Analysis of Stress Concentration and Deflection in Isotropic and Orthotropic Rectangular Plates with Central Circular Hole under Transverse Static Loading

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Abstract—The distributions of stresses and deflection in rectangular isotropic and orthotropic plates with central circular hole under transverse static loading have been studied using finite element method. The aim of author is to analyze the effect of D/A ratio (where D is hole diameter and A is plate width) upon stress concentration factor (SCF) and deflection in isotropic and orthotropic plates under transverse static loading. The D/A ratio is varied from 0.01 to 0.9. The analysis is done for plates of isotropic and two different orthotropic materials. The results are obtained for three different boundary conditions. The variations of SCF and deflection with respect to D/A ratio are presented in graphical form and discussed. The finite element formulation is carried out in the analysis section of the ANSYS package.

Keywords—Finite Element Method, SCF, Deflection, Plate, Boundary conditions

I. INTRODUCTION

A rectangular isotropic or orthotropic plate with central circular hole under transverse static loading, have found widespread applications in various fields of engineering such as aerospace, marine, automobile and mechanical. For design of plates with hole, accurate knowledge of deflection, stresses and stress concentration are required. Stress concentration arises from any abrupt change in geometry of plate under loading. As a result, stress distribution is not uniform throughout the cross section. Failures such as fatigue cracking and plastic deformation frequently occur at points of stress concentration.

Paul and Rao [1,2] presented a theory for evaluation of stress concentration factor of thick and FRP laminated plate with the help of Lo-Christensen-Wu higher order bending theory under transverse loading. Shastry and Raj [3] have analyzed the effect of fibre orientation for a unidirectional composite laminate with finite element method by assuming a plane stress problem under in plane static loading.


In this article a study of rectangular isotropic and orthotropic plate with central circular hole upon the effect of D/A ratio on SCF under transverse static loading is made. The purpose of this research work is to investigate the effect of D/A ratio on SCF for normal stress in X, Y directions (σx, σy) and shear stress in XY plane (τxy), and on deflection in transverse direction (Uz). The Uz for different ratio of D/A is compared with deflection in transverse direction in plate without hole (Uz0). Results are obtained for three different boundary conditions. The analytical treatment for such type of problem is very difficult and hence the finite element method adopt for whole analysis.

II. DESCRIPTION OF PROBLEM

To study the influence of D/A ratio upon deflection and SCF for different stresses, a rectangular plate of dimension 200 mm × 100 mm × 1 mm with a circular hole of diameter D at centre under uniformly distributed static loading of P = 0.02 Newton in transverse direction for all cases is analyzed by finite element method. Fig. 1 shows the basic model of the problem. The entire dimensions are also shown in Fig.1. The D/A ratio is varied from 0.1 to 0.9.
III. FINITE ELEMENT ANALYSIS

An eight nodded Structural 3-D Shell Element (specified as Shell93 in ANSYS package) with element length of 2 mm, was selected based on convergence test and used throughout the study. Each node has six degrees of freedom, making a total 48 degrees of freedom per element. In order to construct the graphical image of the geometries of models for different D/A ratios, rectangular isotropic and orthotropic plates examined using the ANSYS (Advanced Engineering Simulation). Mapped meshing are used for all models so that more elements are employed near the hole boundary. Due to the symmetric nature of different models investigated, it was necessary to discretize the quadrant plate for finite element analysis. Main task in finite element analysis is selection of suitable element type. Numbers of checks and convergence test are made for selection of suitable element type from different available elements and to decide the element length. Fig. 2 provides the example of the discretized models for D/A =0.2, used in study.

![Fig. 1 Basic model of the problem](image)

![Fig. 2 Typical example of finite element model for D/A=0.2](image)

IV. RESULTS AND DISCUSSION

Numerical results are presented for isotropic and orthotropic rectangular plates with a central circular hole. The material properties of different used materials are shown in following Table I. Where; E, G and μ represent modulus of elasticity, modulus of rigidity and poisson’s ratio respectively.

