</td>
<td>8239.44</td>
<td>8234.07</td>
<td>300.27</td>
<td>149.73</td>
</tr>
<tr>
<td>EPSO-GM</td>
<td>8235.32</td>
<td>8234.07</td>
<td>300.27</td>
<td>149.73</td>
</tr>
</tbody>
</table>

### Case study 1: 3-unit system

For this case, the proposed EPSO-GM is aimed at optimizing the schedule of generation to meet a single power demand of 850 MW, while parameters used in the implementation are: the agents’ size = 20, maximum number of generations =300, respectively. From the literature review, it was presented in [34] that the global best solution found for this case study is $8234.07$.

Table II shows the simulation results of both PSO algorithms, the genetic algorithm (GA) [17], and the improved evolutionary programming (IEP) [19], respectively. Although the total power obtained from various methods satisfy power demand constraint, the EPSO and the proposed EPSO-GM algorithms can obtain the global solution (best cost), whilst the response of the GA and IEP can not. In addition, the EPSO-GM algorithm can achieve better result than the conventional EPSO method when the average cost is taken into consideration.

### Table II

<table>
<thead>
<tr>
<th>Method</th>
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<th>Generation schedule (MW)</th>
<th>Total Power (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GA [17]</td>
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<td>300.00</td>
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</tr>
<tr>
<td>IEP [19]</td>
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<tr>
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<td>8235.32</td>
<td>8234.07</td>
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<td>149.73</td>
</tr>
</tbody>
</table>

### Table III

<table>
<thead>
<tr>
<th>Methods</th>
<th>Average cost ($)</th>
<th>Best cost ($)</th>
<th>Generation schedule (MW)</th>
<th>Total Power (MW)</th>
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<td>8239.44</td>
<td>8234.07</td>
<td>300.27</td>
<td>149.73</td>
</tr>
</tbody>
</table>

Case study 2: 10-unit 24-hour system

Instead of scheduling the generation to meet a single power demand as shown in the previous case study, the proposed EPSO-GM, in this case, is intended to determine the schedule of generation to meet a certain period of time power demands (i.e. 24 hr) from 1036 MW to 2220 MW. The parameters used in this implementation are: the agents’ size = 20, and maximum number of generations =20000, respectively.

Table III lists the statistic data that include the average cost, the best cost, the maximum cost and the standard deviation of the average costs obtained from the evolutionary programming (EP) [2], the hybrid method between evolutionary programming and sequential quadratic programming (EP-SQP) [2], the modified hybrid EP-SQP (MHEP-SQP) [35], the hybrid method between PSO and SQP (PSO-SQP) [3], the PSO-SQP method with the “crazy” particle (PSO-SQP(C)) [3], the deterministically guided PSO (DGPSO) [36], the EPSO [31], as well as the proposed EPSO-GM. From the simulation results show that the EPSO-GM method outperforms in finding the better solutions, while considering the population size and the maximum number of generations compared with other algorithms.

Note: As presented in [33], $\phi$ is generally set to 4.1, both $c_1$ and $c_2$ are set to 2.05 and $k$ is 0.729, where $k$ - constriction factor, $c_1$, $c_2$ - acceleration constants, $w_{\text{max/min}}$ - max/min inertia weight, and $R_m$ - mutation rate.
Table IV shows the frequencies of reaching the final solution over 30 different runs obtained from the methods considered. Regarding the number of reaching the best cost in the range of $1,020,000-$1,025,000 the proposed EPSO-GM methodology is superior to the other selected algorithms. In addition, the EPSO-GM shows the higher performance in terms of achieving the higher range of the optimal cost. Again, it can be seen that the EPSO-GM reveals its superiority to all the other methods in regard to reliability of the solutions. The best solution obtained from the EPSO-GM is also shown in Table V.


crazy particle is re-initialization the velocity of the particle randomly when a random number (0,1) is less than or equal to the predefined probability.

VII. Conclusion

In this paper, a hybrid EPSO-GM is proposed for solving the DED problem. The proposed EPSO-GM is a method of combining an enhanced Particle Swarm Optimization (EPSO) with Gaussian mutation (GM) so as to increase the global search capability. Concerning the EPSO, it also employs a modified heuristic approach for handling various operating constraints and increasing the searching performance instead of using the standard PSO alone. To validate the capability of
the proposed EPSO-GM, it is applied to solve DED problem considering many non linear characteristics of the generator i.e. non-smooth cost function characteristic and generator ramp rate limit. It can be concluded from the simulation results that the EPSO-GM shows its superiority over other methods in regard to obtaining higher quality solution.

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