The Role of Periodic Vortex Shedding in Heat Transfer Enhancement for Transient Pulsatile Flow Inside Wavy Channels

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Abstract—Periodic vortex shedding in pulsating flow inside wavy channel and the effect it has on heat transfer are studied using the finite volume method. A sinusoidally-varying component is superimposed on a uniform flow inside a sinusoidal wavy channel and the effects on the Nusselt number is analyzed. It was found that a unique optimum value of the pulsation frequency, represented by the Strouhal number, exists for Reynolds numbers ranging from 125 to 1000. Results suggest that the gain in heat transfer is related to the process of vortex formation, movement about the troughs of the wavy channel, and subsequent ejection/destruction through the converging section. Heat transfer is the highest when the frequencies of the pulsation and vortex formation approach being in-phase. Analysis of Strouhal number effect on Nu over a period of pulsation substantiates the proposed physical mechanism for enhancement. The effect of changing the amplitude of pulsation is also presented over a period of pulsation, showing a monotonic increase in heat transfer with increasing amplitude. The 60% increase in Nusselt number suggests that sinusoidal fluid pulsation can an effective method for enhancing heat transfer in laminar, wavy-channel flows.

Keywords—Vortex shedding, pulsating flow, wavy channel, CFD.

I. INTRODUCTION

Enhancing heat transfer characteristics of surfaces is relevant to a wide range of engineering applications. But while the majority of engineering devices operate in the turbulent regime, many devices do operate in the laminar regime such as some polymeric extruders, compact heat exchangers, and food processing devices. As a result, the study of creative ways to accomplish the heat transfer enhancement has been the subject of much research for the past few decades. Goldstein and Sparrow [1] were among the first to experimentally measure local and average heat transfer coefficients in a corrugated channel, in laminar, transitional and turbulent flow regimes. Their results showed secondary flows inside the triangular waves which resulted in as much as a 300% increase in the averaged heat transfer in turbulent regimes. Wang and Vanka [2] numerically studied the heat transfer for periodic arrays of wavy channels. They observed self-sustained oscillatory flow for Reynolds numbers higher than about 180. These self-sustained oscillations lead to the destabilization of the laminar boundary layers which enhanced the mixing between the core and the near-wall fluid. However, friction factors for wavy channels were noticed to be nearly double those of flat-plate channels in the laminar regime.

Zhang et al. [3] numerically studied laminar forced convection in two-dimensional wavy-plate-fin channels with sinusoidal wall corrugations. They observed that re-circulation cells develop in the wavy-wall troughs as the axial flow separates and reattaches. Considerable increases in heat transfer were accomplished for certain sets of Reynolds number, wall-corrugation severity, and fin spacing. Sawyers et al. [4] studied the heat transfer enhancement in three-dimensional corrugated channel laminar flow both analytically and numerically. Sinusoidal corrugations in two orthogonal directions lead to a higher heat transfer than a flat plate as well as corrugation in one direction because of the presence of multiple recirculation zones and accompanying stagnation points. Alteration of the flow structure from that of flat plates was not limited to sinusoidally varying channels either; Bahaidarah et al. [5] studied heat and momentum transfer in both sinusoidal and arc-shaped channels. They predicted good heat transfer enhancement for some cases. Not limited by the shape of the channel, Herman and Kang [6] used other strategies to enhance heat transfer. The small cylinders and guiding vanes placed above inter-block grooves proved effective in increasing heat transfer from the heat sources.

In nearly all studies of flow in corrugated channels, heat transfer enhancements relied mostly on complicated geometric modifications, careful optimization of corrugation severity and Reynolds numbers, transitional components of the flow, and/or the inclusion of a new phenomena such as natural or mass convection. In most of these techniques, the desired thermal mixing was often accompanied by large pressure drops and increased skin friction. Otherwise, improvements in heat transfer were largely marginal. The need still stands for a technique that delivers appreciable increases in laminar flow heat transfer, with moderate pressure drop and skin friction increases, across a wider range of flow conditions with minimal interference and complication. This paper addresses the effect of superimposing a sinusoidal component on the flow through wavy channels on heat transfer and skin-friction.

