Performance of Subcarrier- OCDMA System with Complementary Subtraction Detection Technique


Abstract—A subcarrier - spectral amplitude coding optical code division multiple access system using the Khazani-Syed code with Complementary subtraction detection technique is proposed. The proposed system has been analyzed by taking into account the effects of phase-induced intensity noise, shot noise, thermal noise and inter-modulation distortion noise. The performance of the system has been compared with the spectral amplitude coding optical code division multiple access system using the Hadamard code and the Modified Quadratic Congruence code. The analysis shows that the proposed system can eliminate the multiple access interference using the Complementary subtraction detection technique, and hence improve the overall system performance.

Keywords—Complementary subtraction, Khazani-Syed code, multiple access interference, phase-induced intensity noise

I. INTRODUCTION

The increasing demand for higher data-rates has inspired tremendous interest in combining subcarrier multiplexed (SCM) and optical code division multiple access (OCDMA) to the fiber optic communication systems [1]. The basic idea of combining the two techniques has been widely studied and published in [2-4]. However the researches of [2-4] were based on the combination of SCM and wireless CDMA using bipolar codes such as Gold [5, 6] and Kasami [6] codes which are suitable for radio frequency (RF) or wireless communication environment. None has been attempted for SCM spectral amplitude coding optical code division multiple access (SAC-OCDMA), a combination between SCM and spectral amplitude coding based optical CDMA. Thus, in this study, a SCM SAC-OCDMA using the Khazani-Syed (KS) code is proposed. KS-code is a unified code construction based on Double Weight (DW) and Modified Double Weight (MDW) codes [7, 8]. The SCM technique is widely used in broadcast system [9], optical wireless [10], label switch routing [11] and local area networks [12]. In SCM, the operation of multiplexing and demultiplexing of the subcarriers is carried out electronically. This gives an advantage over the WDM, due to the lower cost of the electrical multiplexer and demultiplexer if compared with the optical multiplexer [6, 12]. Alternatively, OCDMA offers numerous advantages when applied to photonic networks because of its unique combination of qualities: asynchronous transmission; the potential of secure communications; on-demand soft capacity; and a high degree of scalability [13]. SAC-OCDMA is a good candidate for optical multi-access networks over other OCDMA techniques because of its ability to cancel the multiple access interference (MAI) [14]. Based on the merits of SCM and SAC-OCDMA, it is hypothesized that the proposed SCM SAC-OCDMA can be taken advantage of towards improving the transmission bandwidth and to provide some robustness against interference. The maximum permissible number of simultaneous users can be enhanced by increasing the SCM and/or the SAC-OCDMA code words.

II. SCM SAC-OCDMA SYSTEM ARCHITECTURE

The proposed SCM SAC-OCDMA system architecture with complementary subtraction detection technique is shown in Fig. 1. The data are modulated with microwave carrier at different frequencies. These subcarriers are combined electrically and then they are optically modulated onto a code sequence using Mach Zehnder optical external modulator (OEM). Each code sequence is assigned with a different code word. Then m modulated code sequences are combined together via an optical combiner and transmitted through the optical fiber. At the receiver, an optical splitter is used to split the modulated code sequences to various paths according to the number of receivers. Only the matched code sequences are decoded while the unmatched code sequences will be filtered out. Through the decoder which is based on the complementary subtraction, the decoded signal is detected and the MAI from unmatched transmitters can be completely canceled. An electrical splitter and electrical bandpass filter (BPF) are used to split the SCM signals and to reject the unwanted signals, respectively. In order to retrieve the original transmitted data, the incoming signal is electrically mixed with a local microwave frequency f_l and filtered using a low-pass filter (LPF).
Each user is assigned a particular code sequence $c_i$ and subcarrier frequency $f_i$, where the pair of $(c_i, f_i)$ is unique with respect to every other user. Only the intended receiver with the matched pair of $(c_i, f_i)$ is able to correctly demodulate the detected signal, made possible by the decoding scheme. Other signals are ignored.

### A. Complementary Subtraction Detection Technique

The complementary subtraction detection technique was first proposed by Kavehrad and Zaccarin [15]. The implementation of a SCM SAC-OCDMA system using complementary subtraction detection technique is shown in Fig. 2.

For example, let us consider the KS code sequences as shown in Table I.

