Retail Inventory Management for Perishable Products with Two Bins Strategy

Madhukar Nagare, Pankaj Dutta, and Amey Kambli

Abstract—Perishable goods constitute a large portion of retailer inventory and lose value with time due to deterioration and/or obsolescence. Retailers dealing with such goods require considering the factors of short shelf life and the dependency of sales on inventory displayed in determining optimal procurement policy. Many retailers follow the practice of using two bins - primary bin sales fresh items at a list price and secondary bin sales unsold items at a discount price transferred from primary bin on attaining certain age. In this paper, mathematical models are developed for primary bin and for secondary bin that maximizes profit with decision variables of order quantities, optimal review period and optimal selling price at secondary bin. The demand rates in two bins are assumed to be deterministic and dependent on displayed inventory level, price and age but independent of each other. The validity of the model is shown by solving an example and the sensitivity analysis of the model is also reported.

Keywords—Retail Inventory, Perishable Products, Two Bin, Profitable Sales.

I. INTRODUCTION

PERISHABLE inventory constitute a large portion of retailer business which include virtually all foodstuffs, pharmaceuticals, fashion goods, electronic items, periodicals, digital goods and these goods lose value with time due to deterioration and/or obsolescence. Deterioration refers to damage, spoilage, vaporization, depletion, decay (e.g. radioactive substances), degradation (e.g. electronic components) and loss of potency (e.g. pharmaceuticals and chemicals) of goods. Retailers dealing with perishable goods ought to consider the factors of short shelf life and the dependency of sales volume on the amount of inventory displayed in determining optimal procurement policy.

As perishable inventory constitute a substantial part of total inventory; it has attracted the attention of researchers and the problem is studied extensively. Excellent review on inventory management is provied by Nahmias [1], Raafat [2], Goyal and Giri [3]. Researchers also observed that demand rate is directly related to the amount of inventory displayed in the retail store. Wolfe [4], Levin and McLaughlin [5] and Silver and Peterson [6] provided empirical evidence on dependence of retail sale on inventory displayed. These observations have attracted the attention of many researchers and practitioners to investigate the situation where the demand rate is dependent on inventory level in a store. Gupta and Vrat [7] developed an inventory model where demand rate is dependent on initial stock-level (rather than instantaneous inventory level). Baker and Urban [8] was the first one to develop model with the idea of diminishing demand rate with inventory level throughout the cycle. Datta and Pal [9] modified the concept of Baker and Urban [8] with assumption that demand rate would decline with inventory up to a certain level, and then remain constant for the rest of the cycle and cycle terminates with zero stock. In their later formulation, Padmanabhan and Vrat [10] used selling rate dependent on current inventory with backlogging. Soni and Shah [11] formulated optimal ordering policies model for retailer where demand is partly dependent on stock with credit facility to retailer. Prior studies assumed that cycle ending inventory as zero but noting the positive relation between sales and higher inventory levels, Urban [12] relaxed the terminal conditions of zero ending inventories. Larson and DeMarais [13] referred the displayed inventory that stimulate the demand as ‘psychic stock’ and suggested ‘full-shelf merchandising’ policy. Smith and Achabal [14] refer to this stock as “fixtures fill” inventory which is considered as minimum on-hand inventory for adequate presentation by a retailer. In a recent article, Nagare and Dutta [15] addressed this issue through reserve inventory and find an optimal procurement policy for a retailer that deals with continuously deteriorating product and inventory dependent demand rate with non-zero cycle ending inventory.

It is observed that perishable goods display a significant drop in sales with passage of time and loss of freshness i.e. demand decreases significantly with reduction in remaining shelf life (RSL). The consumer behavior of preferring fresh goods to those relatively old having shorter RSL is amply clear at supermarkets where the buyer can pick from the assortment of items on the shelf. It is observed that the customers prefer fresher items over those stale having a shorter RSL [16]. Effective inventory control of these goods having short RSL is vital to enhance profits. Typically, supermarkets try to salvage these products through discount sales or bundling it with other products. This idea motivates us to develop a two-bin perishable inventory model.