![Table I](image)

**TABLE I**

<table>
<thead>
<tr>
<th>Properties</th>
<th>Isotropic</th>
<th>Orthotropic-1 (E-glass/epoxy)</th>
<th>Orthotropic-2 (Boron Aluminum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E_x</td>
<td>39 GPa</td>
<td>39 GPa</td>
<td>235 GPa</td>
</tr>
<tr>
<td>E_y</td>
<td>39 GPa</td>
<td>8.6 GPa</td>
<td>137 GPa</td>
</tr>
<tr>
<td>E_z</td>
<td>8.6 GPa</td>
<td>8.6 GPa</td>
<td>137 GPa</td>
</tr>
<tr>
<td>G_xy</td>
<td>3.8 GPa</td>
<td>3.8 GPa</td>
<td>47 GPa</td>
</tr>
<tr>
<td>G_yz</td>
<td>3.8 GPa</td>
<td>3.8 GPa</td>
<td>47 GPa</td>
</tr>
<tr>
<td>G_zx</td>
<td>3.8 GPa</td>
<td>3.8 GPa</td>
<td>47 GPa</td>
</tr>
<tr>
<td>μ_xy</td>
<td>0.3</td>
<td>0.28</td>
<td>0.30</td>
</tr>
<tr>
<td>μ_yz</td>
<td>0.28</td>
<td>0.28</td>
<td>0.30</td>
</tr>
<tr>
<td>μ_zx</td>
<td>0.28</td>
<td>0.28</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Plates with three different boundary conditions, as plate (a), (b) and (c) are analyzed. In plate (a); all edges are simply supported, in plate (b); all edges are fixed, in plate (c); two edges are fixed and other two are simply supported. Fig. 3 provides the boundary conditions at all edges of plates (a), (b) and (c).

![Fig. 3 Boundary conditions at all edges of plates (a), (b) and (c)](image)

Stresses and deflections are obtained for uniformly distributed loads P = 0.02 Newton for all cases and D/A ratios. The stresses and deflection in transverse direction (U_z*) for full plate (a), (b) and (c), made of different materials under uniformly distributed load of 0.02 Newton are listed in Table II.

![Fig. 4](image)

Fig. 4 shows the effect of D/A ratio on SCF (for σ_x, σ_y and τ_xy) and U_z/U_z* in plates (a), (b) and (c) of isotropic material. Following observations can be made from these results.
Variation of SCF for σx with respect to D/A ratio observed, maximum in case of plate (a) and significant in case of plates (b) and (c). In case of plate (a); SCF for σx increased from 1.65 to 2.11 with increase of D/A ratio from 0.01 to 0.06 and decreased from 2.11 to 0.89 with increase of D/A ratio from 0.08 to 0.9. In case of plate (b); SCF for σx almost unchanged with increase of D/A ratio from 0.01 to 0.5 and slight increased from 1.07 to 1.30 with increase of D/A ratio from 0.5 to 0.9. In case of plate (c); SCF for σx increased from 1.00 to 1.20 with increase of D/A ratio from 0.01 to 0.1. Variation of SCF for σx with respect to D/A ratio observed, maximum in case of plate (c), significant in case of plates (a) and almost negligible in case of plate (b). In case of plate (a); SCF for σx increased from 1.44 to 1.81 with increase of D/A ratio from 0.01 to 0.08 and decreased from 1.81 to 1.31 with increase of D/A ratio from 0.1 to 0.9. In case of plate (b); SCF for σx almost unchanged with increase of D/A ratio. In case of plate (c); SCF for σx increased from 1.44 to 1.82 with increase of D/A ratio from 0.01 to 0.1 and decreased from 1.82 to 0.88 with increase of D/A ratio from 0.1 to 0.9. Variation of SCF for τxy with respect to D/A ratio observed, maximum in case of plate (b) and significant in plates (a) and (c). In case of plate (a); SCF for τxy fluctuated between 0.9 to 1.25 with increase of D/A ratio from 0.01 to 0.9. In case of plate (b); SCF for τxy increased from 1.33 to 2.37 with increase of D/A ratio from 0.01 to 0.1, decreased from 2.37 to 1.31 with increase of D/A ratio from 0.1 to 0.6 and again increased from 1.31 to 1.61 with increase of D/A ratio from 0.6 to 0.9. In case of plate (c); SCF for τxy increased from 0.98 to 1.81 with increase of D/A ratio from 0.01 to 0.1 and decreased from 1.81 to 1.06 with increase of D/A ratio from 0.1 to 0.9. Variation of Uz/Uz* with respect to D/A ratio observed, maximum in case of plate (a), significant in plate (c) and minimum in plate (b). In case of plate (a); the ratio of Uz/Uz* increased from 1.00 to 1.33 with increase of D/A ratio from 0.01 to 0.6 and decreased from 1.33 to 1.26 with increase of D/A ratio from 0.6 to 0.9. In case of plate (b); the ratio of Uz/Uz* increased from 1.00 to 1.10 with increase of D/A ratio from 0.01 to 0.2, decreased from 1.10 to 0.95 with increase of D/A ratio from 0.2 to 0.7 and again increased from 0.95 to 0.99 with increase of D/A ratio from 0.7 to 0.9. In case of plate (c); the ratio of Uz/Uz* increased from 1.00 to 1.24 with increase of D/A ratio from 0.01 to 0.4 and decreased from 1.24 to 0.90 with increase of D/A ratio from 0.4 to 0.9. It is clear from table 1 that in case of isotropic material, for plates (a) and (b) σx attained more in compare to other stresses i.e. it is remembered that SCF for σx is more important in plates (a) and (b) and SCF for σy is more important in plate (c).

