Abstract—In recent years, tuned mass damper (TMD) control systems for civil engineering structures have attracted considerable attention. This paper emphasizes on the application of particle swarm optimization (PSO) to design and optimize the parameters of the TMD control scheme for achieving the best results in the reduction of the building response under earthquake excitations. The Integral of the Time multiplied Absolute value of the Error (ITAE) based on relative displacement of all floors in the building is taken as a performance index of the optimization criterion. The problem of robustly TMD controller design is formulated as an optimization problem based on the ITAE performance index to be solved using the PSO technique which has a story ability to find the most optimistic results. An 11-story realistic building, located in the city of Rasht, Iran is considered as a test system to demonstrate effectiveness of the proposed method. The results analysis through the time-domain simulation and some performance indices reveals that the designed PSO based TMD controller has an excellent capability in reduction of the seismically excited example building.

Keywords—TMD, Particle Swarm Optimization, Tall Buildings, Structural Dynamics.

I. INTRODUCTION

A critical aspect in the design of civil engineering structures is the reduction of response quantities such as velocities, deflections and forces induced by environmental dynamic loads (i.e., wind and earthquake). Structural control methods are the most recent strategies for this purpose, which can be classified as active, semi-active, passive, and hybrid control methods [1]. In the last three decades or so, the reduction of structural response, caused by dynamic effects, has become a subject of research, and many structural control concepts have been implemented in practice. Tuned mass dampers (TMDs) are the oldest structural vibration control devices in existence. The concept of vibration control using a mass damper dates back to the year 1909, when Frahm invented a vibration control device called a dynamic corresponding vibration absorber [2]. Although active tuned mass damper systems nowadays has received considerable attention from many researchers [3-5], a passive control technique is still considered due to its simplicity. Moreover, many passive control devices such as TMD successfully installed in the real application to reduce the lateral motion of high rise buildings [2,6-8]. In this paper, a TMD system is considered to be applied to multi-degree-of-freedom structure, without specifying which mode should be controlled. Thus, there is no need to transfer the structure to a single-mode model as have been done in the available research.

Several cost or objective functions have been developed to meet a specified performance in the optimization process. In practice, many performance indices can be chosen, as the objective functions resulted in a different result of the optimization. In the active vibration control area, there are also many optimization criteria which have been used by researchers. The most common ones are LQR, LQG, H∞, sliding mode control, pole placement, independent model space control and so on [1,9-13]. It should be noted that in the active control optimization there is a trade off between the responses to be minimize the structural responses while maintaining the control energy to be used, the passive control optimization is free from balancing the two parameters.

In this paper, the Integral of Time multiplied Absolute value of Error (ITAE) based on the displacement of all building's floors is taken as a performance index of the optimization criteria. Usually, the traditional gradient-based search methods are used for solution of this problem. Unfortunately, the optimization requires computations of sensitivity factors and eigenvectors at its iteration process. This gives rise to heavy computational burden and slow convergence. Moreover, there is no local criterion to decide whether a local solution is also the global solution. Thus, conventional optimization methods that make use of derivatives and gradients are, in general, not able to locate or identify the global optimum, but for real-world applications, one is often content with a good solution, even if it is not the best. Consequently, heuristic methods are widely used for the global optimization problem. Recently, global optimization techniques like genetic algorithm (GA) have been used for TMD and Active TMD (ATMD) parameter optimization [14-16]. Although, the GA technique seems to be good method for the solution of TMD parameter optimization problem, however, when the system has a highly epistemic objective function (i.e. where parameters being optimized are highly correlated), and number of parameters to be optimized is large, then they have degraded efficiency to obtain global optimum solution and also simulation process use a lot of computing time. In order to overcome these drawbacks, a Particle Swarm Optimization (PSO) based TMD (PSOTMD) is proposed for the control of structural dynamics in this paper. PSO is a novel population based met heuristic, which utilize the swarm intelligence generated by the cooperation and competition between the particle in a swarm and has emerged as a useful tool for engineering optimization. It has also been found to be

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robust in solving problems featuring non-linearity, non-differentiability and high dimensionality [17-20]. PSO has been motivated by the behavior of organisms, such as fish schooling and bird flocking. Unlike the other heuristic techniques, it has a flexible and well-balanced mechanism to enhance the global and local exploration abilities. Also, it suffices to specify the objective function and to place finite bounds on the optimized parameters.