One of the first attempts to analyze the gain in heat transfer using pulsating fluid motion was that of Perkins et al. [7] who superimposed a sinusoidal pulse during half of the cycle on an otherwise steady flow over a series of rectangular cavities. Kim et al. [8], [9] numerically modeled heat dissipation from constant temperature blocks, of equal and different heights, to a pulsatile channel flow. Cases were run for a series of Reynolds numbers, Strouhal numbers, pulsation amplitudes and inter-block spacing. The unsteady effect of the pulsation substantially destabilized the eddy between the blocks; the temperature field was accordingly altered. The effect of
pulsation on heat transfer in triangular grooved channel was experimentally investigated by Jin et al. [10]. PIV results showed that pulsation caused strong mixing by the repeating sequence of vortex generation, growth, expansion and ejection from the grooves to the main flow.

The problem might be stated as follows: for flow in the laminar regime, enhance the heat transfer from the walls of a channel, more than marginally, without relying on roughness elements, transition to turbulence, complicated geometry, mass or natural convection, while keeping the rise in friction to acceptable levels. Here, we propose the use of a sinusoidally-pulsating flow component, superimposed on the mean channel flow, to aid in the hydrodynamic modification of the flow structure discussed above.

II. PROBLEM FORMULATION

A. Geometry and flow variables

The geometry and properties of the problem are nondimensionalized using the average height of the channel, \( H \), and the fluid density, \( \rho \), specific heat, \( c_p \), dynamic viscosity, \( \mu \), and thermal conductivity, \( k \). Hydrodynamic variables are nondimensionalized using the \( y \)- and \( t \)-averaged inlet velocity, \( U \), while temperature is nondimensionalized using the surface and fluid inlet temperatures, \( T_s \) and \( T_{in} \), respectively. In non-dimensional terms, the height of the channel modeled changes sinusoidally with an average, minimum and maximum of 1, 0.75 and 1.25, respectively. In other words, the sine wave describing the wall has a dimensionless amplitude of \( A = 0.125 \), so the dimensionless position, \( y \), of the lower surface is described by the function

\[
y = a \sin(\pi x'/H)
\]

where \( x' \) is the channel \( x \)-coordinate starting from the wavy section. The 10\( H \)-long, 5-period wavy section is preceded by \( 3H \) and followed by \( 12H \) straight sections. Taking advantage of the symmetry of the problem, only the lower half of the channel is numerically modeled. A schematic of the computational domain showing the wave part of the meshed channel is shown in figure 1 with close-ups of one complete wave period and part of the refined boundary shown in the inserts. The Prandtl number of the fluid was taken to be 0.7.

B. Governing equations and boundary conditions

The laminar flow considered is that of a constant-properties, periodically fully-developed, incompressible, Newtonian fluid in a two-dimensional sinusoidally-wavy channel. The flow is governed by the Navier-Stokes and energy equations. In dimensionless forms, the Navier-Stokes equations are,

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial p}{\partial x} = \frac{1}{Re} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} = \frac{1}{Re} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\]

and the energy equation is,

\[
\frac{\partial T}{\partial t} + \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \frac{1}{Re Pr} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

where the quantities discussed in §II-A were used to nondimensionalize the variables above according to,

\[
x = \frac{x}{H}, \quad y = \frac{y}{H}, \quad u = \frac{u}{U}, \quad v = \frac{v}{U}, \quad p = \frac{p}{\rho U^2}, \quad t = \frac{t}{H}, \quad T = \frac{T - T_s}{T_{in} - T_s}
\]

where primes signify dimensionless quantities. Also, the Reynolds, Prandtl, Nusselt and Strouhal numbers are defined as,

\[
Re = \frac{\rho U H}{\mu}, \quad Pr = \frac{c_p}{\rho U H}, \quad Nu = \frac{\rho U H}{k}, \quad St = \frac{\nu U}{H}
\]

where \( \tau_w \) is the wall shear stress in N/m². The Strouhal number controls the periodic pulsation, and is equivalent to the frequency \( \nu \) in this study. The fully-developed (parabolic) inlet velocity profile was varied sinusoidally according to

\[
u(y, t) = 6\nu(1 - y) \left[ U + A\sin(2\pi \nu t) \right]
\]

where \( A \) and \( \nu \) are the amplitude and frequency of the velocity pulsation, respectively. Pressure outlet boundary conditions was used at the exit boundary where a static (gauge) pressure of zero was specified. At the wavy plate surface the no-slip boundary condition was used for velocities. Thermally constant temperature was used at the surface. At the symmetry line, zero-gradient and adiabatic boundary conditions were used for the velocities and temperature, respectively. Mathematically,