Fig. 5 shows the effect of D/A ratio on SCF (for σx, σy and τxy) and Uz/Uz* in plates (a), (b) and (c) of orthotropic material-1. Following observations can be made from these results. Variation of SCF for σx with respect to D/A ratio observed, maximum in case of plate (a) and significant in case of plates (b) and (c). In case of plate (a); SCF for σx decreased from 5.19 to 1.18 with increase of D/A ratio from 0.01 to 0.4 and increased from 1.18 to 1.35 with increase of D/A ratio from 0.4 to 0.9. In case of plate (b); SCF for σx increased from 0.70 to 1.00 with increase of D/A ratio from 0.01 to 0.9. In case of plate (c); SCF for σx decreased from 1.84 to 0.82 with increase of D/A ratio from 0.01 to 0.1, decreased from 0.82 to 0.91 with increase of D/A ratio from 0.1 to 0.5 and again decreased from 0.91 to 0.83 with increase of D/A ratio from 0.5 to 0.9. Variation of SCF for σy with respect to D/A ratio observed, maximum in case of plates (a) and (c) and significant in case of plates (b). In case of plate (a); SCF for σy continuously decreased from 6.25 to 1.39 with increase of D/A ratio from 0.01 to 0.9. In case of plate (b); SCF for σy decreased from 1.50 to 1.26 with increase of D/A ratio from 0.01 to 0.03, increased from 1.26 to 1.29 with increase of D/A ratio from 0.03 to 0.1 and again decreased from 1.29 to 0.73 with increase of D/A ratio from 0.1 to 0.9. In case of plate (c); SCF for σy continuously decreased from 7.10 to 0.52 with increase of D/A ratio from 0.01 to 0.9. Variation of SCF for τxy with respect to D/A ratio observed, significant for all boundary conditions. In case of plate (a); SCF for τxy decreased from 4.06 to 2.84 with increase of D/A ratio from 0.01 to 0.07 and increased from 2.84 to 3.78 with increase of D/A ratio from 0.07 to 0.9. In case of plate (b); SCF for τxy decreased from 3.81 to 3.12 with increase of D/A ratio from 0.01 to 0.05, increased from 3.12 to 3.18 with increase of D/A ratio from 0.05 to 0.2 and again decreased from 3.18 to 1.64 with increase of D/A ratio from 0.2 to 0.9. In case of plate (c); SCF for τxy decreased from 5.25 to 1.91 with increase of D/A ratio from 0.01 to 0.2, increased from 1.91 to 1.98 with increase of D/A ratio from 0.2 to 0.4 and again decreased from 1.98 to 1.47 with increase of D/A ratio from 0.4 to 0.9. Variation of Uz/Uz* with respect to D/A ratio observed, maximum in case of plate (a), significant in plate (c) and minimum in plate (b). In case of plate (a); the ratio of Uz/Uz* increased from 0.89 to 1.26 with increase of D/A ratio from 0.01 to 0.4 and decreased from 1.26 to 1.14 with increase of D/A ratio from 0.4 to 0.9. In case of plate (b); the ratio of Uz/Uz* increased from 0.51 to 0.72 with increase of D/A ratio from 0.01 to 0.8 and decreased from 0.72 to 0.70 with increase of D/A ratio from 0.8 to 0.9. In case of plate (c); the ratio of Uz/Uz* increased from 0.95 to 1.17 with increase of D/A ratio from 0.01 to 0.2 and decreased from 1.17 to 0.57 with increase of D/A ratio from 0.2 to 0.9. It is clear from table 1 that in case of orthotropic material-1, for all plates (a), (b) and (c) σx attained more in compare to other stresses i.e. it is remembered that SCF for σx is more important than SCF for σy and τxy.

