Optimal Route Policy in Air Traffic Control with Competing Airlines

Siliang Wang, Minghui Wang

Abstract—This work proposes a novel market-based air traffic flow control model considering competitive airlines in air traffic network. In the flow model, an agent based framework for resources (link/time pair) pricing is described. Resource agent and auctioneer for groups of resources are also introduced to simulate the flow management in Air Traffic Control (ATC). Secondly, the distributed group pricing algorithm is introduced, which efficiently reflect the competitive nature of the airline industry. Resources in the system are grouped according to the degree of interaction, and each auctioneer adjusts the price of one group of resources respectively until the excess demand of resources becomes zero when the demand and supply of resources of the system changes. Numerical simulation results show the feasibility of solving the air traffic flow control problem using market mechanism and pricing algorithms on the air traffic network.

Keywords—Air traffic control, Nonlinear programming, Market mechanism, Route policy.

I. INTRODUCTION

In the research of ATC, much work has been reported recently for the air traffic management. Flow scheduling is proved quite a difficult task for air traffic control based on sector occupancy, which is presented in [1]. A continuous model based on airway links is proposed in [2]. Partial differential equation is also applied in ATC [3-4]. From these works, how to realize flow allocation properly is proved the main objection in air traffic network. From the point of resource allocation, flow allocation in proper is also equal to finding optimal route policies for all flights by keeping their schedule. Airlines can make the flight plan properly to solve the problem of finding optimal route policy for each flight in ATC.

This paper is organized as follows. The following section is the problem formulation, then, section 3 presents optimal route policy in ATC. In section 4, numerical examples on small-size network are carried out to demonstrate the applications of the proposed model and algorithms. The final section then concludes the paper.

II. PROBLEM FORMULATION

A. Network initialization

The air traffic network is defined as a directed graph \( G(N, A) \), with \( N \) being the set of nodes and \( A \) being the set of links, respectively. It treats airports or waypoints as nodes and airways as links. The link capacity fluctuates with the weather. The network is discretized with time steps indexed by \( t \in T \). Each airline, \( i \in I \), can plan flights from origin nodes \( r_i \in N \) to destination nodes \( d_i \in N \). \( \{o, d\} \in OD \) is origin-destination pair. Each flight can be started at any point in time \( T \), and it means the ground holding strategy is allowed in the ATC. Each link-time pair \( j = (l, t) \in J \), is defined as a resource. Resources \( (R^j) \) are mainly constrained by link capacity. The link capacity satisfies the vector \( C^e \in R^J \). The path, \( p \in P \) is defined as the resources consumed by the flights at the special time. Each resource is assigned to the path at the time of entry onto a link. The path-resource matrix is \( A \in P \times J \).

The airport departure and arrival flow can be defined by summing all OD pairs that arrive or depart from a specific airport. Let \( A_o \in \{0,1\}^{P \times J} \), \( A_d \in \{0,1\}^{P \times J} \) be the corresponding matrices. The airport capacities are constrained by

\[
\begin{align*}
A_o f^o \leq C^o \\
A_d f^d \leq C^d \\
f^o \leq S f
\end{align*}
\]

(1)

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where $C^o \in R^{T \times n_r}$, $C^d \in R^{T \times n_d}$, are the constraints on safe operation, and $f$ is the allocated flow and $S$ is the scheduled flow in requirement. From equation (1), the initial flow allocation of the network is deduced as

$$\min||S^d - F^d||^2$$

$$s.t. Af \leq C$$

$$f \geq 0$$

(2)

Here $S^d$ is the cumulative scheduled arrival flow, $F^d$ is the arrival flow allocated in practice, $A$, is the set of $A^o$, $A^d$, $A^e$, and $C$ is the set of $C^o$, $C^d$, $C^e$. Through the computation, the initial global information about the network is formed, which can be used in preparation for subsequent research on optimal route policy.

B. Market-based resource allocation mechanism

In view of the initial global information of the network, we can build a market-based resource allocation mechanism, which can efficiently match resource supply and demand. According to the definition of the network, it is easily concluded that the link/time is the resource in ATC, while the flight plans are defined as the demand in market theory. Therefore, the optimal flow scheduling is formulated as finding equilibrium between supply and demand in competing with the resources. Moreover, it is equal to finding optimal route policy under time-dependent environment. From the perspective of economics, the market mechanism ideally suits for the problem of resource allocation, and is also proved a distributed self-decision mechanism in economic activities, namely, the changes between supply and demand reflects the pricing in the market [7]. The optimal resource allocation is achieved as the supply meets the demand.

The market-based resource allocation mechanism in ATC is divided into two steps.

1) Establish the equilibrium price of all resources through central pricing management.

2) Develop resource allocation and optimal route policy with resource pricing and allocation strategies applied into bidding local resource.