In this study, the problem of robust TMD design is formulated as an optimization problem and the TMD parameters are automatically tuned with optimization the ITAE based fitness function by the PSO technique to get the maximum reduction in displacement response of the building’s floors, such that the stability problem is guaranteed and the time domain specifications concurrently secured. The effectiveness of the proposed method is tested on an 11-story realistic building, located in the city of Rasht, Iran, under different earthquakes loads through the time domain simulation and some performance indices. The result evaluation shows that the proposed method achieves good robust performance aspect response reduction.

II. STRUCTURAL MODEL

Consider an N-story shear structure with mass damper installed at top floor as shown in Fig. 1. The equations of motion of the structural system under earthquake load can be written as:

\[ M \ddot{x} + C \dot{x} + Kx = e \ddot{x}_g \]  

Where M, C and K are mass, damping, and stiffness matrices, respectively; e and \( \ddot{x}_g \) are the matrix induced ground acceleration, and ground acceleration, respectively. If the x in Eq. (1) is taken as the relative displacement with respect to the ground, the mass matrix for a tall building structure, with the assumption of masses lumped at floor levels, is a diagonal matrix in which the mass of each story is sorted on its diagonal, as given in the following:

\[ M = diag(m_1, m_2, ..., m_N, m_d) \]  

Where, \( m_i \) is mass of ith floor (i=1,2,\ldots,N) in building and \( m_d \) is mass of damper.

The structural damping matrix [C] is assumed to be proportional to the mass and stiffness matrices as [21]:

\[ C = a_o M + b_o K \]  

\[ a_o = \xi_i \frac{2 \omega_i \omega_j}{\omega_i + \omega_j}; \quad b_o = \xi_j \frac{2}{\omega_i + \omega_j} \]  

Where, \( a_o \) and \( b_o \) are the proportional coefficients; \( \omega_i \) and \( \omega_j \) are the structural modal frequencies of modes i and j, respectively; and \( \xi_i \) and \( \xi_j \) are the structural damping ratios for modes i and j.

The structural stiffness matrix [K] is developed based on the individual stiffness, \( k_i \), of each floor, and is given in Eq. (5).

\[ K = \begin{bmatrix} (k_1 + k_2) & -k_2 & -k_2 & \ldots & -k_N \\ -k_2 & (k_2 + k_3) & -k_3 & \ldots & \ldots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_N & \ldots & -k_N & (k_N + k_d) & -k_d \\ \end{bmatrix} \]  

Where, \( k_i \) is stiffness of ith story (i=1,2,\ldots,N); \( k_d \) is stiffness of damper.

The \( e \) and \( x \) vectors are as follows:

\[ e = [-m_1, -m_2, ..., -m_N, -m_d]^T \]  

\[ X = [x_1, x_2, ..., x_N, x_d]^T \]  

Where, \( x_i \) is displacement of damper relative to ground (i=1,2,\ldots,N); \( x_d \) is displacement of damper relative to ground and \( x_g \) is ground displacement due to earthquake.

The equations of motion can then be converted to state-space realization as follows:

\[ \dot{z} = Az + Ei \]  

Where,

\[ A = \begin{bmatrix} 0_{(N+1)} & I_{(N+1)} \\ -M^{-1}K & -M^{-1}C \end{bmatrix}; \quad E = \begin{bmatrix} 0_{(N+1)x1} \\ M^{-1}e \end{bmatrix} \]

Note that by transforming the equations of motion Eq. (1) to state-space equation, we have transformed the second-order differential equation to the first-order one. Note also that the size of the matrices in the state-space equation is 2 times larger than that of the ordinary equation of motion.

The tuned mass damper is a classical engineering control device consisting of a mass, a spring and a viscous damper attached to a vibrating main system in order to attenuate undesirable vibration. Usually the natural frequency of a TMD

![Fig. 1. Example realistic building model and TMD mounted on its top floor.](image-url)
is tuned to a frequency close to one of the natural frequencies of the main system. Thus, there are three main parameters in a TMD system: MMD mass, TMD stiffness coefficient and TMD damping ratio. Consequently, the objective is to find the optimum value of these parameters that involve state matrix, \( A \) in Eq. (8). Usually, this problem is converted to a problem of single-degree-of-freedom (SDOF) structure where the parameter of the structure is at a specified mode (usually the first mode) to be chosen. In view of seeking a more realistic model, this paper is used the multi-degree-of-freedom (MDOF) model as a structural model without specifying to special mode. By considering the structure as an MDOF structure the optimization process is coming more difficult to solve. Moreover, in case of an inherent damping presence in the structure the optimization process is coming more difficult to solve. Therefore, the PSO technique which is a useful tool for solving the problem of MDOF structures with inherent damping. For this reason, the PSO technique which is a useful tool for engineering optimization is being used to find optimal parameter of TMD for reduction of the structural vibrations.

