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Arriving at an Optimum Value of Tolerance Factor for Compressing Medical Images

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Abstract**—**Medical imaging uses the advantage of digital technology in imaging and teleradiology. In teleradiology systems large amount of data is acquired, stored and transmitted. A major technology that may help to solve the problems associated with the massive data storage and data transfer capacity is data compression and decompression. There are many methods of image compression available. They are classified as lossless and lossy compression methods. In lossy compression method the decompressed image contains some distortion. Fractal image compression (FIC) is a lossy compression method. In fractal image compression an image is coded as a set of contractive transformations in a complete metric space. The set of contractive transformations is guaranteed to produce an approximation to the original image. In this paper FIC is achieved by PIFS using quadtree partitioning. PIFS is applied on different images like , Ultrasound, CT Scan, Angiogram, X-ray, Mammograms. In each modality approximately twenty images are considered and the average values of compression ratio and PSNR values are arrived. In this method of fractal encoding, the parameter, tolerance factor $T_{max,}$ is varied from 1 to 10, keeping the other standard parameters constant. For all modalities of images the compression ratio and Peak Signal to Noise Ratio (PSNR) are computed and studied. The quality of the decompressed image is arrived by PSNR values. From the results it is observed that the compression ratio increases with the tolerance factor and mammogram has the highest compression ratio. The quality of the image is not degraded upto an optimum value of tolerance factor, $T_{max,}$ equal to 8, because of the properties of fractal compression.

Keywords**—**Fractal image compression, IFS, PIFS, PSNR, Quadtree partitioning.

I. INTRODUCTION

FRACTAL compression and fractal encoding uses the property of self-similarity of fractal objects. Exact self-similarity means that the fractal object is composed of scaled down copies of itself that are translated, stretched and rotated according to a transformation. Such a transformation is called affine transformation. A fully automatic Fractal based image compression technique of digital monochrome image was first proposed by Jacquin [1]. The encoding process of an image is given by approximating the smaller range blocks, from the larger domain blocks through some transformations. The domain block approximates the range blocks until an affine contractive map is found. The measure of approximation is given by Root Mean Square (RMS) error. The number of transformations required to approximate the given image depends upon the image complexity.

Fisher [5] has suggested quadtree partitioning or Horizontal-Vertical (HV) partitioning to encode the given image to achieve more compression by keeping the number of range blocks fixed upto a prefixed level. In this paper the fractal image compression is obtained by quadtree partitioning method.

The fractal method offers high compression ratio, good image quality of the decoded image. In FIC encoding process takes more time than decoding since it is difficult to find a suitable approximation of an image by a set of affine contractive maps.

In this paper the fractal image compression has been applied on different imaging modalities like Ultrasound, CT Scan, Angiogram, X-Ray, Mammogram. The compression ratio and PSNR, are obtained for the images by varying the tolerance factor of quadtree partitioning and they are studied.

II. FRACTAL IMAGE COMPRESSION

Fractal encoding is a mathematical process used to encode any given image as a set of mathematical data that describes the fractal properties of the image. Fractal encoding relies on the fact that all objects contain information in the form of similar, repeating patterns called an attractor.

Fractal encoding is largely used to convert the image into fractal codes. In the decoding it is just the reverse, in which a set of fractal codes are converted to image. The encoding process has intense computation, since large number of iterations is required to find the fractal patterns in an image. The decoding process is much simpler as it interprets the fractal codes into the image. Fractal image compression (FIC) is achieved either by using Iterated Function Systems (IFS) or by Partitioned Iterated Function Systems (PIFS).

A. Iterated Function Systems

This is a random iteration algorithm, introduced by Michael Barnsley[5], is based on collage theorem. An iterated function system consists of a collection of contractive transformation $\{wi: \Re^2 \rightarrow \Re^2 \mid i=1 \text{ to } n\}$, which map the plane $\Re^2$ to itself. The collection of transformation defines a map.

$$W(.)= \bigcup_{i=1}^{n} w_i(.) \tag{1}$$

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Given an input set S, we can compute w_i(s) for each *i*, considering the union of these sets ,a new set **W(s)** is obtained.
(i) when the w_i are contractive in the plane, the **W** is contractive in a space of subsets of the plane.
(ii) if a contractive map **W** is given, on a space of images, then there is a special image called the attractor.