<table>
<thead>
<tr>
<th>Code Sequence</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
<th>$\lambda_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Y</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Note that $\lambda_i$, where $i$ is 1, 2, ..., $N$, represents the spectral position of the chips in the KS code sequence.

Optical bandpass filters are used as the encoders and decoders for the detection techniques. The number of filters needed is equal to the number of ‘ones’ in the code sequence. Assumption is made here that separate filters are used for different chip wavelength, as been widely proposed in SAC-OCDMA implementation [14, 16]. In the KS code, two chips (wavelengths) always occur next to each other, whereas in other codes [14, 16-18] two adjacent wavelengths are separated. Since chip ‘ones’ always occurs in pairs, one filter of a larger bandwidth (doubled) could be used instead of using two filters to represent two distinct ‘ones’. Hence, using this property, the number of filters at the encoder and decoder for a system employing KS code is reduced by half [19].

The optical pulses are encoded according to the KS code sequences denoted as $X = (110)$ and $Y = (011)$. The SCM signal is then optically modulated with the KS code sequence as shown in Fig. 2. The outputs of the two OEMs are combined and transmitted through an optical fiber. Referring to Receiver 1, the signals are split and decoded separately by two Complementary decoders. The outputs from the complementary filters are detected by the two photodetectors (PIN) connected to a subtractor. To eliminate the MAI from undesired users, an attenuator $T$ is required at PIN 2 and PIN 4.
III. THEORETICAL DEVELOPMENT OF SCM SAC-OCDMA SYSTEM

In the theoretical development, only those relevant parameters such as shot noise, thermal noise, phase-induced intensity noise (PIN) noise and inter-modulation distortion of subcarrier channels (IMD) noise are being considered based on the implementation assumption. Since the proposed system is influenced by the SCM, the IMD is taken into account for the total noise. Positive-intrinsic-negative (PIN) photodetectors are used and the dark current is assumed negligible [20]. The spacing of optical carriers is assumed to be sufficiently wide so that the effect of crosstalk from adjacent optical channels is negligible. The subcarrier channels are equally spaced. The following assumptions are made [14, 17]: (i) the light source spectra is ideally unpolarized and has flat PSD over the optical source bandwidth; (ii) each power spectral component has identical spectral width; (iii) each user has equal power at the receiver and (iv) each bit stream from each user is synchronized.

The above assumptions are important so that the system performance can be easily analyzed using Gaussian approximation. The source coherence time \( \tau_c \) is expressed as [21, 22]:

\[
\tau_c = \frac{\int G^2(v)dv}{\int G(v)dv},
\]

where \( G(v) \) is the power spectral density (PSD) of the thermal source.

Let \( C_i(t) \) denote the \( i \)th element of the \( k \)th KS code sequence with \( \overline{C}_i(i) \) as its complement. The code properties for the Complementary detection technique can be written as:

\[
\begin{align*}
\sum_{l} C_{i}(i)\overline{C}_{i}(i) &= \begin{cases} W, & \text{for } k = l \text{ in the same sector} \\ 1, & \text{for } k \neq l \text{ in the same sector} \\ 0, & \text{for } k \neq l \text{ not in the same sector} \end{cases} \\
\sum_{l} C_{i}(i)\overline{C}_{i}(i) &= \begin{cases} W-1, & \text{for } k = l \text{ in the same sector} \\ 0, & \text{for } k \neq l \text{ not in the same sector} \end{cases}
\end{align*}
\]

The condition of \( k \) and \( l \) in the same sector meaning that both code sequences are in (C(1), C(2) or C(M)) as shown in Eq. (4). And for the condition of \( k \) and \( l \) not in the same sector meaning that one of the code sequence might be in C(1) and the other code sequence is in C(2) or C(M).

\[
c(m) = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

The matrix size of each 0 and C(m), \( 1 < m \leq M \) is the same as C(1) matrix size. For a fixed value of weight \( W \), when the number of mapping is increased, the code size is extended by \( (m \times K_B) \) and the basic code is extended diagonally as shown in (4). \( K_B \) is the basic code’s row size or the basic number of codes which is given by [8]:

\[
K_B = \frac{W}{2} + 1
\]