Inlet \((x = -3, 0 < y < 0.5)\):

\[
u = 6\nu(1 - y) \left[ U + A\sin(2\pi \nu t) \right]
\]

Exit \((x = 22, 0 < y < 0.5)\):

\[
p_{\text{gauge}} = 0
\]

Surface \((-3 < x < 22, y = a \sin(\pi x'/H))\):

\[
u = v = 0
\]

Symmetry \((-3 < x < 22, y = 0.5)\):

\[
\frac{\partial u}{\partial y} = v = \frac{\partial T}{\partial y} = 0
\]

The whole lower surface, including the upstream and downstream flat sections, are held at a constant dimensionless temperature of 1. Holding the \( 3H \) straight section preceding the wavy section at the same temperature as the wavy section is just sufficient to deliver a thermally fully developed flow to the beginning of the wavy section. Meanwhile, holding the rear straight section at that temperature eliminates any unneeded thermal effect at the last wavy period. The inlet fluid has a dimensionless temperature of 0.
C. Numerical algorithm

The segregated, transient solver of the Fluent\textsuperscript{TM} finite volume-based software was used to run all cases. The SIMPLE [12] scheme was used to provide the pressure-velocity coupling. Under relaxation values of 0.3 and 0.7 were used for pressure (continuity) and momentum, respectively. A convergence criterion of $10^{-8}$ was set for all equations residuals. A limit of 180 iterations per time step was imposed on the time stepper which was usually enough to bring residuals down to this threshold. The small number of times the solution did not fully converge to $10^{-8}$ (usually between 0.55 and 0.7 of the period of pulsation) it still came close to it, down to below $5 \times 10^{-8}$. Also, 100 time steps were used to simulate each pulsation period to ensure smooth variations of computed quantities and preclude any possible instabilities.

D. Grid independence and validation

A series of test cases were run to ensure mesh-independent solutions. The mesh was gradually refined, with about 10% increase at a time, until the averaged Nusselt number changed by less than 0.1%. The adapt feature in Fluent\textsuperscript{TM} was also utilized to refine the wavy surface in a way that ensured acceptable gradients for velocity and temperature on all boundaries. The final result was comprised of 600 cells for the upstream straight section, 1200 cells for the downstream straight section, and 6900 cells for the middle wavy section, graded and refined near the lower boundary. The mesh grading effect and the result of the boundary mesh adaption are shown in figure 1.

Similar cases involving pulsatile flow over wavy surfaces are not available in the literature. The algorithm is validated against a case of steady flow in a wavy channel. Specifically, the results of Wang and Chen [13] are used. That study was for a channel of similar sinusoidally-wavy channel that has six periods, an amplitude-wave length ratio of 0.2, and a fluid Prandtl number of 6.93. A few modifications were made to the current geometry to ensure compatibility. Specifically, one more period was added, the geometry was scaled by $1/2$ in the $y$-direction to account for the different nondimensionalization factor used, and the Prandtl number was changed.

A comparison of the local Nusselt number for the wavy section is shown in figure 2. As can be seen, the agreement is very good. One final note about the small observed difference is that the current study uses a total of 8700 graded, adapted and refined cells while that of Wang and Chen used a total of 3600 graded cells.

III. RESULTS AND DISCUSSION

A number of runs were performed on series of pulsation amplitudes, Strouhal numbers and Reynolds numbers. Results, represented by temporal behavior of $N_u$ over a complete channel (fourth) wave period, are obtained for amplitudes $A = 0.1, 0.2, 0.4, 0.6$ and $0.8$. Results are also obtained for Strouhal numbers (which are equivalent to frequency, $r$) of $0.1, 0.4, 0.45, 0.5$ and $0.7$, for a complete wave period. In addition to varying the amplitude and frequency of pulsation, the effect of Reynolds number on the wavy-surface-integrated $N_u$ is also obtained for Reynolds numbers 125, 250, 500 and 1000.

