World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:1, No:9, 2007

Time-Delay Estimation Using Cross-Ψ_B-Energy Operator

Z. Saidi, A.O. Boudraa, J.C. Cexus, and S. Bourennane

Abstract—In this paper, a new time-delay estimation technique based on the cross Ψ_B-energy operator [5] is introduced. This quadratic energy detector measures how much a signal is present in another one. The location of the peak of the energy operator, corresponding to the maximum of interaction between the two signals, is the estimate of the delay. The method is a fully data-driven approach. The discrete version of the continuous-time form of the cross Ψ_B-energy operator, for its implementation, is presented. The effectiveness of the proposed method is demonstrated on real underwater acoustic signals arriving from targets and the results compared to the cross-correlation method.

***Keywords*—**Teager-Kaiser energy operator, Cross-energy operator, Time-Delay, Underwater acoustic signals.

I. Introduction

Time-delay (TD) estimation between signals in the presence of noise and interference is a problem of importance in areas such as communications (radio transmitter,…), sonar, radar, biomedicine and geophysics [1]-[4]. For example, in sonar signal processing the TD between the observed signals is used to estimate the sources (targets) ranges and bearings. TD is often referred to by TD estimation (TDE), time of arrival (TDOA), and time-of-arrival difference (TOAD). A common method for measuring the TD involves cross correlating the receiver outputs; an estimate of the TD is given by the argument that maximizes the cross-correlation (CC) function. The CC is linear similarity measure. However, often it is not possible to avoid nonlinearities of the sensors used. Furthermore, in practice the interaction between the signals, generated by sensors, may not be linear, and thus the maximum of CC does not, necessarily, corresponds to maximum of interaction. Consequently the resulting TD value is problematic. To tackle this problem we present a similarity method that takes into account the nonlinearity of the signals and their interaction. This method is based on cross Ψ_B-energy operator witch is a nonlinear measure, recently proposed in [5]. Furthermore this operator is well suited for nonstationary signals [6]. Ψ_B-energy operator is derived from a second energy-like function, called cross Teager-Kaiser energy operator (CTKEO) [7]-[8], which measures the interaction between two real time functions. Ψ_B measures how much a signal is present in another one. Based on a nonlinear operator, Ψ_B, the method may be viewed as a nonlinear matched filtering.

In this paper we propose to use the Ψ_B method in the acoustic array processing to estimate the TD. The proposed method is compared to the experimental, considered as the standard, and cross-correlation TD estimation methods on real underwater acoustic data.

Z. Saidi, A.O. Boudraa, and J.C. Cexus are with IRENav, Ecole navale, Lanvéoc-Poulmic, BP 600, 29240, Brest-Armées, France (corresponding author: A.O. Boudraa, e-mail: boudra@ecole-navale.fr).

S. Bourennane is with Institut Fresnel, UMR 6133, Equipe GSM, Domaine universitaire de Saint Jérôme, 13013 Marseille Cedex 20, France.

II. Ψ_B-Based Time Delay Estimation

A. Cross-Ψ_B-Energy Operator

Recently the CTKEO has been extended to complex-valued signals and an operator called Ψ_B introduced [5]. Let x and y be two complex signals, Ψ_B is defined as follows [5]:

$$\begin{aligned}\Psi_B(x,y) &= 0.5\left[\Psi_{\mathbb{C}}(x,y)+\Psi_{\mathbb{C}}(y,x)\right]\\ &= 0.5\left[\dot{x}^*\dot{y}+\dot{x}\dot{y}^*\right]-0.25\left[x\ddot{y}^*+x^*\ddot{y}+y\ddot{x}^*+y^*\ddot{x}\right]\end{aligned}$$

where $\Psi_{\mathbb{C}}(x,y)=0.5\left[\dot{x}^*\dot{y}+\dot{x}\dot{y}^*\right]-0.5\left[x\ddot{y}^*+x^*\ddot{y}\right]$. It has been shown that the cross Ψ_B-energy operator of x and y is equal to the cross-Teager energies of their real and imaginary parts [5]:

$$\Psi_B(x,y)=\Psi_B(x_r,y_r)+\Psi_B(x_i,y_i)$$

$$\Psi_B(x_k,y_k)=\dot{x}_k\dot{y}_k-0.5\left[x_k\ddot{y}_k+\ddot{x}_k y_k\right],\ k\in\{r,i\}$$

