Roller Guide Design and Manufacturing for Spatial Cylindrical Cams
Yuan L. Lai, Jui P. Hung, and Jian H. Chen

Abstract—This paper was aimed at developing a computer aided design and manufacturing system for spatial cylindrical cams. In the proposed system, a milling tool with a diameter smaller than that of the roller, instead of the standard cutter for traditional machining process, was used to generate the tool path for spatial cams. To verify the feasibility of the proposed method, a multi-axis machining simulation software was further used to simulate the practical milling operation of spatial cams. It was observed from computer simulation that the tool path of small-sized cutter were within the motion range of a standard cutter, no occurrence of overcutting. Examination of a finished cam component clearly verifies the accuracy of the tool path generated for small-sized milling tool. It is believed that the use of small-sized cutter for the machining of the spatial cylindrical cams can generate a better surface morphology with higher accuracy. The improvement in efficiency and cost for the manufacturing of the spatial cylindrical cam can be expected through the proposed method.

Keywords—Cylindrical cams, Computer-aided manufacturing, Tool path.

I. INTRODUCTION

The cam mechanisms have been widely used in automatic apparatus, such as machine tools, textile machine, automotive engine, wire stretcher, etc. due to the fact that the cam mechanism is a compact construction with few moving parts and easily designed to achieve complicated and precise motion [1]. However, in order to increase the operation efficiency, the rotation speed of the cam has to be adequately raised and the mating parts should be kept in close contact with high precise and smooth motion. Under this condition, the kinematical characteristics should be effectively improved to avoid positioning errors in motion [2]. To this purpose, the cam type associated with the motion curve of the followers should be selected carefully to avoid manufacturing errors [3-6].

Spatial cam is a type of cam mechanism with three-dimensional configuration in motion morphology. The common used cams include cylindrical cams, edge cams, spherical cams, and toothed cam. For cylindrical cams, the cam profile curve was a groove curve generated around a cylinder which can rotate about its longitudinal axis, by which the roller follower was driven to move along the guided groove. The motion mode of the driven roller can be classified as straight reciprocating motion and oscillating arc motion. For reciprocating follower, its motion direction is parallel to the rotation axial direction; for oscillating follower, it will swing about the fixed pivot within a certain angle of orientation, as shown in Fig. 1.

The guided groove of cylindrical cam was usually machined by using a standard milling cutter with the same diameter as that of the roller follower. This would cause some problems in practical applications; for example, when standard cutter of the same size could not be available, a new one should be purchased timely from cutter manufacturer. Another situation might be happened that when a wider guided groove of cylindrical cams was designed, it would be difficult to find another standard cutter of equal-diameter for machining. All of these unexpected restrictions would lead to the uncertainty in manufacturing planning. To reduce the incontinency in process planning of the cam components, this paper proposed another machining method by using a nonstandard cutter in milling operation. The nonstandard cutter has a diameter less than that of the standard cutter, usually referred small-sized cutter herein. It is believed that the use of small-sized cutter for machining spatial cylindrical cams can avoid these drawbacks encountered in manufacture programming and generate a surface morphology with higher accuracy. In addition, the improvement in production and cost for the manufacturing of the spatial cam can be expected through the proposed method.

Fig. 1 A spatial cylindrical cam with followers

II. GENERATION OF CYLINDRICAL CAM PROFILE

A. Motion Curve

In cam mechanism, a cam is the mechanical element used to drive another element, referred follower, through a specified motion.
motion locus. The mathematic equation used to describe motion status of follower is called motion curve, which characterizes the displacement ($s$), velocity ($v$), acceleration ($a$) and jerk ($j$) of the followers with the time during operation. The fundamental motion curve is displacement curve, which can be expressed as the displacement ($s$) function of the follower with respect to the rotational angle ($\theta$) of the cam shaft. Lots of essential motion curves such as harmonic curve, cycloid curve, and polynomial curve, modified sine curve, modified trapezoidal curve, modified constant velocity curve are suitable for follower to achieve a desired motion. Once the displacement curve was prescribed, the motion characteristics of the follower could be obtained by differentiating the displacement function with respect to the cam angle. For example, the simple harmonic curve can be expressed as:

$$s = \frac{H}{2}(1 - \cos \frac{\pi \theta}{\beta})$$

In above, the $H$ represents the maximum rise displacement or lift of the follower, $\beta$ is the rotation angle of cam to rise period. The other kinematical characteristics can then be obtained as follows:

$$v = \frac{ds}{d\theta} = \frac{\pi H}{2\beta} \sin \frac{\pi \theta}{\beta}$$

$$a = \frac{dv}{d\theta} = \frac{\pi^2 H}{2\beta^2} \cos \frac{\pi \theta}{\beta}$$

$$j = \frac{da}{d\theta} = -\frac{\pi^3 H}{2\beta^3} \sin \frac{\pi \theta}{\beta}$$

B. Cam Profile

The roller guide can be considered as a groove circled around the surface of cylinder following the motion curve of roller follower. Among the motion curves mentioned in above, the modified sine function with better kinematical characteristics was a well-recognized motion curve used for cam mechanism. The roller groove was usually generated by milling machine with a cutting. The cutter has a diameter same as that of roller, but has to meet the following requirements:

1. The working surface of roller follower contacting with the guided groove should be a rectangular shape.
2. The diameter of cutter must be equal to the width of roller guide.
3. The width of the guide at specific working depth should be identical to the diameter of the working depth of roller, ensuring the ensuring the roller always contacting with the guided groove.

Theoretical profile of a cylindrical cam can be considered as the trajectory surface left on the cylinder when the roller follower moved along the roller guide. It is also the tracks of all points of roller centre line. Theoretical profile can be built by unfolding the spatial cam profile into a two-dimensional curve as shown in Fig. 2. Based on this concept, the tool path to generate the guided groove can then be obtained by converting the tool path calculated in unfolded 2D space onto the cylinder surface. The actual profile of the cylindrical cam is the representation of curved surface in 3D space when the roller moves along the center line of the guide. For a oscillating follower roller, its center line is perpendicular to the enveloping surface formed by roller location family during the oscillating motion of the follower link. Since the contact line between roller and actual groove was changing with the rotation of the cam, the actual surface could not be perfectly spread out on 2D plane. As a result, division errors are produced in actual unfolding operation. Such errors, however, should be properly controlled within allowable precision range by the designers.

$$\theta_i = \frac{\Delta \beta}{\theta}$$

Where $\theta_i$ is the division angle (i=1~n), $\Delta \beta$ is the rise or return angle, n is the specified division numbers. The angle division may cause errors in chord height. Such errors ($\Delta \epsilon$) will influence the precision of motion curve as shown in Fig. 4. In the figure, $\theta$ is the angle of each line segment. It is noted that an oversize error margin ($\Delta \epsilon$) would lead to distorted curve; therefore the error margin ($\Delta \epsilon$) should be controlled within certain range. The error margin ($\Delta \epsilon$) is expressed as
\[ \Delta \epsilon = r[1 - \cos(\frac{\theta}{2})] \]  

(6)

In above formula, \( r \) is the radius of cylindrical cam.

III. TOOL PATH GENERATION

A. Standard Cutter

To machine the spatial cam, the tool path should be first derived. Generally, the tool path is defined by the cutter location, position of the tool center and orientation of tool axis. The cylindrical cam was usually machined by envelope method, or generating method. According to the mating relationship between the cylindrical cam and roller follower, the roller guided groove can be generated in motion by the cutter with the same geometry as that of the roller follower. When the cutter and roller has identical geometric attributes, the contact relationship between the cutter and workpiece (spatial cam) is same as that between the roller and cam guided groove, as shown in Fig. 6. Therefore, the groove contour of the cylindrical cams could be machined precisely based on this concept.

The method for generating the tool path of a standard cutter for cylindrical cam is suitable for 2D or 3D space. In 2D space, the rotation angle (\( \theta \)) of cam shaft can be used to calculate the unfold arc length value (S). The cutter location X, S is determined by:

1. for translation follower

\[ X = f(\theta) \quad \text{and} \quad S = D \cdot \theta / 2 \]  

(9)

In above, \( f(\theta) \) represents the motion function describing the displacement of the roller follower, \( D \) is the diameter of cam shaft and \( \theta \) is the rotational angle of cam.