The PSD of the received optical signals can be written as [14]:

\[
r(v) = \frac{P}{A} - \sum_{i=1}^{\infty} \frac{c_i}{i} \text{rect} \left( \frac{t}{i} \right)
\]

where \( P_{sd} \) is the effective power of a source at the receiver, \( N_{u} \) is the number KS optical code sequences, each carrying their corresponding subcarrier. \( N \) is the KS code length. \( d_{n}(t) \) represents the modulated data of \( n \)th subcarrier channel on the \( k \)th optical code expressed as [8]

\[
d_{n}(t) = \sum_{m=1}^{N} u_{n,k}(t)\frac{m_{n,k}}{\cos(\omega_{0}t)}
\]

\( u_{n,k}(t) \) is the normalized digital signal at the \( n \)th subcarrier channel of the \( k \)th code, where 0 and 1 are represented as digital signal “0” and “1”, respectively. \( \omega_{0} \) is the angular subcarrier frequency, \( m_{n,k} \) is the modulation index of the \( n \)th subcarrier of the \( k \)th code and \( N_{u} \) is the number of subcarrier channels on each code. Assuming an identical modulation index for all subcarrier channels, it is necessary that \( 0 < m_{n,k} \leq \frac{1}{N_{u}} \) [12].

The rect \( (i) \) function in (6) is given by:

\[
\text{rect}(I) = \begin{cases} v - \nu_{v} - \frac{\nu_{v}}{2N}(-N+2i-2) & \text{if } v \geq \nu_{v} - \frac{\nu_{v}}{2N}(-N+2i-2) \\ v - \nu_{v} - \frac{\nu_{v}}{2N}(-N+2i-2) & \text{if } v < \nu_{v} - \frac{\nu_{v}}{2N}(-N+2i-2) \end{cases}
\]

where \( u(v) \) is the unit step function expressed as:

\[
u(v) = \begin{cases} 1, & \text{if } v \geq 0 \\ 0, & \text{if } v < 0 \end{cases}
\]

From (2) and (3), the weight of \( C_{i}(i)\overline{C}_{i}(i) \) is larger than \( C_{i}(i)C_{i}(i) \) when \( k \neq l \). In order to eliminate the MAI, the complement cross correlation \( C_{i}(i)\overline{C}_{i}(i) \) must be multiplied by an attenuator with a transmission coefficient, \( T \) so that the cross correlation \( C_{i}(i)\overline{C}_{i}(i) \) subtracted from \( C_{i}(i)C_{j}(i) \) is equal to zero. The value for \( T \) can be derived as:

\[
\begin{align*}
\sum_{i} C_{i}(i)C_{i}(i) - T \sum_{i} C_{i}(i)\overline{C}_{i}(i) &= 0, \text{ for } k \neq l \\
I(K_{g} - 1) - T \left[ \left( \overline{W}-1 \right)K_{g} - 1 + WK_{g}(m-1) \right] &= 0 \\
I(K_{g} - 1) &= T \left[ \left( \overline{W}-1 \right)K_{g} - 1 + WK_{g}(m-1) \right] \\
T &= \frac{1}{\left( \overline{W}-1 \right) + \frac{WK_{g}(m-1)}{K_{g} - 1}}
\end{align*}
\]

Eq. (11) is when the number of mapping is greater than 1. Without the mapping or \( m = 1 \), (11) becomes

\[
T = \frac{1}{\left( \overline{W}-1 \right)}
\]

Therefore, the subtraction can be written as:
\sum_{0}^{\infty} C_{l}(i)C_{l}(i) - T \sum_{0}^{\infty} C_{l}(i)\overline{C}_{l}(i) = \begin{cases} W, & k = l \\ 0, & k \neq l \end{cases}

Equation (13) shows that when the code sequence is matched, the weight W is obtained. However when the code sequences are unmatched, the weight is zero, meaning that the MAI is eliminated in this system.

The PSD at the PIN photodiode, PIN1 and PIN2 as in Fig. 2 of the 6th receiver during one data bit period can be written as:

\[ G_{1}(v) = \frac{P_{m}}{N} \sum_{i=1}^{N} d_{i}(i) \sum_{j=1}^{N} C_{l}(i)C_{l}(i) \{ \text{rect}(i) \} \]

(14)

and

\[ G_{2}(v) = T \frac{P_{m}}{\Delta v} \sum_{i=1}^{N} d_{i}(i) \sum_{j=1}^{N} C_{l}(i)\overline{C}_{l}(i) \{ \text{rect}(i) \} \]

(15)