**TABLE II**

<table>
<thead>
<tr>
<th>Material Type</th>
<th>Plate</th>
<th>σx (N/m²)</th>
<th>σy (N/m²)</th>
<th>τxy (N/m²)</th>
<th>Uz/Uz*</th>
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<tbody>
<tr>
<td>Isotropic</td>
<td>(a)</td>
<td>2803.10</td>
<td>6122.60</td>
<td>2386.90</td>
<td>2.85E-07</td>
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<td></td>
<td>(b)</td>
<td>3415.00</td>
<td>4969.80</td>
<td>3634.29</td>
<td>7.10E-08</td>
</tr>
<tr>
<td></td>
<td>(c)</td>
<td>7151.00</td>
<td>5224.80</td>
<td>2061.30</td>
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<tr>
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<td>(a)</td>
<td>6499.60</td>
<td>5301.40</td>
<td>2809.90</td>
<td>1.20E-06</td>
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<tr>
<td></td>
<td>(b)</td>
<td>7345.10</td>
<td>4541.90</td>
<td>577.27</td>
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<tr>
<td></td>
<td>(c)</td>
<td>13725.00</td>
<td>3224.80</td>
<td>1534.00</td>
<td>7.23E-07</td>
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<tr>
<td>Orthotropic-2</td>
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<td>3721.80</td>
<td>6146.00</td>
<td>2608.70</td>
<td>8.42E-08</td>
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<tr>
<td></td>
<td>(b)</td>
<td>4473.70</td>
<td>4956.60</td>
<td>536.16</td>
<td>2.09E-08</td>
</tr>
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<td></td>
<td>(c)</td>
<td>9290.90</td>
<td>4807.50</td>
<td>1699.10</td>
<td>6.42E-08</td>
</tr>
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</table>
Fig. 4 Effect of $D/A$ ratio on SCF (for $\sigma_x$, $\sigma_y$, and $\tau_{xy}$) and $U_{z}/U_{z}^*$ in plates (a), (b) and (c) of isotropic material