2. for oscillating follower

\[ X = b \cos \phi \quad \text{and} \quad S = D \cdot \theta / 2 \]  

(10)

Where \( \phi = f(\theta) \) is angular displacement of the roller follower, \( b \) is the link length of the oscillating follower, \( D \) is the diameter of cam shaft and \( \theta \) is the rotational angle of cam.

For milling machine with four positioning axises, the cutter location is defined by two linear axises X, Y and one rotational axis A, in which the coordinate A can be obtained by converting the coordinate S in 2D space into rotational angle. Therefore the tool path of a cutter with the same diameter as the width of the guide of spatial cylindrical cam can be generated based on this
concept. The procedure are followed: 1. Establish the cam profile curve by setting the kinematical parameters for cam motion, 2. Calculate the cutter location X, S in 2D space, and then convert it into the machine coordinates X, Y, A (rotation axis) for a vertical milling machine. The tool path of a standard-sized cutter can then be obtained.

B. Small-Sized Cutter

When a milling cutter with diameter smaller than roller groove was adopted in the operation, the processing path should be generated by offsetting the centerline of roller groove to the right and left along the normal direction by corresponding distance. Fig. 7 shows the relationship between machining path and centerline of roller groove for small-sized cutter used in NC vertical milling machine. This can be done by firstly converting cylindrical cam from 3D space to X-S plane in 2D space and then generating the tool path of standard-sized cutter, namely, the centerline of the roller guided groove (Y coordinate). After that, by offsetting the centerline to the right and left sides of the groove with a distance, the tool path of small-sized cutter can be generated, which are labeled as CL and CR in the figure, respectively. A detail calculation of the machining path of small-sized standard cutter was given in below.

Fig. 7 Tool path of standard cutter and small-sized cutter

1. Unit normal-vector

\[
\begin{align*}
p_i &= p_o + \left( \frac{BW}{2} - r_s \right) \cdot \hat{n} \\
p_r &= p_o - \left( \frac{BW}{2} - r_s \right) \cdot \hat{n} \\
\hat{n} &= \frac{(s_{i+1} - s_i) \cdot \left( x_{i+1} - x_i \right)}{\sqrt{(x_{i+1} - x_i)^2 + (s_{i+1} - s_i)^2}}
\end{align*}
\]  

where \( S_i \) represents the s value at i point, \( S_{i+1} \) is s value of downward i point, x is the x value of i point, \( X_{i+1} \) is the x value of downward i point. The related calculation steps are:

\[
\begin{align*}
p_i(x_i) &= x_i + \left( \frac{BW}{2} - rs \right) \cdot Y_i \\
p_i(s_i) &= s_i + \left( \frac{BW}{2} - rs \right) \cdot (-X_i) \\
p_i(x_{i+1}) &= x_{i+1} + \left( \frac{BW}{2} - rs \right) \cdot Y_i \\
p_i(s_{i+1}) &= s_{i+1} + \left( \frac{BW}{2} - rs \right) \cdot (-X_i)
\end{align*}
\]  

2. Generation of tool path CL and CR

For CL path

\[
\begin{align*}
p_i(x_i) &= x_i + \left( \frac{BW}{2} - rs \right) \cdot Y_i \\
p_i(s_i) &= s_i + \left( \frac{BW}{2} - rs \right) \cdot (-X_i) \\
p_i(x_{i+1}) &= x_{i+1} + \left( \frac{BW}{2} - rs \right) \cdot Y_i \\
p_i(s_{i+1}) &= s_{i+1} + \left( \frac{BW}{2} - rs \right) \cdot (-X_i)
\end{align*}
\]  

In the above expression, \( p_i(x_i) \) is the x value of i point on the left side and \( p_i(x_{i+1}) \) is the x value of downward i point on the left side. \( p_i(s_i) \) is the s value of i point on the left side and \( p_i(s_{i+1}) \) is the s value of downward i point on the left side.

For CR path

\[
p_r = p_o - \left( \frac{BW}{2} - r_s \right) \cdot \hat{n}
\]  

By substituting parameter x and s in \( P_0 \) in (15), the CR path can then be obtained.