where $x(t)=x_r(t)+jx_i(t)$ and $y(t)=y_r(t)+jy_i(t)$. Where j denotes the imaginary unit. To discretize Ψ_B, two-sample backward difference is used. The aim is to obtain a discrete operator, noted Ψ_{Bd}, closely related to continuous version of Ψ_B and operating on discrete-time signals $x(n)$ and $y(n)$. Thus, we replace t by nT_S (T_S is the sampling period), $x(n)$ with $x(nT_S)$ or simply $x(n)$, $\ddot{x}(t)$ with $\left[x(n)\text{-}2x(n\text{-}1)+x(n\text{-}2)\right]/T_S$ and $\dot{x}(t)$ with $\left[x(n)\text{-}x(n\text{-}1)\right]/T_S$. Then, for $k\in\{r,i\}$ we have:

$$\Psi_B\left(x_k(t),y_k(t)\right)\mapsto x_k(n\text{-}1)\,y_k(n\text{-}1)/T_S^2$$
$$-0.5\left[x_k(n)\,y_k(n\text{-}2)+y_k(n)\,x_k(n\text{-}2)\right]/T_S^2$$
$$\Psi_B\left(x_k(t),y_k(t)\right)\mapsto \Psi_{Bd}\left(x_k(n\text{-}1),y_k(n\text{-}1)\right)/T_S^2$$

where $\mapsto$ denotes the mapping from continuous to discrete. Thus from Ψ_B we obtained Ψ_{Bd} shifted by one sample to the right and scaled by T_S^{-2}. If we ignore the one-sample shift and the scaling by T_S^{-2}, one can transform $\Psi_B\left(x(t),y(t)\right)$ into $\Psi_{Bd}\left(x(n),y(n)\right)$ as follows:

$$\Psi_B\left(x(t),y(t)\right)\mapsto \Psi_{Bd}\left(x_r(n),y_r(n)\right)+\Psi_{Bd}\left(x_i(n),y_i(n)\right)$$
$$\Psi_{Bd}\left(x_k(n),y_k(n)\right)=x_k(n)\,y_k(n)$$
$$-0.5\left[x_k(n{+}1)\,y_k(n\text{-}1)+x_k(n\text{-}1)\,y_k(n{+}1)\right]$$

It is easy to see that the two-sample forward difference gives the same result. However, three-sample symmetric difference leads to more complicated expression . Note that if $x=y$, the operator Ψ_{Bd} is reduced to the discrete Teager-Kaiser operator $\Psi_{Bd}\left(x(n),x(n)\right)=x^2(n)-x(n\text{-}1)\,x(n{+}1)$.

B. Time-Delay Estimation Problem

Consider the signal from a remote source being received in the presence of noise at two spatially separated receivers. The time histories of the receiver outputs, denoted by $r_m(t)$ and $r_k(t)$, are given by

$$r_m(t)=s(t)+n_m(t)$$
$$r_k(t)=A.s\left(t\text{-}(k\text{-}m)D\right)+n_k(t)$$

where $s(t)$ is the signal waveform, $n_m(t)$ and $n_k(t)$ are the noise waveforms at the respective receivers, A is an attenuation factor and D is the difference in the wavefront arrival times at two consecutive receivers $(k=2,m=1)$, TD.

Proposition:

Suppose that the noises $n_m(t)$ and $n_k(t)$, and the signals $r_m(t)$ and $r_k(t)$ are mean square differentiable and mutually uncorrelated. Then

$$E\left[\Psi_B(r_m(t),r_k(t))\right]=A.E\left[\Psi_B(s(t),s(t\text{-}\tau))\right] \qquad (1)$$

where $\tau=(k\text{-}m).D$ and $E[.]$ is the expectation operator.

Proof:

$n_m(t)$ and $r_m(t)$ have zero-mean and are uncorrelated. Then,

$$E\left[n_m(t).n_k(t)\right]=0\Rightarrow E\left[n_m(t).\frac{dn_k(t)}{dt}\right]=\text{-}E\left[n_k(t).\frac{dn_m(t)}{dt}\right]$$

$s(t)$ and $n_m(t)$ are independent. Then

$$E\left[s(t).n_k(t)\right]=E\left[s(t)\right].E\left[n_k(t)\right]=0\ \Rightarrow$$
$$E\left[\frac{ds(t)}{dt}.\frac{dn_k(t)}{dt}\right]=-\frac{1}{2}E\left[s(t).\frac{d^2n_k(t)}{dt^2}\right]-\frac{1}{2}E\left[n_k(t).\frac{d^2s(t)}{dt^2}\right]$$