Two bin practice followed by many retailers has been studied by researchers. Kar et al. [17] developed an inventory model for several deteriorating items sold from two bins with
limitations on investment and total floor-space area. Das and Maiti’s [18] model allowed shortages with full backlogged in two bins. Hwang et al. [19] provided a seminal work on fixed life time inventory model which subsequently extended to two-bin by Han et al. [20] under LIFO issuing policy and under mixed policy of FIFO and LIFO [21].

However, this paper attempts to model the real world scenario into a mathematical model and studies the impact of some important factors on the profit function under a two-bin framework with inventory dependent demand rate and non-zero cycle ending inventory for primary bin; wherein selling price dependent demand rate for secondary bin. This paper develop inventory models for perishable products with a two-bin framework in which (a) first, we consider the fresh inventory scenario (primary bin) to find an optimal procurement policy and optimal review period for a retailer that deals with inventory dependent demand rate with non zero cycle ending inventory; then (b) in the second scenario, we consider the situation in which the deteriorating goods with its RSL are transferred from the fresh bin to the discount bin. In the later case, as sales depends upon discounted price and hence a selling price (decision variable) dependent demand is constructed. The remainder of this paper is organized as follows. Section II describes the background of the work, notations and assumptions used in this paper, and then two mathematical models considering two different scenarios that maximizes retailer’s profit. In Section III, results of the proposed models are explained using numerical example. Finally, concluding remarks appear in Section IV.

II. MODEL DEVELOPMENT: TWO-BIN PERISHABLE INVENTORY MODEL

A. Background

When two-bin inventory system is used by retailers to manage perishable goods that lose value and freshness with time, keeping in mind different type of customers, one type of customers value freshness and ready to pay premium price; while other type-price sensitive customers prefer aged goods at discounted price. The demand diminishes significantly with loss of freshness with time i.e.as its remaining shelf life (RSL) reduces. It is observed that the customers prefer fresher items over those that have a shorter RSL. A recent survey by Dutta et al. [16] suggests that such behavior is stronger in case of edible perishable items. Effective inventory control of these items with short RSL is vital to enhance profits. Typically, supermarkets try to salvage these products through discount sales or packaging it with other products.

The two-bin inventory model specifically designed for sales of perishable items where First in First out (FIFO) policy cannot be implemented. This inability to control the issuing policy of inventory leads to reduction in demand for perishable items over time. The resulting stock-piling of items with short RSL is difficult to sale due to competition from the fresh inventory. Supermarkets salvage such inventory through promotions like discount pricing or bundling it with other products. This model captures this behavior by dividing the inventory into two bins and tracking the sales of each inventory separately through factors critical for that type of inventory.

The two-bin perishable inventory model breaks up the single assortment of a given item with fixed usable shelf-life (USL) into two based on its RSL:

I. Fresh Inventory: Newly bought and having long RSL

II. Discount Inventory: Items that have short RSL

Newly arrived items are generally placed in the fresh inventory for sales. After the review period T, the items on the fresh bin that have less than a threshold USL are moved to the discount bin (Fig. 1). The issuing policy adopted by the model is roughly equivalent to First Expiry First Out (FEFO) with two bins.

The sale of the discount inventory is primarily based on the promotional price than on the stock on display as applicable for the fresh inventory. This assumption has been verified through the customer survey [16]. The deeper the discount, higher is the sales volumes. So, it is essential that the sales price be carefully set based on stock volumes. The RSL of the inventory in the discount bin is also a crucial determinant in the two-bin model. Hence, it is vital to determine the optimum crossover point from Fresh to Discount Inventory. The point at which the inventory crosses over from the fresh bin to the discount bin is crucial factor. It determines a radical change in the inventory control objectives of the control policies. The purpose of Discount Inventory is primarily to reduce loss or redeem the invested capital by salvaging inventory with short RSL. Retailers do not have fixed policies to control such transfer. However, it was generally observed that customers avoid inventory that has lower than 20% RSL. So, we assume the transfer to discount inventory when item has 20% RSL.