3. Coordinate value of Y on the centerline

The \( Y \) coordinate of the tool path CL and CR is followed the coordinate \( Y \) of the point on the centerline to ensure the machining path of a small-sized cutter being within the motion range of the original tool path; otherwise, the interference between cutter and workpiece might occur.
IV. APPLICATION AND VERIFICATION

In this section, a cylindrical cam with an oscillating roller follower was employed to demonstrate the feasibility of the proposed method by generating and comparing the tool path of standard-sized cutter and small-sized cutter, respectively. Besides, a computer-aided design software for spatial cam mechanism was also developed and presented to show the novelty of the method.

Table I shows the kinematical parameters of a cylindrical cam. Fig. 8 shows operation interface of the package developed in this paper for input of the parameters defining the motion curve of the cylindrical cam. The procedure for generating the tool path was illustrated in the below.

A. Tool Path of Standard Cutter

The motion curve was divided based on the angle division scheme, with a angle segment of 0.8 degree. That means the rotation angle of the cam ranging from 0.8, 1.6, 2.4, 3.2..., 360. The motion curve equation and location of the cutter in machine coordinate system are given as follows:

\[
\phi = \theta - \frac{\theta}{\pi} \cos(\pi \frac{\theta}{\phi})
\]
\[
X = b \cos \phi
\]
\[
Y = \sqrt{b^2 - x^2} - a
\]
\[
A = \text{initial angle} + \theta
\]

Substituting the given parameters into (16) to (19)

For \( \theta = 0.8 \) degree

\[\phi = 70.901 - \frac{6.634}{2} \left[ 1 - \cos(\pi \cdot \frac{0.8}{60}) \right] = 70.88909 \text{ (degree)}\]
\[X = 158.922 \times \cos(70.89809) = 52.007129 \text{ (mm)}\]
\[Y = \sqrt{(158.922)^2 - (52.007129)^2} - 150 = 0.17143 (\text{mm})\]
\[A = 60 + 0.8 = 60.8 \text{ (rotational angle for NC machine A-axis)}\]

B. Tool Path of Small-Sized Cutter

The cutting path for small-sized cutter can be converted from NC code of standard-sized cutter. The additional parameters required for this calculation consist of cam diameter, diameter of small-sized cutter, offset… etc, as shown in Table II. Fig. 9 shows the motion diagram and NC code corresponding machining path.
shows operation interface for the input of these parameters. Some calculation processes are listed as below.

Basing on the NC code of a standard cutter, we can recalculate the location of the small-sized cutter following the steps:

\[ P_1 = P_0 + \left( \frac{BW}{2} - r_s \right) \cdot \hat{n} \]  \hspace{2cm} (20)

\[ P_2 = P_0 - \left( \frac{BW}{2} - r_s \right) \cdot \hat{n} \]  \hspace{2cm} (21)

\[ \hat{n} = \frac{(s_{i+1} - s_i)j - (x_{i+1} - x_i)j}{\sqrt{(x_{i+1} - x_i)^2 + (s_{i+1} - s_i)^2}} \]  \hspace{2cm} (22)

1. Converting the rotational angle of NC machine A-axis into arc length, for example, For \( A = -60 \)

\[ s_1 = rA = 75 \times -60 \times \pi / 180 = -78.5398 \]  \hspace{2cm} (23)

\[ s_2 = rA = 75 \times 60.8 \times \pi / 180 = -79.5870138 \]  \hspace{2cm} (24)

2. Calculating the normal unit vector

\[ UL = \sqrt{(52.007 - 52.000)^2 + (79.5870138 - 78.5398)^2} \]
\[ = 1.047234 \]  \hspace{2cm} (25)

\[ UX = \frac{(52.007 - 52.000)}{1.047234} = 0.00668424 \]  \hspace{2cm} (26)

\[ UY = \frac{(-79.5870138 - 78.5398)}{1.047234} = -0.9999777 \]  \hspace{2cm} (27)

\[ \hat{n} = 0.00668424i + 0.9999777j \]  \hspace{2cm} (28)

### TABLE II

<table>
<thead>
<tr>
<th>Cam type</th>
<th>Oscillating follower</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motion curve</td>
<td>Simple harmonic</td>
</tr>
<tr>
<td>Diameter of cam</td>
<td>150 mm</td>
</tr>
<tr>
<td>Diameter of standard cutter</td>
<td>47 mm</td>
</tr>
<tr>
<td>Depth of roller guide</td>
<td>12 mm</td>
</tr>
<tr>
<td>Diameter of small-sized cutter</td>
<td>12 mm</td>
</tr>
<tr>
<td>Offset</td>
<td>17.5 mm</td>
</tr>
</tbody>
</table>