Similarly, for $s(t\text{-}\tau)$ and $n_m(t)$ we can write

$$E\left[\frac{ds(t)}{dt}.\frac{dn_m(t)}{dt}\right]=-\frac{1}{2}E\left[s(t\text{-}\tau).\frac{d^2n_m(t)}{dt^2}\right]-\frac{1}{2}E\left[n_m(t).\frac{d^2s(t)}{dt^2}\right]$$

where $\dfrac{ds(t\text{-}\tau)}{dt}=\dfrac{ds(t)}{dt}$

$$\Psi_B\left(r_m(t),r_k(t)\right)=\frac{d\left[s(t)+n_m(t)\right]}{dt}.\frac{d\left[a.s(t\text{-}\tau)+n_k(t)\right]}{dt}$$
$$-\frac{1}{2}\left[\left(s(t)+n_m(t)\right)\left(a.\frac{d^2s(t)}{dt^2}+\frac{d^2n_k(t)}{dt^2}\right)+\left(a.\frac{d^2s(t)}{dt^2}+\frac{d^2n_m(t)}{dt^2}\right).\left(a.s(t\text{-}\tau)+n_k(t)\right)\right]$$

$$E\left[\Psi_B(r_m(t),r_k(t))\right]=A.E\left[\Psi_B(s(t),s(t\text{-}\tau))\right]+$$
$$2.E\left[\frac{dn_k(t)}{dt}.\frac{ds(t)}{dt}\right]+2A.E\left[\frac{dn_m(t)}{dt}.\frac{ds(t)}{dt}\right]+2E\left[\frac{dn_m(t)}{dt}.\frac{dn_k(t)}{dt}\right]$$

$$E\left[\frac{dn_m(t)}{dt}.\frac{dn_k(t)}{dt}\right]=E\left[\lim_{h\to 0}\frac{n_m(t+h)-n_m(t)}{h}.\lim_{h'\to 0}\frac{n_k(t+h')-n_k(t)}{h'}\right]$$
$$=E\left[\lim_{h\to 0}\left(n_k(t+h).n_m(t+\alpha.h)\right)\right]\text{-}E\left[\lim_{h\to 0}\left(n_k(t+h).n_m(t)\right)\right]$$
$$-E\left[\lim_{h\to 0}\left(n_k(t).n_m(t+\alpha.h)\right)\right]=\text{-}E\left[n_k(t).n_m(t)\right]=0$$

where $h'=\alpha.h$ and α is a constant.

Similarly,

$$E\left[\frac{dn_m(t)}{dt}.\frac{ds(t)}{dt}.\right]=E\left[\frac{dn_k(t)}{dt}.\frac{ds(t)}{dt}.\right]=0$$

and (1) results.

Let $t_0=(k\text{-}m).D$ be the unknown time representing the delay of the received signal. Let $T=\left[T_{min},T_{max}\right]$ be the possible range of values for t_0. For any given $t_0\in T$ the maximum of interaction between $r_m(t)$ and $r_k(t)$ measured with the cross Ψ_B-energy operator is given by

$$T_B=\arg\max_{t\in T}\left[\int_T \Psi_B\left(r_m(t),r_k(t)\right)\right] \qquad (2)$$

Thus the quadratic detector, Ψ_B, calculates the interaction between the received signal and all possible time-shifted versions of the transmitted signal and picks the largest energy interaction as the decision. The location of the peak is the estimate of the unknown parameter $T_B=\Delta d/c$ where and $\Delta d=d_k\text{-}d_m$ is the path length difference (Figure 1).

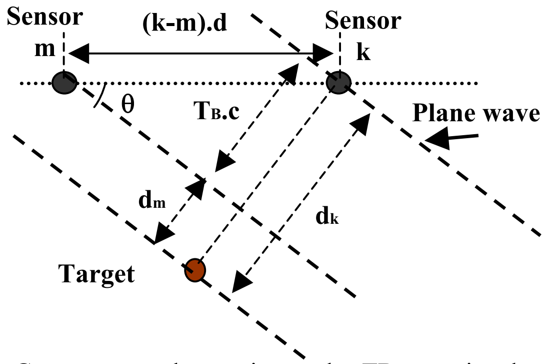

Figure 1: Geometry used to estimate the TD associated with plane waves.

