Piezoelectric Transducer Modeling: with System Identification (SI) Method

Nora Taghavi, and Ali Sadr

Abstract—System identification is the process of creating models of dynamic process from input-output signals. The aim of system identification can be identified as "to find a model with adjustable parameters and then to adjust them so that the predicted output matches the measured output". This paper presents a method of modeling and simulating with system identification to achieve the maximum fitness for transformation function. First by using optimized KLM equivalent circuit for PVDF piezoelectric transducer and assuming different inputs including: sinuside, step and sum of sinuses, get the outputs, then by using system identification toolbox in MATLAB, we estimate the transformation function from inputs and outputs resulted in last program. Then compare the fitness of transformation function resulted from using ARX, OE(Output-Error) and BJ(Box-Jenkins) models in system identification toolbox and primary transformation function form KLM equivalent circuit.

Keywords—PVDF modeling, ARX, BJ(Box-Jenkins), OE(Output-Error), System Identification.

I. INTRODUCTION

System identification is the process of creating models of dynamic process from input-output signals. The aim of system identification can be identified as "to find a model with adjustable parameters and then to adjust them so that the predicted output matches the measured output". Two important questions are:

- Which model parameterization is to be used
- How to know if the fitted model is good

Most of SID techniques have their roots in statistical methods like Least square fitting, maximum likelihood estimation etc.

It should be noted that there are non-parametric techniques for system identification like spectral analysis, correction analysis and transient analysis.

The various steps in system identification are:
1. Experiment setup and data collection
2. Data preprocessing
3. Model structure selection
4. Parametric estimation
5. Validation

There are generally 2 methods:

- First principles modeling
- System identification

In first principles modeling the mathematical model of the system identification is constructed from knowledge of system working. The parameters of the mathematical model are determined by iterative process.

In situations where system is a black box we go for system identification procedures where we will use various model structures before selecting one that gives optimum fit for test data.

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A. Stages of System Identification

![Diagram of system identification stages]

B. System Identification Structure

![System identification structure diagram]

B. SID Experiment Set up

System identification is done using the input output test data. The sole information is test data from which all parameters of system transfer function is to be identified. The test data must incorporate all the properties of the system. So the way system identification experiment is performed is very crucial. Following points must be noted:

The input signal must expose all relevant properties of the system. Suppose it is pure sinoside of a particular frequency. It will give information of system frequency response at the frequency. The input signal must have as many input frequencies as the order of the system. Following inputs can be considered:

- Step/Pulse Inputs
- Gaussian White noise
- Random Binary Signal (RBS)
- Pseudo-Random Binary Signal (PRBS)
- Multi-level Pseudo-Random Signals
- Multisine Inputs

A variety of model structures are available to assist in modeling a system. The choice of model structure is based upon an understanding of the system identification method and insight and understanding into the system undergoing identification [5,6]. Even then it is often beneficial to test a number of structures to determine the best one. Model types are as following:

- Parametric Model Structures
- AR Model
- ARX Model
- ARMAX Model
- Box-Jenkins Model
- Output-Error Model
- State-Space Model

Here we assume ARX, BJ, OE models, and describe them as following:

1. Parametric Model Structures

Parametric models describe systems in terms of different equations and transfer functions. A general-linear polynomial model or the general-linear model is as following.

\[ y(n) = q^{-1}G(q^{-1}, \theta)u(n) + H(q^{-1}, \theta)e(n) \]

\( u(n) \) and \( y(n) \) are the input and output of the system respectively. \( e(n) \) is zero-mean white noise, or the disturbance of the system. \( G(q^{-1}, \theta) \) is the transfer function of the deterministic part of the system. \( H(q^{-1}, \theta) \) is the transfer function of the stochastic part of the system.

The general-linear model structure is shown in Fig. 3.
Simpler models that are a subset of the General Linear model structure are possible. By setting one or more of $A(q)$, $B(q)$, $C(q)$ or $D(q)$ polynomials equal to 1, you can create these simpler models such as ARX, Box-Jenkins, and output-error structures[7].

2. **ARX Model**

The ARX model, shown in Fig. 4, is the simplest model incorporating the stimulus signal. The estimation of the ARX model is the most efficient of the polynomial estimation methods because it is the result of solving linear regression in analytic form. Moreover, the solution is unique. In other words, the solution always satisfies the global minimum of the loss function. The ARX model therefore is preferable, especially when the model order is high. The disadvantage of the ARX model is that disturbances are part of the system dynamics.

![Fig. 4 ARX Model Structure](image)

3. **Box-Jenkins Model**

The Box-Jenkins (BJ) structure provides a complete model with disturbance properties modeled separately from system dynamics.

![Fig. 5 Box-Jenkins Model Structure](image)

4. **Output-Error Model**

The Output-Error (OE) model structure describes the system dynamics separately. No parameters are used for modeling the disturbance characteristics [4,8].

![Fig. 6 OE Model Structure](image)

### III. PIEZOELECTRIC MATERIAL

There are two classes of piezoelectric materials used in vibration control, ceramics and polymers. The best well-known piezoceramic is Lead Zirconate Titanate, widely used as an actuator and sensor including ultrasound applications that is good for high-accuracy. Generally piezopolymers used as a sensors which the best well-known is Lead Lantanum Zirconate Titanate (PVDF).

There are several ways to show and analysis the piezoelectric transducers equivalent models, we choose KLM optimized equivalent circuit to simulate and modeling the transducer. Thus, here is a brief description of theoretical equation of it. Usually it’s hard to design transducers with wide-boundary operation and also predict their operation without referring to numerical calculation. But it’s possible to provide some physical aspects of this type of transducers by using another equivalent circuit that is well-known as a KLM equivalent circuit.