The two-bin perishable inventory model is significantly different from the multi-bin system in two aspects: 

1. The inventory is divided into two bins: fresh and discount.
2. Each bin has a fixed usable shelf-life (USL). 

B. Assumptions and Notations

All the following assumptions and notations are used in the paper.

i) Inventory system consists of only one item.

ii) The item has a known fixed usable shelf-life (USL).

iii) Item cost does not vary as per order size.

iv) Item shortage is not considered.

v) The demand for the item reduces as the remaining shelf
life reduces.

vi) The total inventory is divided into two bins: fresh inventory and discount inventory.

vii) Segregating the items does not affect the demand for fresh inventory.

viii) The demand for the two bins is independent.

ix) Holding cost for inventory in the same bin is considered to be determined by unit sales price. Hence, the demand for the discount inventory. This purchase decision is based on the volume of the discount. Hence, the demand for the discount inventory is considered to be determined by unit sales price.

III. SCENARIO I: INVENTORY MODEL WITH PRIMARY BIN

In order to develop the model for fresh inventory, the following assumptions are made.

i) The demand is stock-dependent and deterministic.

\[ D_1(t) = \alpha_1 + \beta_1 I_1(t) \]

where \( \alpha_1 \) is the average demand of the item (\( \alpha_1 > 0 \)), \( \beta_1 \) is the factor that determines the fluctuation of demand based on stock (\( \beta_1 > 0 \)), \( I_1(t) \) is inventory level at time \( t \), \( D_1(t) \) is demand for the fresh inventory.

ii) Replenishment rate is infinite and lead time is zero.

iii) Backordering is not permitted as observed in a typical Indian supermarket.

iv) Deterioration rate \( \theta_1 \) is deterministic and constant.

(\( 0 < \theta_1 < 1 \))

v) Unit cost of new item \( C \), Selling price per unit \( S_1 \), Holding cost per unit \( H_1 \), Ordering cost per order \( A \) are known and constant.

The differential equation representing the inventory level at time \( t \) may be represented as:

\[ \frac{dI_1(t)}{dt} = -\theta_1 I_1(t) - D_1(t) \]

Substituting the demand function, it yields

\[ \frac{dI_1(t)}{dt} = -\theta_1 I_1(t) - (\alpha_1 + \beta_1 I_1(t)) \]

Using Boundary Condition, \( I_1(T) = I \), and on solving we get,

\[ I_1(t) = \frac{\left(\frac{\alpha_1(\theta_1 + \beta_1)}{\theta_1(\theta_2 + \beta_1)}\right)}{\left[ e^{(\theta_1 + \beta_1)(T-t)} - \frac{\alpha_1}{\theta_1(\theta_2 + \beta_1)} \right]} - I \]  \( (1) \)

Sales of inventory, \( S_1 \)

\[ S_1 = S \int_0^T (\alpha_1 + \beta_1 I_1(t)) \, dt \]

Substituting (1) in above equation,

\[ S_1 = S \int_0^T \left( \alpha_1 + \beta_1 \left(\frac{\alpha_1(\theta_1 + \beta_1)}{\theta_1(\theta_2 + \beta_1)}\right) e^{(\theta_1 + \beta_1)(T-t)} - \frac{\alpha_1}{(\theta_2 + \beta_1)} \right) \, dt \]

\[ = S \left( \alpha_1 T \left[ 1 - \frac{\beta_1}{\theta_1(\theta_2 + \beta_1)} + \frac{\beta_1(\theta_1 + \beta_1)(\theta_1 + \beta_2)}{(\theta_1(\theta_2 + \beta_1)^2)} e^{(\theta_1 + \beta_1)(T-1)} \right] \right) \]  \( (2) \)

Cost of purchase of new items, \( C_1 \)

\[ C_1 = \frac{I_1(T = 0^+) - I_1(T = 0^-)}{C} \]  \( \text{at } t = 0 \)

Holding cost of inventory (\( CH_1 \))

\[ CH_1 = \int_0^T I_1(t) \, dt = H \left[ \frac{\alpha_1}{\theta_1(\theta_2 + \beta_1)} e^{(\theta_1 + \beta_1)(T-1)} - 1 \right] \]  \( (3) \)