3. Calculation of cutting path for \( CL \), and then \( CR \)

For \( CL \)

Substituting \( x \) and \( s \) into (20) to (22) for \( P_0 \) and giving

\[ P_{x1} = 52.000 + (47/2 - 12) \cdot (-0.9999777) \approx 34.50039 \]
\[ P_{s1} = -78.5398 - (47/2 - 12) \cdot 0.00668424 = -78.65679 \]
\[ P_{x2} = 52.007 + (47/2 - 12) \cdot (-0.9999777) \approx 34.50739 \]
\[ P_{s2} = -79.5870138 - (47/2 - 12) \cdot 0.00668424 = -79.70399 \]

For \( CR \)

Substituting \( x \) and \( s \) into (20) to (22) for \( P_0 \) and giving

\[ P_{x1} = 52.000 - (47/2 - 12) \cdot (-0.9999777) \approx 69.49961 \]
\[ P_{s1} = -78.5398 + (47/2 - 12) \cdot 0.00668424 = -78.42283 \]
\[ P_{x2} = 52.007 - (47/2 - 12) \cdot (-0.9999777) \approx 69.50661 \]
\[ P_{s2} = -79.5870138 + (47/2 - 12) \cdot 0.00668424 = -79.47004 \]

4. Converting the \( s \) value in step (3) into rotational angel \( A \) for A-axis of NC machine

For \( CL \)

\[ A_1 = \frac{P_{s1}}{r} \times 75 \times \pi = -60.089 \]  \hspace{2cm} (29)

\[ A_2 = \frac{P_{s2}}{r} \times 75 \times \pi = -60.889 \]  \hspace{2cm} (30)

For \( CR \)

\[ A_1 = \frac{P_{s1}}{r} \times 75 \times \pi = -59.91064 \]  \hspace{2cm} (31)

\[ A_2 = \frac{P_{s2}}{r} \times 75 \times \pi = -60.71064 \]  \hspace{2cm} (32)

5. The \( Y \) coordinate is same as that of the center line of the roller guide, that is,

\[ y_{11} = 0.174 \quad y_{12} = 0.171 \quad \text{For } CL \]
\[ y_{21} = 0.174 \quad y_{22} = 0.171 \quad \text{For } CR \]

6. Reordering the coordinates of cutter location for NC machine

For \( CL \)

\[ A_1 = 60.089 \quad P_{x1} = 34.50039 \quad y_{11} = 0.174 \]
\[ A_2 = 60.889 \quad P_{x2} = 34.50739 \quad y_{12} = 0.171 \]

For \( CR \)

\[ A_1 = 59.91064 \quad P_{x1} = 69.49961 \quad y_{11} = 0.174 \]
\[ A_2 = 60.71064 \quad P_{x2} = 69.50661 \quad y_{12} = 0.171 \]

The NC code corresponding to the tool path for small-sized cutter can be obtained following above calculations. By loading the NC program code into commercialized multi-spindle NC simulation software (AlphaCUT), we can simulate the machining process, as shown in Fig. 11. Finally, a finished product after the machining is shown in Fig. 12.

**V. CONCLUSION**

According to the proposed method, the tool path for machining the roller guide groove was accurately generated for a cutter with a small diameter, instead of the standard cutter. A
cylindrical cam with an oscillating roller follower was employed to demonstrate the feasibility of the presented method by generating the tool path of standard-sized cutter and small-sized cutter, respectively. Prior to the practical machining, a computer simulation of practical milling of spatial cams was performed by using a multi-axis NC milling simulation software. It was observed from computer simulation that the tool path of small-sized cutter were within the motion range of a standard cutter, no interference being found. The feasibility of the use of a small-sized cutter for the machining of a spatial cam was fully confirmed after examining the finished cam component.

![Fig. 11 Machining simulation for cylindrical cam using a small-sized cutter](image1)

(a) simulation image  (b) finished cam component

![Fig. 12 Comparison of the spatial cam from machining simulation and finished component](image2)

REFERENCES