B. Partitioned Iterated Function Systems

Normally, the natural images are not self similar. They do not contain affine transformation of it. By partitioning the images into pieces, partitioned self-similarity can be achieved.

In PIFS, a fixed point, called an attractor, is an image *f* that satisfies W(*f*)=*f*; that is, when we apply the transformations to the image, the original image can be obtained. The contractive mapping theorem says that the fixed point of *W* will be the image we get, when we compute the sequence $W(f_0), W(W(f_0)), W(W(W(f_0)))\ldots$, where f_0 is any image. The contractive mapping theorem can be applied to W^{om} , so it is sufficient for W^{om} to be contractive. [5].The transformation W will be contractive, if the distance between any two points P_1,P_2, is

$$d(w(P_1),w(P_2))<s.d(P_1, P_2), \qquad (2)$$

for s<1,where s is the scaling factor and d is the distance.

In fractal image compression a partitioning of the image into range blocks **R** ,is done. The encoding of each range blocks consists of finding the best affine transformation by searching a pool of domain(**D**) blocks. For each range block **R**=(r_{ij}) , we search the domain pool to find the domain block $\hat{D} = \hat{d}_{ij}$ and the transformation τ , such that $\tau(\hat{D})$ provides best matching , that has low root mean square (rms) error when mapped to **R**. The domain blocks are shrunken either by sub-sampling or by pixel averaging to match the range block size.The decoding is obtained by iterating **W** from any initial image. The iterations are repeated until we get close to a fixed point.

III. QUADTREE PARTITIONING

The most popular partitioning mechanism is obtained by partitioning the image in a tree structure. A quad-tree partitioning is a representation of an image as a tree in which each node corresponding to a square portion of the image contains four sub-nodes corresponding to the four quadrants of the square, the root of the tree being the initial image as in Fig. 1.

A. Range Selection in Quadtree Partitioning

The squares at the nodes are compared with domains in the domain pool **D**, which are twice the range size. The pixels in the domain are averaged in groups of four so that the domain is reduced to the size of range and the affine transformation of the pixel vales is found that minimizes the root mean square (RMS) difference between the transformed domain pixel values and the range pixel values. With a tolerance factor given for the rate or the quality, this method will break up into squares, thereby creating additional ranges with corresponding transformation codes and improving the reconstructed image quality until the desired rate or quality is obtained. [4][5].

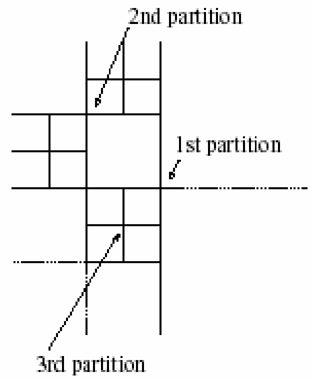


Fig. 1 Quadtree Partitioning

B. Significance of Tolerance Criteria

In this work the tolerance factor T_{max} is assumed to vary from1,2,…10. The tolerance factor is compared with the RMS factor between the transformed domain and the range in the image. The effect of the factor are given in condition 1,2.

Condition 1:
$T_{max} < RMS$ says that the difference between the domain and range is a large value and hence we have to partition the domain further into subnodes.

Condition 2:
$T_{max} > RMS$ says that the difference between the domain and the range is reduced and we have found the appropriate domain from the domain pool **D** for the range and the partitioning of that node is stopped. This domain is considered as w_i which is a part of **W**, the encoded image.

C. Decoding

Decoding process is done by iterating the set of transformations on an arbitrary initial image and the quadtree partition is used to determine the range in the image. For each range block, the size of the domain block that maps to it is shrunk by 2x2 pixel averaging. The pixel values of the shrunken domain block are then placed in the location in the range determined by the orientation information after scaling and offsetting. Computing all the range blocks constitutes one iteration. After several iterations, the decompressed image will be very close to the original image.