Fig. 5 Effect of $D/A$ ratio on SCF (for $\sigma_x$, $\sigma_y$, and $\tau_{xy}$) and $U_{z}/U_{z}^*$ in plates (a), (b) and (c) of orthotropic material-1
Fig. 6 Effect of $D/A$ ratio on SCF (for $\sigma_x$, $\sigma_y$ and $\tau_{xy}$) and $U_z/U_z^*$ in plates (a), (b) and (c) of orthotropic material-2.
Fig. 6 shows the effect of $D/A$ ratio on SCF (for $\sigma_x$, $\sigma_y$ and $\tau_{xy}$) and $U_x/U_y$ in plates (a), (b) and (c) of orthotropic material-2. Following observations can be made from these results. Variation of SCF for $\sigma_x$ with respect to $D/A$ ratio observed, maximum in case of plate (a) and significant in case of plates (b) and (c). In case of plate (a); SCF for $\sigma_x$ decreased from 3.51 to 0.82 with increase of $D/A$ ratio from 0.01 to 0.5 and increased from 0.82 to 0.85 with increase of $D/A$ ratio from 0.5 to 0.9. In case of plate (b); SCF for $\sigma_x$ continuously increased from 0.92 to 1.22 with increase of $D/A$ ratio from 0.01 to 0.9. In case of plate (c); SCF for $\sigma_x$ decreased from 1.25 to 0.92 with increase of $D/A$ ratio from 0.01 to 0.1 and increased from 0.92 to 1.07 with increase of $D/A$ ratio from 0.1 to 0.9. Variation of SCF for $\sigma_y$ with respect to $D/A$ ratio observed, maximum in case of plates (a) and (c) and significant in case of plates (b). In case of plate (a); SCF for $\sigma_y$ continuously decreased from 2.52 to 1.36 with increase of $D/A$ ratio from 0.01 to 0.9. In case of plate (b); SCF for $\sigma_y$ increased from 1.05 to 1.07 with increase of $D/A$ ratio from 0.01 to 0.1, decreased from 1.07 to 0.83 with increase of $D/A$ ratio from 0.1 to 0.7 and again increased from 0.83 to 0.85 with increase of $D/A$ ratio from 0.7 to 0.9. In case of plate (c); SCF for $\sigma_y$ continuously decreased from 2.64 to 0.76 with increase of $D/A$ ratio from 0.01 to 0.9. Variation of SCF for $\tau_{xy}$ with respect to $D/A$ ratio observed, significant for all boundary conditions. In case of plate (a); SCF for $\tau_{xy}$ increased from 1.87 to 2.27 with increase of $D/A$ ratio from 0.01 to 0.03, decreased from 2.27 to 1.65 with increase of $D/A$ ratio from 0.03 to 0.2 and again increased from 1.65 to 2.11 with increase of $D/A$ ratio from 0.2 to 0.9. In case of plate (b); SCF for $\tau_{xy}$ increased from 2.71 to 3.21 with increase of $D/A$ ratio from 0.01 to 0.03, decreased from 3.21 to 1.56 with increase of $D/A$ ratio from 0.03 to 0.5, again increased from 1.56 to 1.86 with increase of $D/A$ ratio from 0.5 to 0.8 and again decreased from 1.86 to 1.80 with increase of $D/A$ ratio from 0.8 to 0.9. In case of plate (c); SCF for $\tau_{xy}$ increased from 2.43 to 2.95 with increase of $D/A$ ratio from 0.01 to 0.03 and decreased from 2.95 to 1.24 with increase of $D/A$ ratio from 0.03 to 0.9. Variation of $U_x/U_y$ with respect to $D/A$ ratio observed, maximum in case of plate (a), significant in plate (c) and minimum in plate (b). In case of plate (a); the ratio of $U_x/U_y$ first increased from 0.93 to 1.26 with increase of $D/A$ ratio from 0.01 to 0.5 and decreased from 1.26 to 1.21 with increase of $D/A$ ratio from 0.5 to 0.9. In case of plate (b); the ratio of $U_x/U_y$ increased from 0.79 to 0.89 with increase of $D/A$ ratio from 0.01 to 0.2, decreased from 0.89 to 0.86 with increase of $D/A$ ratio from 0.2 to 0.5 and again increased from 0.86 to 0.92 with increase of $D/A$ ratio from 0.5 to 0.9. In case of plate (c); the ratio of $U_x/U_y$ increased from 0.94 to 1.16 with increase of $D/A$ ratio from 0.01 to 0.3 and decreased from 1.16 to 0.76 with increase of $D/A$ ratio from 0.3 to 0.9. It is clear from table 1 that in case of orthotropic material-2, for plates (a) and (b) $\sigma_x$ attained more in compare to other stresses and in case of plate (c) $\sigma_x$ attained more in compare to other stresses i.e. it is remembered that SCF for $\sigma_x$ is more important in plates (a) and (b) and SCF for $\sigma_y$ is more important in plate (c).

V. CONCLUSIONS

In general; for plates (a) and (c), the maximum stress concentration is always occurred on hole boundary and in case of plates (b), the maximum stress concentration is occurred on supports. The SCF for $\sigma_x$ and $\sigma_y$ varied maximum in plates (a) and (c) and minimum in plate (b), where the SCF for $\tau_{xy}$ varied significant in plates (a), (b) and (c) with respect to $D/A$ ratio for all materials. The variation of SCF for all stresses with respect to $D/A$ ratio observed more in orthotropic plates as compare to isotropic plates for all boundary conditions. The SCF for all stresses is achieved more in orthotropic plate as compare to isotropic plate for respective boundary conditions. It is also observed that variation of SCF for all stresses with $D/A$ ratio; highly depends on elastic constants and differ with material to material. For all materials, stress concentration for $\sigma_x$ and $\sigma_y$ occurred maximum in plates (a) and (c) and minimum in plate (b), where Stress concentration for $\tau_{xy}$ occurred significant in plates (a), (b) and (c), hence the SCF for $\sigma_x$ and $\sigma_y$ plays an important role in design of plates (a) and (c) and a minor role in design of plate (b), where the SCF for $\tau_{xy}$ plays an important role in design of plates (a), (b) and (c). Maximum deflection in transverse direction occurred for plate (a) and minimum deflection in transverse direction occurred for plate (b) for all cases. The variation of $U_x/U_y$ with $D/A$ ratio has maximum in plate (a), significant in plate (c) and minimum in plate (b) for all materials. It is also observed that the trend of variation of $U_x/U_y$ with $D/A$ ratio is almost same for all materials for respective boundary conditions. For all materials; in case of plate (a) and (c), the $U_x/U_y$ first increased with increase of $D/A$ ratio and after some increase, decreased with increase of $D/A$ ratio, but in case of plate (b) the $U_x/U_y$ fluctuated between a small range with respect to $D/A$ ratio.

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REFERENCES


