Abstract—A zero-field ferromagnetic Ising model is utilized to simulate the propagation of infection in a population that assumes a square lattice structure. The rate of infection increases with temperature. The disease spreads faster among individuals with low $J$ values. Such effect, however, diminishes at higher temperatures.

Keywords—Epidemiology, Ising model, lattice models

I. INTRODUCTION

One of the simplest models in epidemiology separates the population into two discrete states: $S$ for susceptible and $I$ for infected or infective [1]. In the SI model, the time rate of change in number of susceptible and infected individuals varies given by the following equations:

\[
\frac{dS}{dt} = -\beta SI \quad \text{(1)}
\]

and

\[
\frac{dI}{dt} = \beta SI \quad \text{(2)}
\]

where $\beta$ is the infection rate. To consider a closed system, we consider a constant population size. Thus at any given time, the sum of susceptibles and infectives is equal to some constant, say $N$. That is, $S(t) + I(t) = N$. In which case, the above equations yield the following solutions:

\[
S(t) = \frac{N}{1 + e^{-\beta(t-t_0)}} \quad \text{(3)}
\]

and

\[
I(t) = \frac{N}{1 + e^{-\beta(t-t_0)}} \quad \text{(4)}
\]

Eq. 3 and 4 are logistic or S-curves. The point of inflection occurs at the critical time $t_c$.

In this work, we adapt the Ising model [2] framework to study dynamics of disease spread. Similar to [3], we confine our investigation to a square lattice and impose periodic boundary conditions. We observe the infection rate, $\beta$, varies with parameters such as temperature $T$ and neighbor interactions.

II. THE MODEL

Consider a closed community described by the square lattice in Fig. 1. Each site is occupied by one individual which can either be susceptible (spin down, $\sigma = -1$) or infective (spin up, $\sigma = +1$). The edges of the lattice are connected, thus forming a torus or a donut. Associate with each lattice configuration is a Hamiltonian which takes the form

\[
H = -J \sum_{i,j} \sum_{x,y} \sigma_i^x \sigma_j^y \quad \text{(5)}
\]

Where $(x,y) \in \{(i+1, j), (i-1, j), (i, j+1), (i, j-1)\}$ describes the Von Neumann neighborhood and $J$ is the interaction parameter which described the (coupling) strength between spins.

Initially, the lattice is filled with susceptibles (all spin down). A random site is then selected and the corresponding spin flipped (from down to up). This is the first infective. For succeeding iterations, a randomly chosen site changes state if the change in energy, $\Delta H = H_{\text{new}} - H_{\text{previous}}$, is less than or equal to zero. If $\Delta H > 0$, the flip is accepted according to the probability

\[
p = e^{-\Delta H / T} \quad \text{(6)}
\]
where $T$ is the scaled temperature. Flipping is allowed only in one direction (up $\rightarrow$ down, $S \rightarrow I$). Whenever a site is infected, it remains that way until the end of the simulation. Data presented in the next section correspond to the mean of 10 independent trials for a population size $N=100$ (10 $\times$ 10 lattice).

III. RESULTS AND DISCUSSION

Simulation results reveal a logistic type of behavior, which is characteristic of the standard SI model. Fig. 2 shows the propagation of infection when $T=2.20$ and $J=1.00$. At the onset, the spread of the disease approximates an exponential growth. But as time progresses, the growth slows down and saturates at the value of $N$. Since there is no recovery (flipping is one way), the entire community eventually becomes infected. The point of inflection, $t_c$, corresponds to the time when 50% of the population is already affected by the disease.

Estimates of the critical time, $t_c$, and infection rate, $\beta$, are obtained by using Eqn. 4 as a fitting function. Fig. 3 reveals a sudden drop in $t_c$ associated with a slight rise in temperature (from 6000 to $<1000$ when $T$ was changed from 1.0 to 2.0). But as $T$ is increased further, the variation in critical times becomes minimal. The point of inflection occurs at an earlier time at larger $T$ values. Thus, the disease spreads more rapidly at higher temperatures. Calculated values of the rates, $\beta$, are plotted in Fig. 4. As $T$ increases, $\beta$ approaches a constant value. In our Ising-based SI model, the concept of temperature may be related to the level of aggression of a particular virus or associated to external parameters like ambient temperature and humidity. The concept of temperature may also be perceived with a broader scope to include the effects of cultural and socio-economic risk factors.

Next, we present the effect of varying interaction parameter, $J$, on the infection curves. In Fig. 5(a), saturation (100% infective) is achieved fastest when $J=0.25$. The rate of spread of infection decreases with increasing $J$. This parameter may be interpreted as the inverse of the contact time. Lower $J$ values can mean prolonged exposure to the agent or contagion. The effect of the parameter $J$ on the spread of the disease, however, vanishes at higher temperatures. Fig. 5(b) shows overlapping plots, independent of the $J$ value.

IV. SUMMARY AND CONCLUSION

The dynamics of the spread of infection on a two-dimensional square lattice with periodic boundary conditions was analyzed using an Ising-based SI model. Beginning from a single infective, the disease was able to propagate faster at higher temperatures. Increasing the value of the interaction parameter, $J$, slowed down the infection spread. These effects, however, became less evident as the temperature is increased further.
Fig. 5 Effect of varying $J$: (a) $T=2.2$ and (b) $T=10.0$

REFERENCES