Here, we assume the piezoelectric as a material that can emit bi-axes wave. KLM equivalent circuit is shown below. These modes are continuously simulated by the current $(j\omega A)$ in the transducer[2,3].

![Fig. 7 KLM equivalent circuit](image)

#### A. Related Equations

In this model, $V_3$ and $I_3$ are sequentially voltage and current that are used to piezoelectric to produce Acoustic forces $F$ and Particle velocities $V$ in two-side piezo. Model parameters are Transducer thickness $l$, piezo area $A$, Acoustic Transmission line Impedance $Z_c$, $Z_1$ and $Z_2$ Impedances, environment...
Radiation Impedances are in bi-axial crystal. Finally value of circuit elements is calculated as following:

\[ C_0 = \frac{\varepsilon^i A}{l} \]  
(1)

\[ X_0 = -\frac{C_0}{k^2} \frac{1}{\sin(\omega / \omega_0)} \]  
(2)

\[ \varphi = k_T (\pi / \omega_0 C_0 Z_c)^{1/2} \sin(\omega / 2 \omega_0) \]  
(3)

\[ k_T = \sqrt{\frac{\varepsilon^i}{c_p \varepsilon^{S}}} \]  
(4)

In above equation, piezoelectric dielectric constant \( \varepsilon^i \), electromechanical coupling constant \( k_T \), half of transducer frequency wave length (equall with \( \pi c/T \)), stifened elastic constant \( C^D \), piezoelectric force constant \( e_{33} \) is in thickness direction.

By using electrical impedances and an impedance caused by acoustic load that is known as radiation impedance \( Z_a \), total electrical impedance \( Z_\text{in} \), that is seen in electrical input is calculated from below equation:

\[ Z_\text{in} = \frac{1}{j \omega C_0} + \frac{1}{j \omega X_0} + Z_a \]  
(5)

where,

\[ Z_a = \varphi^2 Z_{sh} \]  
(6)

\( Z_a \) is radiation impedance that is converted shant impedance and calculated as following:

\[ Z_a = \frac{Z_{L1}Z_{L2}}{Z_{L1} + Z_{L2}} \]  
(7)

KLM model successfully can be used for transducers that are made from piezo electric elements with small internal losses for example: quartz and piezoceramic material such as PZT.

IV. FIND OUT TRANSMISSION FUNCTION USING SYSTEM IDENTIFICATION

Here we find out the transmission function with programming in MATLAB and using ident to work with Geraphical User Interface (GUI) and simultany compare the result with the transmission fuction of the KLM optimized circuit and find out which model can give us the maximum fitness[9].

- For the single sinuside inputs we change the parameters such as frequency, phase and amplitude and found out whit assuming phase and amplitude as constant parameters and changenig the frequency we don't have describable change on the result.

Examples:

1. Assume that Ap=10e-3, Ph0=pi/6 are constant parameters and we are changing the frequency from f=500e6 to f=700e6, thus Fig. 11 illustrates the results.
2. With assuming amplitude (Ap=10^{-3}) and frequency (f=500e6) as constant parameters and changing phase from Ph0=\pi/6 to Ph0=\pi, we have:

![Graph showing change in results from Ap=10^{-3} to Ap=100^{-3} and to Ap=20^{-3}](image)

3. With assuming frequency (f=500e6) and phase (Ph0=\pi/6) as constant parameters and changing amplitude from Ap=10^{-3} to Ap=100^{-3} and to Ap=20^{-3}, we don't have any change in results:

![Graph showing change in results with multi-sinuside input](image)

- For the multi-sinuside input the results are so different in comparison with the single inputs as with assuming amplitude and as constant parameters and decreasing the phase, the fitness will increase, but increasing the phase for some phases the fitness is decreased and for some others, the fitness is increased as following:

![Graph showing decrease in fitness with increasing phase](image)

![Graph showing increase in fitness with increasing phase](image)

![Graph showing fitness decrease with increasing phase](image)
From above result we find the \( \phi_01=\pi/6 \) as best choice. Now we do the same work for the \( \phi_02 \):

Hence, choosing \( \phi_01=\pi/6 \) and \( \phi_02=\pi/4 \) result in the best fitness for the multi-sinuside input. Corresponding that the KLM transmission function is closed to the OE model, it provides the best fitness compare to the other models.

- For the step input changing the parameters don’t have any affect on the result, thus we use the ident in MATLAB and attempt to find the maximum fitness by changing the coefficients orders of different system models (ARX, BJ, OE) polynomials.

### Table I

<table>
<thead>
<tr>
<th>Model Type</th>
<th>ARX</th>
<th>BJ</th>
<th>Units</th>
<th>OE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order</td>
<td>4 8 1</td>
<td>1 2 8 1 0 1</td>
<td>4 3 1</td>
<td></td>
</tr>
<tr>
<td>Fitness</td>
<td>82/66</td>
<td>84/21</td>
<td>67/09</td>
<td></td>
</tr>
</tbody>
</table>

V. Conclusion

As the result show, it's easy to understand that the OE model is choosed as a best model with achieving %99.7 fitness for the single sinuside input and % 88.7 for the multi-sinuside input, we mentioned earlier that the OE model is supposed to be the best model as its transmission function is closed to the KLM transmission function. Hence we expect it would be best model for every input such as step and it is, but in comparison with the other models here, results shows that the BJ model gives us the higher fitness, so the BJ model is choosed as a best model with %84.21 for step input.

### REFERENCES


[9] MATLAB Toolbox