Profit from fresh inventory,

\[ P_1 = Sales - Ordering cost - Cost of inventory - Holding cost \]

By using \( dP_1/dT \) and \( d^2 P_1/dT^2 \) one can easily determine the optimum value of \( T \). The optimum \( T \) (viz. \( T_{opt} \)) is obtained using \textit{excel solver} by solving (5) and obtained value of \( T_{opt} \) will give values of optimal profit and order quantity.

\[ Q_{opt} = \frac{\left(\frac{\alpha_1(\theta_1 + \beta_1)}{\theta_1(\theta_2 + \beta_1)}\right)}{\left[ e^{(\theta_1 + \beta_1)(T_{opt}-1)} - \frac{\alpha_1}{(\theta_2 + \beta_1)} \right]} - I \]  \( (6) \)

IV. SCENARIO II: INVENTORY MODEL WITH SECONDARY BIN

The following additional assumptions are made in developing the discounted bin inventory model.

i) The demand is price-dependent

\[ D_2(t) = (\alpha_2 - \beta_2 S) e^{-\gamma t} \]

where \( D_2(t) \) is demand for the discount inventory, \( \alpha_2 \) is the peak demand (\( \alpha_2 > 0 \)), \( \beta_2 \) is the price elasticity of demand for the item (\( \beta_2 > 0 \)), \( \gamma \) is exponential decay in demand over time (\( \gamma > 0 \)), \( S \) is the per unit sales price.

ii) Replenishment rate is limited by deterioration rate and total inventory.

iii) Lead time is zero as it only involves changing of shelves.

iv) Backordering is not permitted as observed in a typical Indian supermarket.
v) Deterioration rate $\theta_2$ is deterministic and constant. $(0 < \theta_2 < 1)$. As continuously deteriorating goods with its RSL have to be transferred from the fresh bin to the discount bin, this intermediate movement can be represented by inclusion of an increase in deterioration factor or shift factor ($\delta$) such that the resulting deterioration factor, $\theta_2 = \theta_1 + \delta$; $\delta$ can be referred to as accelerated deterioration rate. For primary bin (Scenario-I), $\delta$ is taken as zero.

vi) Selling price per unit $S_1$, Holding cost per unit $H_2$ are known and constant. The differential equation representing the inventory level at time $t$ can be written as:

$$\frac{dI_2(t)}{dt} + \theta_2I_2(t) = -D_2(t)$$

Substituting the demand function, it yields

$$\frac{dI_2(t)}{dt} + \theta_2I_2(t) = -[(\alpha_2 - \beta_2 S)e^{-\gamma T}]$$

On solving the above differential equation we get,

$$I_2(t) = \frac{[\alpha_2 - \beta_2 S]}{(\gamma + \theta_2)} \left[ e^{-\theta_2 t} - e^{-\theta_1 t} \right]$$

(7)

Sales of fresh inventory, $S_2$

$$S_2 = S \int_{0}^{T} \left( \frac{\alpha_2 - \beta_2 S}{\gamma} \right) dt = \frac{S(\alpha_2 - \beta_2 S)[1 - e^{-\gamma T}]}{\gamma}$$

(8)

Cost of inventory, $C_1 = 0$

Holding cost of inventory, $CH_2$

$$CH_2 = \int_{0}^{T} I_2(t) dt = \frac{H_2(\alpha_2 - \beta_2 S)}{\theta_2} \left[ \frac{1 - e^{-\theta_2 T}}{\theta_2} - T \right]$$

(9)

Ordering cost, $O_2 = 0$

Profit from fresh inventory,

$$P_2(t) = S - O_2 - Cost - Cost of inventory - Holding$$

$$P_2(t) = \frac{1}{T} \left[ \frac{S(\alpha_2 - \beta_2 S)[1 - e^{-\gamma T}]}{\gamma} - \frac{H_2(\alpha_2 - \beta_2 S)}{\theta_2} \left[ \frac{1 - e^{-\theta_2 T}}{\theta_2} - T \right] \right]$$