C. Pseudo-code

Both the reference signal, $s(m)$, and the received one, $r(n)$, are smoothed using Savitzky-Golay filter [9]. The complex form of the signals are obtained using Hilbert transform. The quadratic operator involves the following steps:

Inputs:

Reference signal: $s(m)=s_a(m)+js_b(m), \quad m\in\{1,2,\cdots,w\}$

Received signal: $r(n)=r_a(n)+jr_b(n), \quad n\in\{1,2,\cdots,N\}$

Output: T_B (time-delay)

For k=1 to (N-w+1) // k is the current window

$g^k(n-k+1)\leftarrow r(n), \quad n\in F=\{k,w+k-1\}$

$\Psi_r^l\leftarrow\Psi_{B_d}(s_a(l),g_a^k(l))\ , \quad l\in\{1,\cdots,w\}$

$\Psi_i^l\leftarrow\Psi_{B_d}(s_b(l),g_b^k(l))$

$\Psi_{B_d}(s(l),g^k(l))\leftarrow\Psi_r^l+\Psi_i^l$

Compute the sum $I(k)$ of Ψ_{B_d} values over F:

$$I(k)\leftarrow\sum_{l=k}^{w+k-1}\Psi_{B_d}(s(l),g^k(l))$$

EndFor

$$T_B\leftarrow\underset{1\le k\le M}{\operatorname{argmax}}\left[I(k)\right]$$

where $\leftarrow$ denotes the affectation operation.

III. RESULTS

TD Ψ_B method is tested on underwater acoustic signals. A linear array composed of n uniformly spaced sensors (Figure 1) is used. Each sensor received signals arriving from the sources (targets) supposed sufficiently distant from the receivers. In this case, the wavefronts are assumed to be planar. The TD, T_B, between the wave fronts impinging upon the mth and the kth sensors are given by the experimental method as follows:

$$T_B=(k-m)\,d.\sin\theta/c \tag{3}$$

where d is the distance between two consecutive receivers $(k=2,m=1)$, c is the celerity in the medium and θ is the direction of arrival of a plane wave (Figure 1). TD is estimated on real acoustical data measured in a tank with a linear array of twenty sensors (n=20) where air-full cylindrical objects are buried under the sand. Two cylinders for Data 1 and one cylinder for Data 2: c=1485ms^{-1}, d=2mm, $\theta_{Data1}=22^\circ$, $\theta_{Data2}=64.15^\circ$, sample number=2000 points, sampling rate=10MHz. 2D plots, samples-sensor, of Figure 2(b) and 3(b) represent sensors output of signals (echoes) arriving from two (Data 1) and one (Data 2) cylinders respectively. Signals are smoothed using a third-order Savitzky-Golay filter over a moving window of width 51. Both the Ψ_B and the CC methods, applied to filtered signals, are implemented in the time-domain. 1D plots, amplitude-samples, of Figure 2(a) and 3(a) represent the output of the first sensor, selected as reference signal s(n), for Data 1 and Data 2 respectively. The Experimental, CC and Ψ_B estimated time delays, for all array sensors, are plotted, as a function of the position indexes of the sensors along the array, in Figures 4(a)-(b) for Data 1 and Data 2 respectively. In each case delay estimation is performed between the first sensor of the array and the remaining ones. Root mean square error (RMSE) between pair of sensors, calculated using equation (4), for Data 1 and Data 2 is reported in Table I.

$$RMSE_{(\psi_B\text{-}Exp)}=\sqrt{\frac{\sum_{i=1}^{n}\left(TD_{\psi_B}(i)-TD_{Exp}(i)\right)^2}{n-1}} \tag{4}$$

As shown in Figure 4(b), for Data 2 there is a parfait agreement (except for sensor 2 where the error is one sample) between the Ψ_B and the experimental method (Eq. (3)). This is confirmed by the RMSE value (0.0526). The RMSE of the CC method is 3.42 times higher than that of the Ψ_B method. Figure 4(a) shows that, for Data 1, the estimated TD by the Ψ_B and the CC deviate moderately from TD values given by Eq. (3). Note that the Ψ_B performs slightly better than the CC with RMSE value of 0.217. For both Data 1 and Data 2, globally, the Ψ_B performs better than the CC method. This may suggest, even partially, the nonlinear relationship between the signals that linear method such as the CC cannot account for. The mismatch between expected TD values and the Ψ_B TD ones may be due to the error in the estimation of the bearing angles θ_{Data1} and θ_{Data2}, and the c value which depends on the temperature of the propagation media. It is important to keep in mind that there is no a golden method for evaluating the TD estimation. The method based on Eq. (3) can be used, in the far field case, as reference method if we have good measures of θ and d.