III. PSO TECHNIQUE AND OPTIMIZATION OF TMD PARAMETERS

Particle swarm optimization algorithm, which is tailored for optimizing difficult numerical functions and based on metaphor of human social interaction, is capable of mimicking the ability of human societies to process knowledge [18]. It has roots in two main component methodologies: artificial life (such as bird flocking, fish schooling and swarming); and, evolutionary computation. Its key concept is that potential solutions are flown through hyperspace and are accelerated towards better or more optimum solutions. Its paradigm can be implemented in simple form of computer codes and is computationally inexpensive in terms of both memory requirements and speed. It lies somewhere in between evolutionary programming and the genetic algorithms. As in evolutionary computation paradigms, the concept of fitness is employed and candidate solutions to the problem are termed particles or sometimes individuals, each of which adjusts its flying based on the flying experiences of both itself and its companion. It keeps track of its coordinates in hyperspace which are associated with its previous best fitness solution, and also of its counterpart corresponding to the overall best value acquired thus far by any other particle in the population. Vectors are taken as presentation of particles since most optimization problems are convenient for such variable presentations. In fact, the fundamental principles of swarm intelligence are adaptability, diverse response, proximity, quality, and stability. It is adaptive corresponding to the change of the best group value. The allocation of responses between the individual and group values ensures a diversity of response. The higher dimensional space calculations of the PSO concept are performed over a series of time steps. The population is responding to the quality factors of the previous best individual values and the previous best group values. The principle of stability is adhered to since the population changes its state if and only if the best group value changes [18, 19]. As it is reported in [17], this optimization technique can be used to solve many of the same kinds of problems as GA, and does not suffer from some of GAs difficulties. It has also been found to be robust in solving problem featuring non-linearity, non-differentiability and high-dimensionality. The PSO is the search method to improve the speed of convergence and find the global optimum value of fitness function.

PSO starts with a population of random solutions ‘particles’ in a D-dimension space. The ith particle is represented by \( X_i = (x_{i1}, x_{i2}, \ldots, x_{iD}) \). Each particle keeps track of its coordinates in hyperspace, which are associated with the fittest solution it has achieved so far. The value of the fitness for particle \( i \) (\( pbest \)) is also stored as \( Pi = (p_{i1}, p_{i2}, \ldots, p_{iD}) \). The global version of the PSO keeps track of the overall best value (\( gbest \)), and its location, obtained thus far by any particle in the population. PSO consists of, at each step, changing the velocity of each particle toward its \( pbest \) and \( gbest \) according to Eq. (9). The velocity of particle \( i \), is represented as \( V_i = (v_{i1}, v_{i2}, \ldots, v_{iD}) \). Acceleration is weighted by a random term, with separate random numbers being generated for acceleration toward \( pbest \) and \( gbest \). The position of the \( j \)th particle is then updated according to Eq. (10) [18].

\[
v_{ij} = w \times v_{ij} + c_1 \times rand() \times (P_{ij} - x_{ij}) + c_2 \times rand() \times (P_{gij} - x_{ij}) \quad (9)
\]

\[
x_{ij} = x_{ij} + c_1 \times v_{ij} \quad (10)
\]

Where, \( P_{ij} \) and \( P_{gij} \) are \( pbest \) and \( gbest \). Several modifications have been proposed in the literature to improve the PSO algorithm speed and convergence toward the global minimum. One modification is to introduce a local-oriented paradigm (\( lbest \)) with different neighborhoods. It is concluded that \( gbest \) version performs best in terms of median number of iterations to converge. However, \( pbest \) version with neighborhoods of two is most resistant to local minima. The PSO algorithm is further improved via using a time decreasing inertia weight, which leads to a reduction in the number of iterations [17]. Figure 2 shows the flowchart of the proposed PSO algorithm.

This new approach features many advantages; it is simple, fast and easy to be coded. Also, its memory storage requirement is minimal. Moreover, this approach is advantageous over evolutionary in many ways. First, PSO has memory. That is, every particle remembers its best solution (local best) as well as the group best solution (global best). Another advantage of PSO is that the initial population of the PSO is maintained, and so there is no need for applying operators to the population, a process that is time and memory-storage-consuming. In addition, PSO is based on “constructive cooperation” between particles.

A. TMD design using PSO

Tuned mass damper consisting of a mass, a damping, and a spring is an effective and reliable structural vibration control device commonly attached to the vibrating primary system for suppressing the undesirable vibrations induced by winds and earthquake loads. In the proposed method, we must find the TMD parameters optimally to get maximum reduction in building responses due to the different earthquake excitations. Thus, in optimization process the simulated earthquake using Kanai-Tajimi filter (Eq. (11)) is shown in Fig. 3 by MATLAB.
In this work, $\gamma$ and $\xi$ are considered as 37.3 and 0.3, which are for usual soils, respectively.