IV. IMAGE QUALITY

Signal to Noise Ratio (SNR) estimates of the quality of a reconstructed image compared with the original image [2][3][5] . Considering $\left(f(i,j)\right)$ as original image and $\left(F(i,j)\right)$ as reconstructed image and N as number of pixels in the image, the mean square error is computed using (3)

$$\text{MSE} = \sum \frac{[f(i,j) - F(i,j]^2}{N^2} \quad (3)$$

The quality of the image is obtained by computing the peak signal- to- noise ratio (PSNR), of the reconstructed image is obtained using (4)

$$\text{PSNR} = 20 \log_{10}\left(\frac{N}{RMSE}\right)\text{dB} \quad (4)$$

where N is the largest possible value of the signal in the image and RMSE is the root mean square error. Typical PSNR values range between 20 and 40 for good quality reconstructed image.

V. Compresssion Ratio

Data compression is the process of encoding information using fewer bits than an unencoded representation. The essential figure of merit for data compression is the compression ratio, or ratio of the size of a compressed file to the original uncompressed file.

VI. Methodology

A. Encoding

Mammogram, chest images of Ultrasound, CT Scan, Angiogram, X-Ray, of size 256x256 of 8 bit gray scale is considered for the analysis. The tolerance factor T_{max} is varied from 1,2,..10. The parameters minimum tree depth m, maximum tree depth M, bits used for scaling factor s_i and offset factor o_i of quad tree partitioning are fixed as 4, 6 , 5 and 7 respectively. Image is partitioned into four subnodes and is compared with domains from the domain pool **D**. The pixels in the domain are averaged, in groups of four so that the domain is reduced to range size. The root mean square (rms) value between the transformed domain pixel values and the range pixel values is found out as,

rms= $\sqrt{\min E(R,D)/n,}$ where n is the number of pixels in the range R.

If the RMS $\geq T_{max}$ and depth $\leq$ M, image is partitioned further into four sub nodes and is compared with domains from the domain pool **D**. The pixels in the domain are averaged, in groups of four so that the domain is reduced to range size. If the RMS $\leq T_{max}$, the domain is mapped as w_i. The collection of all such maps is given as **W**=$\bigcup w_i$, where **W** is the encoded image.

B. Decoding

Decoding an image consists of iterating **W** from any initial image. For each range R_i,the domain D_i that maps is shrunk by two averaging non-overlapping groups of 2x2 pixels. The shrunken domain pixel values are then multiplied by s_i added to o_i and placed in the location in the range determined by the orientation information. This iteration is done until the fixed point is approximated by maximum number of iterations.

C. Computation of PSNR

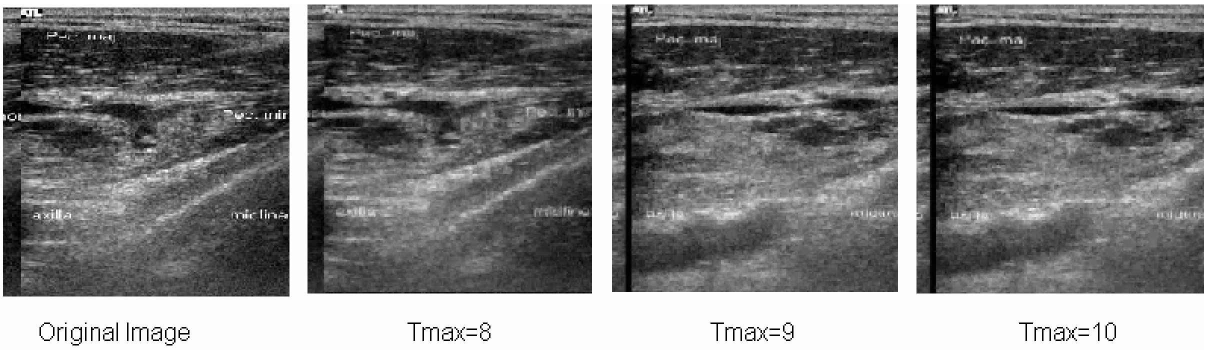
The quality of the reconstructed image is arrived by computing the peak signal- to- noise ratio (PSNR) using (3) and (4).