a) b)

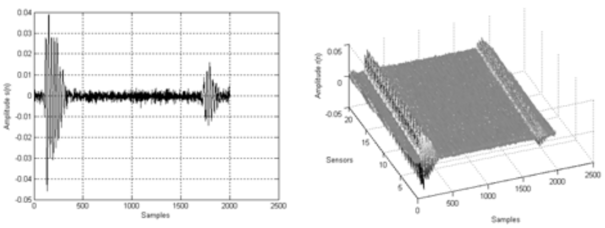

Fig. 2: Two buried targets example. (a) The selected reference signal s(n) . (b) Time-sensors representation of Data 1.

a) b)

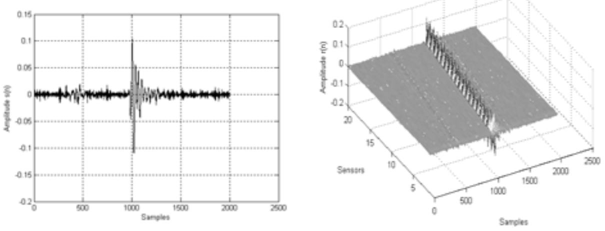

Fig. 3: Two buried targets example. (a) The selected reference signal s(n) . (b) Time-sensors representation of Data 2.

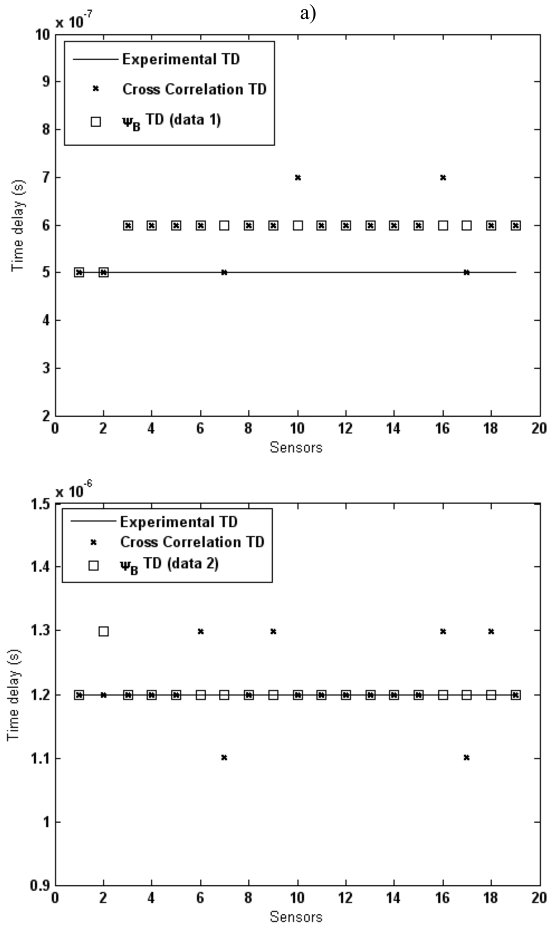

Fig. 4: TD estimations by experimental, CC and Ψ_B methods. Estimated TD versus the sensors position indexes along the array in Data 1 (a) and Data 2 (b) respectively.

Table I

RMSE between Ψ_B and experimental TD, CC and experimental TD for Data 1 and Data 2.

Data 1		Data 2	
$RMSE_{(\Psi_B\text{-Exp})}$	$RMSE_{(CC\text{-Exp})}$	$RMSE_{(\Psi_B\text{-Exp})}$	$RMSE_{(CC\text{-Exp})}$
0.2170	0.250	0.0526	0.180

IV. CONCLUSION

In this paper, a new method for TD estimation, based on Ψ_B operator is introduced. Ψ_B measures how much a signal is present in another one. As the CC method, the Ψ_B is simple and very easy to implement efficiently. A discrete version of the continuous-time of Ψ_B for its implementation is presented. Presented preliminaries results on real underwater acoustic signals are very close to that of the experimental method. The obtained results show that the Ψ_B method globally performs better than the CC method. To confirm the effectiveness of the Ψ_B method must be evaluated with a large class of signals and in different experimental conditions such as high noise levels, sampling rates and sample sizes. As future work, we plan to estimate TD in the case of non uniform array. We also plan to modify the approach to tackle the problem of mutually correlated noises. It is also interesting to use TD estimated by Ψ_B in order to correct sensor gain and phase uncertainties.