Therefore, the total power incident at the input of PIN1 and PIN2 is given by:

\[ \int_{0}^{\infty} G_{1}(v)dv = \int_{0}^{\infty} G_{2}(v)dv = \int_{0}^{\infty} \frac{P_{m}}{\Delta v} \sum_{i=1}^{N} d_{i}(i) \sum_{j=1}^{N} C_{l}(i)C_{l}(i) \{ u \left[ \frac{\Delta v}{N} \right] \} dv \]

(16)

\[ \int_{0}^{\infty} G_{2}(v)dv = \int_{0}^{\infty} \frac{P_{m}}{\Delta v} \sum_{i=1}^{N} d_{i}(i) \sum_{j=1}^{N} C_{l}(i)\overline{C}_{l}(i) \{ u \left[ \frac{\Delta v}{N} \right] \} dv \]

(17)

Consequently, the signal from the desired user is given by the difference of the photodiode current I, expressed as:

\[ I = I_{1} - I_{2} \]

(18)

where

\[ I_{1}, I_{2} \]

is current at PIN1 and PIN2, respectively.

\[ I = \int G_{1}(v)dv - \int G_{2}(v)dv \]

\[ = \int \left[ \frac{P_{m}}{N} \sum_{i=1}^{N} d_{i}(i) \sum_{j=1}^{N} C_{l}(i)C_{l}(i) \{ u \left[ \frac{\Delta v}{N} \right] \} dv \right. \]

\[ \left. - \frac{P_{m}}{N} \sum_{i=1}^{N} u_{i}(i) \cos(\omega_{c}i) \right] \]

(19)

where \( R_{h} \) is the responsivity of the photodetectors given by:

\[ R_{h} = \frac{\eta e}{\hbar v_{c}} \]

(20)

Here, \( \eta \) is the quantum efficiency, \( e \) is the electron’s charge, \( \hbar \) is the Planck’s constant, and \( v_{c} \) is the central frequency of the original optical pulse.

The useful photocurrent signal for the \( k \)th channel is as in (19). At the RF demodulator, this signal coherently mixes with a local oscillator and is filtered out using lowpass filter (LPF). The output signal at the demodulator is

\[ I = \frac{R_{h} P_{m} W}{N} m_{n,k} u_{n,k}(i) \]

(21)

Since the noises in PIN 1 and PIN 2 are independent, the power of noise sources that exist in the photocurrent can be written as:

\[ \langle I_{1}^{2} \rangle = \langle I_{1}^{2} \rangle_{SN} + \langle I_{1}^{2} \rangle_{IN} \]

(22)

where \( I_{1}^{2} \) denotes the shot noise, \( I_{1}^{2} \) is the phase-induced intensity noise (PIN), \( I_{1}^{2} \) is the thermal noise and \( I_{1}^{2} \) is the inter-modulation distortion noise of subcarrier channels.

The shot noise:

\[ \langle I_{1}^{2} \rangle_{SN} = 2eB(I_{1}^{2} + I_{2}) \]

(23)

Hence, (23) is the shot noise for the system using the complementary subtraction detection technique. By using the methodology similar to that in [21] and approximating the summation \( \sum_{N} \), the noise power of PIN can be written as:

\[ \langle I_{1}^{2} \rangle = B I_{1}^{2} \tau_{c} + B I_{2}^{2} \tau_{c} \]

(24)

\[ \langle I_{1}^{2} \rangle_{IN} = B R \int G_{1}(v)dv + \int G_{2}(v)dv \]

(25)

The thermal noise is given as [23]:

\[ \langle I_{1}^{2} \rangle_{SN} = \frac{4K_{B}T_{B}}{R_{i}} \]

(26)

The inter-modulation distortion noise is given as [24, 25]:

\[ \langle I_{1}^{2} \rangle_{IN} = \frac{2K_{B}T_{B}}{R_{i}} \]

(27)
\[ \langle I_{\text{IMD}} \rangle = P_m \cdot \text{Re} \cdot \sigma_m \cdot \delta \left[ \frac{D_{111}}{128} + \frac{D_{21}}{128} \right] \]  
(27)

where \( D_{111} \) is the three-tone third order inter-modulation at \( f_i + f_k - f_l \), and \( D_{21} \) is the two-tone third order inter-modulation at \( 2f_i - f_k \).