A. Periodic vortex shedding

To observe how the flow is behaving hydrodynamically with the inlet velocity pulsation a series of snapshots of the flow and
Fig. 3. The evolution-destruction cycle of the main vortex in a channel wave over one complete period of pulsation. Bottom: velocity streamlines. Top: temperature contours. Results are for $Re = 500$, $A = 0.2$ and $St = 0.45$. 
temperature fields are shown in figures 3. These figures are for the case of $Re = 500$, $A = 0.2$ and $St = 0.45$. The 10 double-naps which span one complete period of pulsation show the generation (d), expansion (e)-(a), and ejection (b)-(c) of a vortex into the main flow. It is immediately noticed that the vortex takes a little more time to form and grow than it takes to be ejected. Because of the momentary high static pressure in the trough of the wave the fluid separates and reattaches, creating a dynamic vortex. The presence of the vortex incurs two stagnation points which are also continuously moved along the surface. Stagnation points effect better mixing between fluid layers since they transport fluid from near the surface to the main flow and vice versa, i.e., enhanced advection.

On the other hand, the parallel flows upstream and downstream of stagnation points steepen the thermal gradients, resulting in enhanced diffusion from the surface. These two effects contribute to the heat transfer enhancement. In addition, the continuous movement of these vortices around the channel prevents the mixing and thin boundary layers (and thus enhancement) from being confined to one place. The momentary absence of a clear separation between the main flow and a well-defined recirculation zone in the valley of the wave is also noticed. The last point contributes directly to heat transfer enhancements for a wide range of Reynolds number and geometries by involving all thermal and velocity zones touched by the vortex as well as the accelerated flow outside of it.

B. Amplitude effect on periodic $Nu$

The integrated normalized Nusselt number over the wavy surface over a complete period of pulsation for a number of different pulsation amplitudes is shown in figure 4. The figure is for cases where $Re=500$ and $St = 0.45$. It can be seen that the highest Nusselt numbers are for the first quarter and the last tenth or so of each period. Comparing these intervals with what happens in figure 3 in terms of vortex activity shows that this corresponds to the vortex being in the converging part of the wave. When this is the case the rear part of the vortex and its stagnation point are in the middle of the wave trough. Hot fluid from that level is therefore advected to the thick part of the channel, which enhances mixing greatly. This continues until the vortex is ejected and the flow is clear of large recirculation zones—from approximately $\pi/3$ until about $5\pi/6$, cases (c)-(i) in figure 3.

The contrary mechanism, when the vortex is upstream of the lowest point in the trough, is not that effective. The reason might be that the mixing resulting from bringing fluid from the core of the wave, by the leading vortex edge and stagnation point, is not as effective. One obvious reason is the higher static pressure confronting the downward flow as it approaches the surface as opposed to the lower static pressure welcoming the upward flow from the trailing stagnation point. Finally, we note that the Nusselt number increases throughout the period with increasing amplitude, but the rate of increase is negative. This might be a factor when deciding when to stop raising the amplitude, namely when the gain in heat transfer ceases to be worth the additional pumping power required.

C. Strouhal number effect on periodic $Nu$

The effect of changing the pulsation frequency (the Strouhal number) on the local, integrated Nusselt number is seen in figure 5. These results are for $Re = 500$ and $A = 0.2$. It can be seen from the figure that the Nusselt number for low values of Strouhal number, e.g., $St=0.1$, is maximum halfway through the pulsation cycle. A single vortex exists in such cases, wiggled around by the pulsation effect. Naturally, around the middle of the pulsation cycle, when the velocity is maximum, the maximum shifting of the vortex takes place. More shifting of the stagnation points brings up more mainstream fluid closer to the bottom of the trough, resulting in relatively better mixing.