The structure of the market-based resource allocation mechanism is depicted in Fig 1.

The subscript of variable $G_i$ denotes the sequence of resources. Therefore, the object function of resource group is to minimize the total dispersion of clusters

$$D = \sum_{k=1}^{m} \sum_{X_i \in C_k} d(X_i, Z_k)$$

(5)

where $Z_k$ is the kth clustering center, and $d(X_i, Z_k)$ is the distance from $X_i$ to the clustering center ($Z_k$) associated with the samples. The objection of (5) is to compute Euclidean distance of $X_i$ and $Z_k$, namely,

$$d(X_i, Z_k) = \sum_{l=1}^{k} d(X_i, \bar{X}_k)$$

(6)

Therefore, equation (5-6) can efficiently depict distribution of resource. We can obtain the cluster $G_i$ and clustering center $Z_k$ by using K-means clustering algorithm[8].

It is obvious that the procedure of finding groups has done the same work as the preprocess in pursuing the optimal route policy. The problem’s dimensions tends to be confined into a single group. Thus, the computation cost of finding the optimal route policy is obviously decreased.

III. OPTIMAL ROUTE POLICY BASED ON MARKET IN AIR TRAFFIC CONTROL

The assumption of airline cooperation is irrational due to the competitive nature of the airline industry. The market mechanism is a method of resource allocation based on distributed self-decision. That is, each participant makes decisions on its preference in accordance with market prices. Simultaneously, the variation of the prices reflect the dynamic of the demand and supply. According to the market mechanism, the central flow optimization can be formulated as a distributed market optimization. The procedure of the transformation consists of two parts: local optimizations performed by the airlines that...
trade off the cost of deviating from the scheduled flow with the cost of purchasing network resources. The other part is, a central pricing mechanism that enforces constraint satisfaction by the airlines [9].

The central pricing is formed with Lagrange multipliers \( \mu \in R^{nw} \) that are referred to as resource prices, and the central flow allocation problem is

\[
P(\mu) = \sum_{i \in I} F_l(i,f_i) + \mu^T (Af - C)
\]

where \( i \) denotes the airline, \( f_i \) is the allocation vector, \( S_i \) is the set of resource tags, \( f_i^v \) is the scheduled flow, \( f_i^t \) is the arrival flow, \( P_m_i \) is the price vector for resource \( i \), and \( P_m_{i+1} \) is the price vector for resource \( i+1 \).

An approximation formula of \( P(\mu_k) \) based on the Taylor series expansion on \( \mu_k^0 \) is formulated as follows

\[
P(\mu_k) \approx P(\mu_k^0) + P'(_{\mu_k}^0) (\mu_k - \mu_k^0) = 0
\]

It is deduced that the expression of new price with (10), that is

\[
\mu_k = \mu_k^0 - [P'(\mu_k^0)]^{-1} P(\mu_k^0)
\]

Here \( P'(\mu_k^0) \) is the first order derivative of vector function \( P(\mu_k) \). According to (11), we obtain the new price \( \mu_k \), then, substitute the temporary price \( \mu_k^0 \) with \( \mu_k \), repeat the procedure until it meets the convergence criteria, \( P(\mu_k) < \xi \). The final new price \( \mu_k^* \) is computed at that time.

4) Compute scope of prices variation as follows,

\[
V = ||\mu_k^* - \mu_k^0||
\]

In accordance with price threshold \( \delta \), we can evaluate the principle of price variation.

\[
v_k^p = \begin{cases} 0, V < \delta \\ 1, V \geq \delta \end{cases}
\]

5) Send new price vector \( \mu_k^p \) and price variation symbol \( v_k^p \) to all agents.

The procedure of price update is

TABLE II

<table>
<thead>
<tr>
<th>Algorithm 2: The procedure of price update</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: For ( k = 1; k &lt; K; k + + )</td>
</tr>
<tr>
<td>2: Evaluate the group ( j )'s price vector ( \mu_j = 1, 2, ..., K, j \neq k )</td>
</tr>
<tr>
<td>3: Compute the additional flow request function ( P_m_i ), and new price in group ( j ) based on price ( \mu_k ) with (10), (11).</td>
</tr>
<tr>
<td>4: Send ( \mu_k ) to auctioneer ( k ).</td>
</tr>
<tr>
<td>5: End while</td>
</tr>
</tbody>
</table>

Definition 1 (Route policy): The route policy is what link \( k \) to take next at each decision epoch, based on the state space \( S = \{s|s = (i,t,In_k)\} \), where \( i \) denotes link \( i \), \( t \) is the discrete time step, and \( In_k = (i,t,tra_i) \) denotes information about travel time of link \( i \), \( tra_i \) and capacity of link \( i \), \( t _i \) under current weather.