(10)

Differentiating with respect to $S$,

$$\frac{dP_2}{dS} = \frac{(\alpha_2 - \beta_2 S)[1 - e^{-\gamma T}]}{\gamma T} + \frac{\beta_2 S(1 - e^{-\gamma T})}{\gamma T}$$

$$+ \frac{H_2(\alpha_2 - \beta_2 S)}{\theta_2} \left[ \frac{e^{-\theta_2 T}}{\gamma} - \frac{e^{-\theta_2 T}}{\theta_2} + \frac{1}{\theta_2} \right]$$

For Maximum Sales, Selling Price

$$S_{max} = \frac{\alpha_2}{2\beta_2} - \frac{H_2\gamma T}{2\theta_2(1-e^{-\gamma T})} \left[ \frac{e^{-\theta_2 T}}{\gamma} - \frac{e^{-\theta_2 T}}{\theta_2} + \frac{1}{\theta_2} - \frac{1}{\gamma} \right]$$

(11)

In the following, we will illustrate the proposed two-bin strategy through numerical illustrations.

V. RESULTS AND DISCUSSIONS THROUGH NUMERICAL EXAMPLE

A. Figures and Tables

Data considered to illustrate the proposed two bin inventory model are as follows:

- Primary Bin Selling Price: Rs. 10/unit
- Cost Price: Rs. 8/unit
- Ordering Cost: Rs. 1000/order
- Holding Cost: Rs. 0.05/unit-day
- Average Demand: 200 unit / day
- Minimum Inventory level: Demand for one time period units
- Optimum cycle time ($T_{opt}$), order quantity ($Q_{opt}$) and profit ($P_1$) are determined using (5) and (6).

Table I shows the effect of $\theta_1$ and $\beta_1$ on profit, time period and order quantity with a constraint that the resulting sales are profitable (i.e. $P_1 > 0$). The effect of $\theta_1$ is pronounced on profit, time period and order quantity when compared with that of $\beta_1$.

In order to demonstrate the effectiveness of the secondary bin inventory model, let us assume the following values of the additional parameters.

- Increase in deterioration factor: 0.01
- Demand for discount inventory: Inventory shifted from fresh to discount inventory
- Deterioration of demand for discount inventory: 0.05
- Deterioration of discount inventory: 0.02
- Total Profit ($P_2$) and Sales Price ($S_{opt}$) are determined using expressions (10) and (11) respectively. To analyze the effect of stock dependent selling rate parameter $\alpha_1$ and rate of deterioration $\theta_1$ the results are tabulated in Table I.

For Maximum Sales, Selling Price

$$S_{max} = \frac{\alpha_2}{2\beta_2} - \frac{H_2\gamma T}{2\theta_2(1-e^{-\gamma T})} \left[ \frac{e^{-\theta_2 T}}{\gamma} - \frac{e^{-\theta_2 T}}{\theta_2} + \frac{1}{\theta_2} - \frac{1}{\gamma} \right]$$
TABLE I
RESULTS OF PRIMARY BIN INVENTORY MODEL

<table>
<thead>
<tr>
<th>θ</th>
<th>P</th>
<th>T</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.035</td>
<td>34,140</td>
<td>3,058</td>
<td>37,549</td>
</tr>
<tr>
<td>0.04</td>
<td>23,497</td>
<td>3,253</td>
<td>35,062</td>
</tr>
<tr>
<td>0.045</td>
<td>20,005</td>
<td>3,095</td>
<td>32,633</td>
</tr>
<tr>
<td>0.05</td>
<td>16,584</td>
<td>2,895</td>
<td>28,633</td>
</tr>
<tr>
<td>0.055</td>
<td>12,618</td>
<td>1,959</td>
<td>25,666</td>
</tr>
<tr>
<td>0.06</td>
<td>8,710</td>
<td>1,105</td>
<td>1,651</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>θ</th>
<th>P</th>
<th>T</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.035</td>
<td>467,036,88</td>
<td>43759,01</td>
<td>42,940,59</td>
</tr>
<tr>
<td>0.04</td>
<td>46641,43</td>
<td>43666,18</td>
<td>42811,31</td>
</tr>
<tr>
<td>0.045</td>
<td>45526</td>
<td>4,3974</td>
<td>7,2904</td>
</tr>
<tr>
<td>0.05</td>
<td>44561,72</td>
<td>43589,72</td>
<td>42589,93</td>
</tr>
<tr>
<td>0.055</td>
<td>4448,62</td>
<td>43439,69</td>
<td>42494,61</td>
</tr>
<tr>
<td>0.06</td>
<td>43702</td>
<td>3,0270</td>
<td>28,562</td>
</tr>
</tbody>
</table>