The TMD frequency ratio to the first model frequency of the building has been chosen $\beta_d$. The mass of the TMD system was chosen to be $m_o\%$ of total mass of the building and its damping ratio ($\xi_d$) was considered to be percent of the critical value. These three parameters ($m_o$, $\beta_d$ and $\xi_d$) of TMD are optimized by evaluating the cost function as given in Eq. (13). In order to acquire better performance, the number of particle, particle size, number of iteration, $c_1$, $c_2$, and $c$ is chosen as 25, 3, 250, 1.47, 2.2 and 1, respectively. Also, the inertia weight, $\omega$, is linearly decreasing from 0.9 to 0.4.

To improve the overall building response in a robust way and optimization synthesis, PSO is used to solve the above optimization problem that search for optimal or near optimal set of TMD parameters.

**IV. CASE STUDY**

In order to investigate the performance of the proposed control strategy in reducing the structural responses under earthquake loads, an 11-story shear building, located in city of Rasht in the north of Iran, is chosen as an a test system. The structure represents a tall building. The structural properties of this building are listed in Table 1 [15].

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Results of TMD parameter set values using the proposed PSO method are given in Table 2. Also, Fig. 4 shows the minimum fitness value evaluation process.
To investigate the effectiveness of the control system for different disturbances, three different seismic motions are used in the numerical simulations. These ground acceleration records are: El Centro 1940, Kobe 1995 and Tabas 1999 earthquakes. The absolute peak ground accelerations (PGAs) of these earthquake records are 0.3417, 0.8178 and 0.9512 g, respectively.

The result of controlled displacement response of the example building top story due to the El Centro NS earthquake calculated by the PSO-based TMD designed systems are compared with the corresponding uncontrolled ones in Fig. 5 and Table 3. Also, Fig. 6 depicted maximum displacement of floors using the proposed method and uncontrolled systems. Fig. 7 shows the ITAE performance index. It can be seen that from the figures and Table, the response reduction ratio (ratio of the controlled to uncontrolled response) for maximum displacement of top floor of the 11-story example building is about 40% for the PSOTMD. Thus, PSO-based designed TMD is effective method.

A. Simulation results

The effectiveness of the controller system and ITAE in reducing the response of the example building due to other earthquakes is also shown for comparison in Figs. 8-13 and Tables 4-5. Almost the same behavior as for the El Centro earthquake can be observed for these earthquakes, too. Moreover, it is seen from Table 4-5 that the Kobe 1995 and Tabas 1999 earthquakes cause the maximum displacement at the top floor of the example building, which is a very high value in comparison with the other earthquakes. This is expected due to fact that the PGA of the two earthquakes is very high and almost remains constant for a long period of time in comparison with the El Centro earthquake.
To demonstrate the robustness effectiveness of the proposed method, the average reduction ratios (controlled to uncontrolled displacement and acceleration ratio) for all three earthquakes on all 11 stories of the example building are shown in Table 6 and 7, respectively. According to these Tables, it can be seen that in general the PSOTMD system are capable of reducing the maximum displacement and acceleration of the building in each story to about 25.5 and 24.5 of the uncontrolled response, respectively. Thus, based on average the proposed method is very effective.
V. CONCLUSIONS

A new PSO based TMD controller is proposed for the reduction of the tall building responses subjected to earthquake excitations in this paper. This study emphasizes to design and optimize the parameters of the TMD control scheme for achieving the best results in the reduction of the building response under earthquake excitations. The problem of robustly TMD controller design is formatted as an optimization problem based on the ITAE performance index to be solved using PSO technique. The PSO algorithm proposed in this paper is easy to implement without additional computational complexity. Thereby experiments this algorithm gives quite promising results. The ability to jump out the local optima, the convergence precision and speed are remarkably enhanced and thus the high precision and efficiency are achieved. Also, it has simple structure and easy to implement and it does not depend on the nature of the function it minimizes and is insensitive to the initial searching point, there by ensuring quality solution for different trial runs.

For the numerical study, an 11-story realistic building is chosen and the problem is solved in state space, as well as a PSO based TMD controller system are also designed to...
control the building response. The system performance characteristics in terms of 'ITAE' and 'the average reduction ratios' reveal that this control strategy is a promising control scheme for the getting the maximum reduction in displacement and acceleration response of the building's stories. It is effective and ensures robust performance for different earthquakes excitations.

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