In the example of Figure 2, the decision at Airport1 can then be described as: when the state is \( (1, t, (2,13m)) \), take link 3 next; when the state is \( (1, t, (1, 15m)) \), take link 1 next, for all \( t \). Note that information \( In_k \) is one component of a state and refers to link travel time realizations based on which the current decision is made, while a reference to information alone is in the general sense. A routing policy \( RP(x) \) is defined as a mapping from states to decisions (next links), \( \pi : RP(x) \rightarrow A \) [10]. For a given current state and a given decision, probabilities of all possible next states can be evaluated from the network probabilistic description \( P \). A statement of the optimal routing policy problem in an air traffic network is to find

\[
\pi^* = \arg \min_{\pi \in \{\pi^1, \pi^2, ..., \pi^n\}} E(t_{xx} - t_{xx})
\]
IV. NUMERICAL EXPERIMENTS AND ANALYSIS OF RESULTS

The purposes of the numerical examples are to illustrate: the effects of optimal routing policy based on market theory, and the cost comparison and resource payment for all airlines. A number of sensitivity tests and investigations will be carried out as follows for these purposes.

A small-size network of seven nodes, ten links, two O-D pairs and twelve paths, as shown in Figure 2, is used to illustrate the application of the proposed model. In Figure 2, node airport1 and airport2 are original airports, and node airport3 and airport4 are destinations while others are waypoints. There are 30 aircrafts take off from each O/D pair during the time T. The research time is divided into 10 time steps, such as t1, t2, ..., t10.

![Fig. 2. Test network with seven nodes and ten links](image)

Here \( t_{x_k} \) denotes path travel time at the destination state, and \( t_{x_k} \) denotes path travel time at the start state, and \( E(t_{x_k} - t_{x_k}) \) is the expected value of the path travel time. Notice \( f_i \in R^K \), where \( K \) is set of OD/time pairs, and resource \((l, t) \) \( \in R \). Therefore, the optimal routing policy can be formulated as

\[
f_i = \arg \min_{f_i \in \{f_1, f_2, ..., f_K\}} \{ F_i(f_i) + \rho_i \}
\]

\[
\pi^* = F(f_1, ..., f_K)
\]

where \( F(\cdot) \) is the mapping function between \( f_i \) and \( \pi \).

Table III

<table>
<thead>
<tr>
<th>Departure Time</th>
<th>O-D</th>
<th>Route Choice</th>
<th>Payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>t1</td>
<td>Airport1-Airport3</td>
<td>1-2</td>
<td>21.2</td>
</tr>
<tr>
<td>Airport2-Airport4</td>
<td>9-10</td>
<td></td>
<td>23.8</td>
</tr>
<tr>
<td>t2</td>
<td>Airport1-Airport3</td>
<td>3-4-2</td>
<td>22.8</td>
</tr>
<tr>
<td>Airport2-Airport4</td>
<td>7-8-6</td>
<td></td>
<td>30.1</td>
</tr>
<tr>
<td>t3</td>
<td>Airport1-Airport3</td>
<td>1-2</td>
<td>24.4</td>
</tr>
<tr>
<td>Airport2-Airport4</td>
<td>7-8-10</td>
<td></td>
<td>23.2</td>
</tr>
<tr>
<td>t4</td>
<td>Airport1-Airport3</td>
<td>3-8-6</td>
<td>19.7</td>
</tr>
<tr>
<td>Airport2-Airport4</td>
<td>9-10</td>
<td></td>
<td>11.3</td>
</tr>
<tr>
<td>t5</td>
<td>Airport1-Airport3</td>
<td>1-2</td>
<td>32.1</td>
</tr>
<tr>
<td>Airport2-Airport4</td>
<td>9-10</td>
<td></td>
<td>36.2</td>
</tr>
<tr>
<td>t6</td>
<td>Airport1-Airport3</td>
<td>1-2</td>
<td>35.2</td>
</tr>
<tr>
<td>Airport2-Airport4</td>
<td>9-10</td>
<td></td>
<td>36.4</td>
</tr>
<tr>
<td>t7</td>
<td>Airport1-Airport3</td>
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<td>40.2</td>
</tr>
<tr>
<td>Airport2-Airport4</td>
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<td>43.2</td>
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<tr>
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<td>18.3</td>
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<tr>
<td>Airport2-Airport4</td>
<td>7-8-10</td>
<td></td>
<td>17.6</td>
</tr>
<tr>
<td>t9</td>
<td>Airport1-Airport3</td>
<td>1-2</td>
<td>19.6</td>
</tr>
<tr>
<td>Airport2-Airport4</td>
<td>9-10</td>
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<td>23.2</td>
</tr>
<tr>
<td>t10</td>
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<td></td>
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</tr>
</tbody>
</table>