ACKNOWLEDGMENT

Authors would like to thank Dr. Jean Pierre Sessarego from CNRS-LMA, Marseille, France, for making data available. We also thank Pr. Thierry Chonavel from ENST-Bretagne, France for his helpful suggestions and valuable comments.

V. REFRENCES

[1] S. Haykin, Adaptive Filter Theory. Prentice-Hall, Inc., New-Jersey, 1996.
[2] B.G. Ferguson, “Improved time-delay estimates of underwater acoustic signals using beamforming and prefiltering techniques,” *IEEE Trans. Oceanic Eng*., vol. 14, no. 3, pp. 238-244, 1989.
[3] S. Chandran, M.K. Ibrahim, “DOA estimation of wide-band signals based on time-frequency analysis,” *IEEE Trans. Oceanic Eng*., vol. 24, no. 1, pp. 116-121, 1999.
[4] R.J. Ulman, and E. Geraniotis, “Wideband TDOA/FDOA processing using summation of short-time CAF’s,” *IEEE Trans. Sig. Proc*., vol. 47, no. 12, pp. 3193-3200, 1999.
[5] J.C. Cexus, and A.O. Boudraa, “Link between cross-Wigner distribution and cross-Teager energy operator,” *IEE Electronics Lett*., vol. 40, pp. 778-780, 2004.
[6] A.O. Boudraa, J.C. Cexus, F. Salzenstein and L. Guillon, "IF estimation using EMD and nonlinear Teager energy operator," *Proc. of First Int. Symp. Control, Commun. and Sig. Process.,* Tunisia, pp. 45-48, 2004.
[7] J.F. Kaiser, “Some useful properties of Teager’s energy operators,” *Proc. ICASSP*, vol. 3, pp. 149-152, 1993.
[8] P. Maragos, and A. Potamianos, “Higher order differential energy operators,” *IEEE Sig. Proc. Lett*.., vol. 2, pp. 152-154, 1995.

[9] A. Savitzky, and M.J.E. Golay, "Smoothing and differentiation, of data by simplified least squares procedures," *Analytical chemistry*, vol. 36, pp. 1627-1639, 1964.

Zineb SAIDI was born in Boumerdes, Algeria. She received the Engineering degree in Electronics from University of Tizi-Ouzou, Algeria and the M.Sc degree in Electrical engineering from Polytechnic School of the University of Nantes, France. She is currently pursuing the PhD degree in signal processing at Institut de Recherche de l'Ecole navale (IRENav), Brest-Armées, France. Her research interests include detection and localization of buried targets using array processing method and blind sources separation.

Abdel-Ouahab BOUDRAA was born in Constantine, Algeria. He received the BS degree in Physics (Electronics Engineering) from Constantine Institute of Physics, University of Constantine, Algeria. He received the M.Sc in Biomedical Engineering from INSA, Lyon, the University degree in Nuclear Magnetic Resonance, the PhD degree in Image Processing and the University degrees in Statistics and Modeling and in Positron Emission Tomography all from the University of Claude Bernard, Lyon 1, France. He is currently Associate Professor of electrical engineering at Ecole navale, Brest, France. His current research interests include computer vision, vector quantization, data structures and analysis, data fusion, time frequency analysis, hard and fuzzy pattern recognition and applications of fuzzy set theory to medical image. Dr. BOUDRAA is recipient of 2003 Varian Prize awarded by the Swiss Society of Radiobiology and Medical Physics for the best published paper impacting Radiation Oncology. Dr. BOUDRAA is Member of IEEE ociety.

ean Christophe CEXUS was born in Hyeres, France. He received the ngineering degree and the M.Sc degree in control from l'Ecole Supérieure les Sciences Appliquées pour l'Ingénieur de Mulhouse (ESSAIM) in control. Ie is currently pursuing the PhD degree in signal processing at Institut de lecherche de l'Ecole navale (IRENav), Brest-Armées, France. His research nterests include time-frequency analysis, Sonar target recognition and neural etworks.

alah BOURENNANE received the PhD degree from the Institut National olytechnique de Grenoble, France. He is currently, Professor of electrical ngineering at Ecole Nationale Supérieure de Physique de Marseille, France. Iis main research include array processing and sources detection and ocalization in underwater acoustics.