VII. Results and Discussions

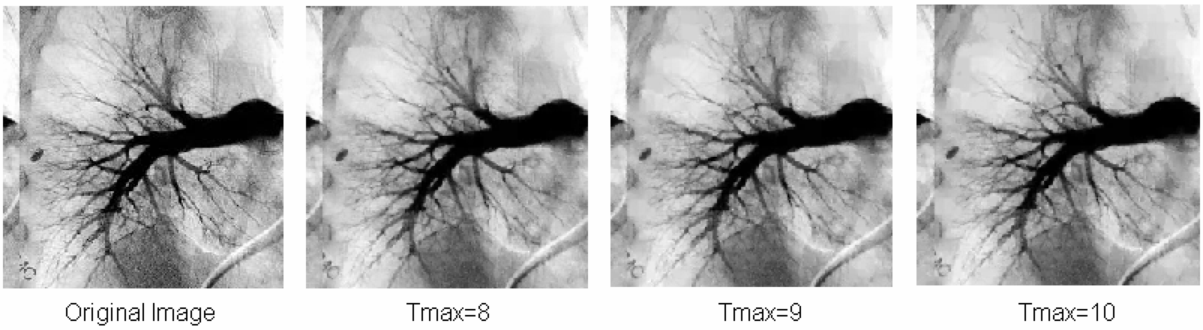
Mammogram images, chest images of Ultrasound, CT Scan, Angiogram, X-Ray, of size 256x256 of 8 bit gray scale is considered for the analysis. The tolerance factor T_{max} is varied from 1,2,..10.

With the tolerance factor T_{max}=1 to T_{max} =10, the improvement in compression ratio for different imaging modalities is observed and tabulated. The compression ratio for ultrasound is observed to be the lowest and that of mammogram is observed to be the highest for different values of $T_{max.}$

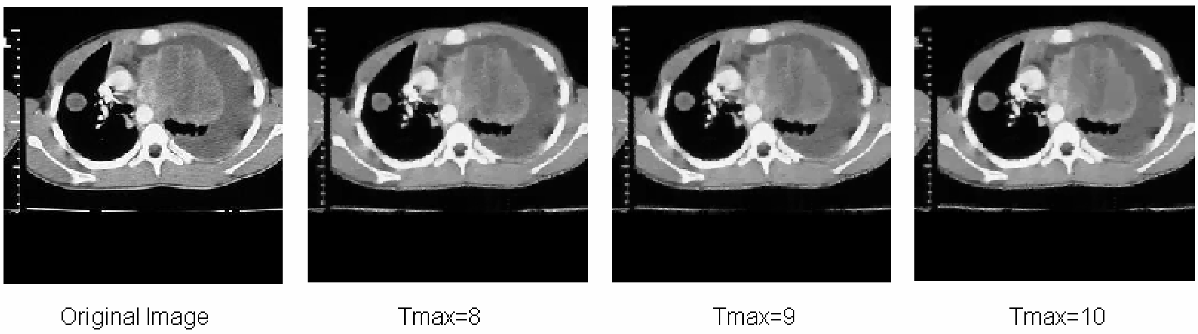
With the value of tolerance factor $T_{max,=}$ 8, the compression ratio for Ultrasound, CT Scan, Angiogram, X-ray, Mammogram is computed as 8.66, 10.92, 17.82, 26.68, 35.92 respectively. The quality of the decompressed image is studied from the PSNR values.