Three curves for Strouhal numbers at and around the optimum (see $\text{subsec:Reynolds number effect below}$) are also shown, $St=0.4$, 0.45 and 0.5. For these frequencies, the eddies do form and get destroyed. The spanning by eddies of all the wavy valley brings the stagnation points near the bottom of the trough at least twice. More importantly, the high fluid acceleration coincides with the acceleration and ejection of the vortex through the converging sections of the wavy
D. Reynolds number effect

To see the effect of pulsation frequency (Strouhal number) on the total heat transfer enhancement, the Nusselt number is integrated over the whole wavy surface and over one complete period of pulsation, and then normalized by the steady state Nusselt number. This was done for four values of Reynolds numbers: 125, 250, 500 and 1000. The result is shown in figure 6. A similar plot, figure 7, shows the effect of Strouhal number on the total skin friction coefficient. Nine values for the Strouhal number were used to approximate the variation: 0.1, 0.3, 0.4, 0.45, 0.5, 0.55, 0.6, 0.7 and 1.

![Fig. 6. Normalized Nusselt number for the whole wavy surface vs. St for different Re values.](image)

At very small Strouhal numbers the flow is close to steady state conditions, therefore the normalized increases in $Nu$ fall to 1 as shown in the figures. The same can be said for the coefficient of friction. At large Strouhal numbers the fast periodic change of the inlet flow does not allow ample time for eddies to form, move along the surface and be ejected. So the Nusselt number increases tend to 1 again. The fast pulsation, however, does not change the fact that the fluid is actually experiencing that fast motion back and forth, with all the friction between the fluid layers it entails. Thus we see that the friction coefficient make a small dip then increase again.

That heat transfer enhancement increases with Reynolds number was somewhat expected because of the larger thermal gradients induced by the higher velocities. However, an optimum value for Strouhal number around $St = 0.45$ can also be seen, almost across all tested Reynolds numbers tested which spanned a whole order of magnitude, dropping only slightly with increasing Re. This suggests that the optimum pulsation frequency is not a strong function of Reynolds number in this range. The decrease in Nusselt number with Strouhal numbers greater than 0.45, while monotonic, experiences a region where it tries to plateau, around $St = 0.6$, before continuing to decrease further. The phenomenon is clearer in the friction coefficient plot and $Re = 1000$, figure 7, where a clear dip in the friction coefficient is observed. Regarding the Strouhal number of the maximal friction coefficients, they correspond almost exactly to the Strouhal number of the maximum Nusselt number, which suggest that this sort of trade-off will always accompany the gain in heat transfer, at least in the $Re$ range tested.

IV. Conclusions

Pulsatile flow inside a wavy channel is investigated via the finite volume method. The objective was to determine the potential heat transfer enhancement for such flow situations. A sinusoidal component was superimposed on a mean fluid entered the channel at a lower temperature than a sinusoidal-wavy surface. The effect of changing the Strouhal number of the flow was shown for different values of Reynolds number. The space-averaged Nusselt number was also shown through a complete period of pulsation for different values of Strouhal numbers and fluid pulsation amplitude.

It was found that, as expected, higher Reynolds numbers meant better heat removal efficiency. More importantly, results show that heat transfer enhancement shows a clear peak at an optimum value of the Strouhal number, and that this value (about $St=0.45$) is nearly the same across a wide range of laminar Reynolds numbers. The enhanced heat transfer was attributed to better mixing of the fluid layers due to the presence of vortices that move around the wavy channel by virtue of pulsation. Moreover, it was proposed that the effectiveness of the vortex formation-movement-ejection sequence increased as its frequency approached that of the pulsating fluid. Averaged Nusselt number, normalized by their corresponding steady state values, shed more light on the changes in $Nu$ when the amplitude and frequency of pulsation change. Snapshots of the velocity and temperature fields visually correlate these key
points in vortex evolution to the harmonics described above for two sample cases: the optimum and another one for which $St$ was too high.

The incorporation of a sinusoidally-varying component into a steady flow in a wavy channel proves to be a good, robust way of enhancing heat transfer. Depending on the Strouhal number, the gain in heat transfer reached up to 60% for higher values of Reynolds number and an amplitude of pulsation of 0.2. Much more enhancement can be attained for larger pulsation amplitudes, as was shown in figure 4, while still staying within the laminar regime. Such gains in heat transfer were limited in the past to flows having transitional components, complicated geometries, or very high friction and pressure drops due to roughness elements. One must be alert, however, to the accompanying friction coefficient increase and account for the cost of this heat transfer enhancement in therm of added pumping power. All in all, the method was shown to be fairly effective and very robust, even though further optimization can be performed once available resources are known to be constrained.

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