The SNR of the system can be written as:

\[ \text{SNR} = \frac{(I_1 - I_2)^2}{\langle I^2 \rangle} \]

\[ \text{SNR} = \frac{n^2 P_{sr}^2 \Delta \pi}{2 \sigma^2 + n^2} \]

\[ \frac{D_{111} + D_{21}}{128} \]

\( (29) \)

Since no pulses are sent for the data bit “0”, the formula used to calculate the bit error rate (BER) using Gaussian approximation can be expressed as [26] – [28]:

\[ \text{BER} = \frac{1}{2} \text{erfc} \left( \frac{\sqrt{\text{SNR}}}{8} \right) \]

(30)

IV. SYSTEM PERFORMANCE

The typical parameters used in the analysis are listed in Table II and they are as those adopted in [8, 14].

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta )</td>
<td>Photodetector quantum efficiency</td>
<td>0.6</td>
</tr>
<tr>
<td>( \Delta v )</td>
<td>Line-width Broadband Source</td>
<td>3.75 THz</td>
</tr>
<tr>
<td>( \lambda_c )</td>
<td>Operating Wavelength</td>
<td>1550 nm</td>
</tr>
<tr>
<td>( B )</td>
<td>Electrical Bandwidth</td>
<td>311 MHz</td>
</tr>
<tr>
<td>( T_n )</td>
<td>Receiver Noise Temperature</td>
<td>300 K</td>
</tr>
<tr>
<td>( R_L )</td>
<td>Receiver Load Resistor</td>
<td>1030 ( \Omega )</td>
</tr>
<tr>
<td>( e )</td>
<td>Electron Charge</td>
<td>( 1.6 \times 10^{-19} ) C</td>
</tr>
<tr>
<td>( h )</td>
<td>Planck’s Constant</td>
<td>( 6.66 \times 10^{-34} ) Js</td>
</tr>
<tr>
<td>( K_b )</td>
<td>Boltzmann’s Constant</td>
<td>( 1.38 \times 10^{-23} ) J/K</td>
</tr>
<tr>
<td>( c )</td>
<td>Speed of Light</td>
<td>( 3 \times 10^8 ) m/s</td>
</tr>
<tr>
<td>( P_e )</td>
<td>Effective Received Power</td>
<td>(-10 ) dBm</td>
</tr>
</tbody>
</table>

The performance of the SCM SAC-OCDMA in terms of BER against the number of codes is shown in Fig. 3. The parameters used to evaluate the performance using other codes are the same as used for KS code. A few code specific parameters are chosen based on the published results for these particular codes [14, 16, 17] for comparison purposes. They are \( m = 4 \) for Hadamard (\( W = 8 \)) and \( p = 7 \) for MQC (\( W = 8 \)) [14]. The number of subcarriers is purposely set at 2 in order to observe the effect of the number of codes on the system performance. The total number of users is determined by the number of subcarriers multiplied with the number of the codes. It is shown that the performance of the SCM SAC-OCDMA using KS code is better compared with the others even though the weight is half of the other codes for the system, which is 4 in this case. For example, it can be seen that at an error rate of less than \( 10^{-9} \), at most 8 codes for SCM SAC-OCDMA using Hadamard code. SCM SAC-OCDMA using MQC code, however up to 14 codes would be possible under the same conditions, and SCM SAC-OCDMA using KS code would permit up to 23 codes. Therefore the total number of users for the system of SCM Hadamard, SCM MQC and SCM KS code are 16, 28 and 46, respectively. Since all other parameters are the same, the major advantage of KS code is due to less weight to achieve a similar BER.

Observation on Fig. 4 shows that the performance of SCM KS code is better compared to SCM MQC and SCM Hadamard system. The BER degrades as the number of subcarriers increases. This is because the optical modulation index per subcarrier decreases linearly with the number of channels. Thus increases the BER of the system.
V. CONCLUSION

In this paper, a SCM SAC-OCDMA system using KS code has been proposed. The detailed system performance analysis has been presented. It has been proven that the performance of the proposed system can eliminate the MAI. Hence, it can accommodate more users to access the network simultaneously. The BER performance shows that the SCM SAC-OCDMA system using the KS code performs better than the SCM SAC-OCDMA using MQC and Hadamard code. This proposed SCM SAC-OCDMA system using the KS code offers a great potential as a flexible and powerful technique.

REFERENCES