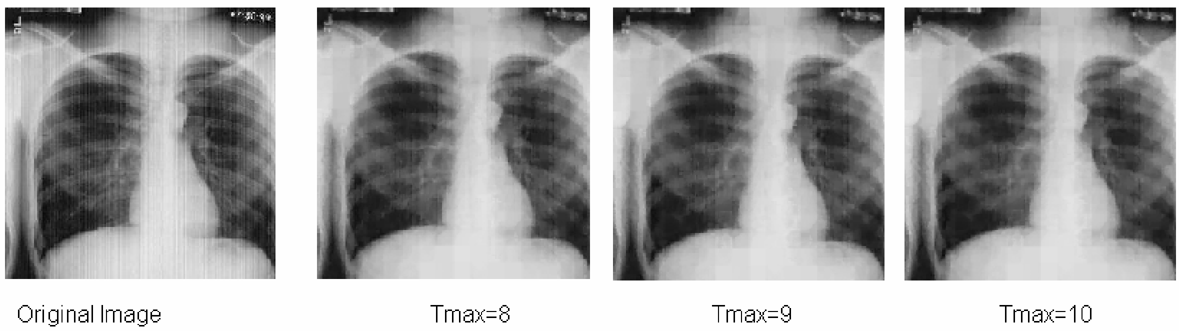
a) Ultrasound



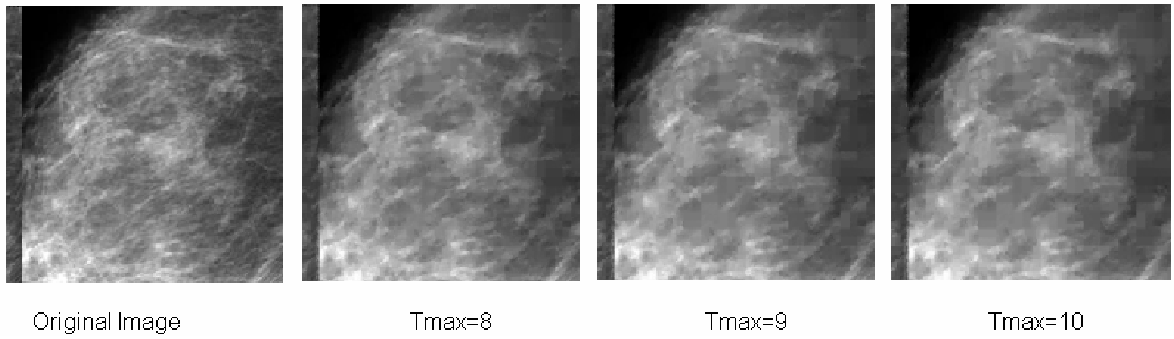
b) Angiogram



c) CT Scan



d) X-Ray



e) Mammogram

Fig. 2 Original images and decompressed images of different modalities obtained by fractal image compression using Quad tree partitioning, for various values of tolerance criterion

TABLE I
COMPRESSION RATIO (C.R.) AND PSNR

Modality of image	T_{max}=5		T_{max}=8		T_{max}=9		T_{max}=10	
	C.R	PSNR	C.R	PSNR	C.R	PSNR	C.R	PSNR
UltraSound	8.02	28.74	8.15	28.45	8.66	25.89	8.89	25.81
CT Scan	8.5	27.93	10.34	26.33	10.92	26.30	11.63	26.57
Angiogram	9.95	30.59	15.59	27.97	17.82	27.18	22.20	28.38
X-Ray	12.15	33.92	20.37	29.60	26.68	29.28	31.68	29.00
Mammogram	19.99	33.21	35.74	31.89	35.92	31.32	47.31	30.55

Table I gives the compression ratios and the PSNR values of different imaging modalities for various values of Tolerance factor T_{max}. From the values we conclude that, even though the PSNR values ranges between 20dB and 40dB , for various values of tolerance criterion of different imaging modalities, the visual inspection of images show that the image quality is found to be detoriating beyond T_{max}=8. This is observed by noticing the decompressed images for T_{max}=9 and T_{max}=10 where the poor image quality is clearly seen. Hence in this work, T_{max}=8 is found to be the optimum value for medical images using Fractal image compression.

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