* P & T in Rs & T in days and Q in units. (Only profitable sales are considered)

TABLE II
RESULTS OF SECONDARY BIN INVENTORY MODEL

<table>
<thead>
<tr>
<th>β</th>
<th>θ</th>
<th>P</th>
<th>T</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.035</td>
<td>671</td>
<td>1,105</td>
<td>1,651</td>
<td>2,246</td>
</tr>
<tr>
<td>0.04</td>
<td>608</td>
<td>1,012</td>
<td>1,522</td>
<td>2,138</td>
</tr>
<tr>
<td>0.045</td>
<td>553</td>
<td>929</td>
<td>1,405</td>
<td>1,983</td>
</tr>
<tr>
<td>0.05</td>
<td>513,4</td>
<td>6,1573</td>
<td>7,0089</td>
<td>7,8684</td>
</tr>
<tr>
<td>0.055</td>
<td>4553,72</td>
<td>43507,77</td>
<td>42589,93</td>
<td>42,178,49</td>
</tr>
<tr>
<td>0.06</td>
<td>45702</td>
<td>3,0270</td>
<td>28,562</td>
<td>27,403</td>
</tr>
</tbody>
</table>

* P, P, and S in Rs. (Only values that result in profitable sales are considered)

B. Findings from the Study

The effect of the crucial parameters on the profit function is discussed below:

1) Deterioration Rate (θ1)

The deterioration rate (θ1) has the biggest impact among all the factors on the profit function of the retailer. The impact of the factor is evident from the fact that a 5% change in the experimental data from 0.2 to 0.21 resulted in doubling of the profit from Rs. 1932 to Rs. 3882. The deterioration rate also directly impacts the review period (T) such that an increase in deterioration rate reduces the review period. So, one can infer that if the deterioration rate of an item can be reduced or controlled through methods like cool storage, then it could have a high positive impact on the overall profitability of the sales.

2) Optimum Order Quantity (Q)

The optimum order quantity is a function of a lot of variables, but its sensitivity is highest for the review period (T). Thus, the sensitivity of optimum order quantity would depend on the inventory based sales factor and relatively lower for deterioration factor.

3) Price Elasticity for Discount Inventory (β2)

Higher value of the price elasticity factor indicates that the sales price will be lower to consume the entire inventory. So, higher value indicates lower profits. The longer the shelf life and review period, the higher the resulting profit.

4) Optimum Sales Price (Sopt)

The sales price of the discount bin is dependent on the deterioration rate and the elasticity of the demand. If the demand is increased through a small decrease in the sales price, the optimum value will be higher.

VI. CONCLUSION

Perishable items have limited shelf life and high cost of expiry when stored for consumption in the Retail stores. A trial and error based ordering policy based on intuition is followed by most retailers. A possibility to deal with the perishable products is to have optimistic stock levels and
move the inventory with short RSL to another bin. Such practices are frequent for items with shelf life of a few weeks. Promotional discounts or bundling with other related items is a common practice to empty stocks with short RSL. This paper attempted to model the real world scenario into a mathematical model and studies the impact of some important factors on the profit function under a two-bin framework with inventory dependent demand rate and non-zero cycle ending inventory for primary bin; wherein selling price dependent demand rate for secondary bin. This study uses deterministic demand to model the profitable sales of the perishable item. It would be a big leap to include random fluctuation in the demand and model for stochastic demand.

REFERENCES