World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:3, No:5, 2009

The use of voltage stability indices and proposed instability prediction to coordinate with protection systems

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***Abstract*—This paper proposes a methodology for mitigating the occurrence of cascading failure in stressed power systems. The methodology is essentially based on predicting voltage instability in the power system using a voltage stability index and then devising a corrective action in order to increase the voltage stability margin.**

The paper starts with a brief description of the cascading failure mechanism which is probable root cause of severe blackouts. Then, the voltage instability indices are introduced in order to evaluate stability limit. The aim of the analysis is to assure that the coordination of protection, by adopting load shedding scheme, capable of enhancing performance of the system after the major location of instability is determined. Finally, the proposed method to generate instability prediction is introduced.

Keywords — Blackouts, Cascading failure, Voltage stability indices, Singular Value Decomposition, Load shedding.

I. Introduction

THE long-term development of power system technology resulted in the formation of large-scale interconnected grids in Europe during the last five decades. The integration of the national grids was motivated mainly by the growth in electricity demand alongside the desire to minimize operational costs of the power system yet safeguarding the high level supply of electricity.

The Union for the Coordination of Transmission of Electricity (UCTE) synchronous system has experienced a chain of severe power system failures suggesting that the present day power systems are being operated quite close to their stability margins. Such would warrant a search for viable methods capable of preventing severe failures or at least decrease the risk of encountering them before blackouts become widespread. Large blackouts widely propagate by a complex sequence of cascading failures. Cascading failure refers to any components that fail as a result of overloading, human error or activating relays as a result of loading transferred from one affected component to another according to the circuit laws. This kind of failure is typically rare given that the highly anticipated failure has already been accounted for in power system planning design and operational routines.

This paper starts with a generic definition of a cascading failure aimed at facilitation of the understanding the propagation mechanism. This is because voltage instability is often triggered by the tripping of transmission or generation equipments whose probability of occurrence is relatively large. This study concentrates on the assessment of voltage instability by using stability indices. These indices will be presented to demonstrate how close to voltage instability that the system can be operated which can lead to blackouts in large parts of the interconnected system. The voltage instabilities are in response to an unexpected raise in the load level or in combination with an inadequate reactive power support at critical areas. Voltage instabilities can be classified into slow and fast characteristics, this paper merely focuses on the slow voltage instability which results from load increment. More details on the phenomena which contribute to voltage instability have been described along with countermeasures can be found in [1].

A Newton-Raphson method can be used to estimate the steady-state stability limits by the convergence of load flow calculation [2]. There are several proposed indices, as a by-product of solution from Newton's method, to predict oscillatory stability, for example eigenvalue index, etc. To be more specific, the use of minimum singular value of power flow Jacobian matrix [3], and an index of the modified power flow matrix will be introduced and their features compared. The purpose of indices is not only to determine operational limit but also significant for developing the coordination of protection systems of the Swedish network.

This paper organizes as the overview of cascading failure in Section II. Then, the voltage instability indices and their products are derived in Section III. The Swedish test system is briefly described in Section IV. Simulations and results are presented in Section V. Finally, in Section VI and Section VII, conclusions and recommendations for further research are then drawn out, respectively.

II. Cascading failure

The cascading failure mechanism is originated after critical component of the system has been removed from service. The removal creates load redistribution to other components which might become overloaded by the accumulation of the initial loading and the additional loading from the failed

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Paper submitted to the International Conference on Electric Power and Energy Systems (ICEPES 2009) in Tokyo, Japan May 27-29, 2009

component. This process sequentially weakens network and generates further failures more likely so that a blackout can propagate to devastate the entire system.

Cascading failures are typically rare and unanticipated because the likely and anticipated failures have already been accounted for in the process of power system design and operation. That is, it is customary in the operation of power systems to ensure that the $N-1$ criterion is fulfilled. In other words, an exhaustive and detailed analysis of these cascading events before the blackouts occur are difficult due to the huge number of possible combinations of unlikely events.

Either voltage or rotor angle stability are involved with many system components. For example, Load modeling is related to both types of stability whereas Load Tap changer is categorized in voltage stability only. However, all of system components can make power system blackouts become widespread by a complicated sequence of cascading failures. As mentioned earlier, protective relays play a central role by the tripping of critical component is the course of cascading events as shown in Fig. 1.

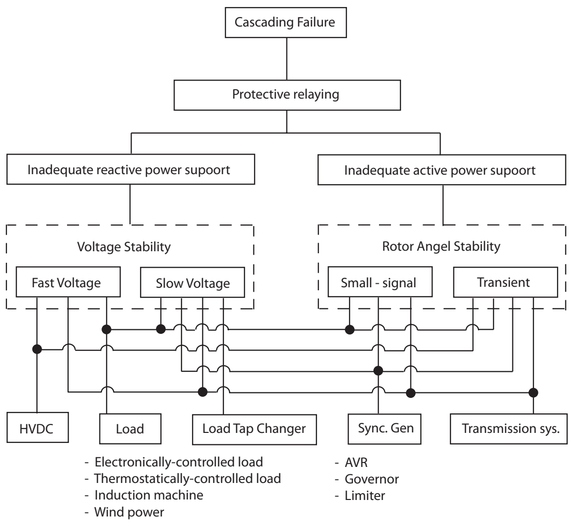

Fig. 1. Flow chart of components leading to cascading failure

III. VOLTAGE STABILITY INDICES

Voltage stability indices are invaluable tools for gauging the proximity of a given operating point to voltage instability. Fast voltage stability indices can be successfully applied to online dynamic voltage stability assessment. The objective of the voltage stability indices is to quantify how close a particular point is to the steady state voltage stability margin. One of significant stability indices is minimum singular value. It was shown in a number of publications that Singular Value Decomposition of the system Jacobian can be used in synthesizing fast voltage stability indices e.g. in [3] and [4]. Since power flow techniques are well-known and described in many publications, only a short description of their main properties is given.

In a power system, Newton-Raphson method can be used to obtain the unknown nodal voltages and angles. This can be done by linearization method calculated from the known active and reactive power, which can be expressed as following.

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} P_\alpha & P_V \\ Q_\alpha & Q_V \end{bmatrix} \begin{bmatrix} \Delta \alpha \\ \Delta V \end{bmatrix} \tag{1}$$

where P_α, P_V, Q_α and Q_V are submatrices which their elements of them are the partial derivatives of active power, P, and reactive power, Q, with respect to the nodal voltage angles, α and nodal voltages V. Hence, the power flow Jacobian can be written as

$$J(\alpha, V) = \begin{bmatrix} P_\alpha & P_V \\ Q_\alpha & Q_V \end{bmatrix} \tag{2}$$

In addition, the Newton-Raphson method has a quadratic convergence and computational time increases linearly with the system size. Eq. (1) can be computed by given power balance equations (using the π-equivalent representation for the transmission lines) as

$$P_{net_i} = V_i \sum_{j=1}^{N} V_j [G_{ij} \cos(\alpha_{ij}) + B_{ij} \sin(\alpha_{ij})] \tag{3}$$

$$Q_{net_i} = V_i \sum_{j=1}^{N} V_j [G_{ij} \sin(\alpha_{ij}) - B_{ij} \cos(\alpha_{ij})] \tag{4}$$

From Eq. (1), the Jacobian matrix can be modified by assuming ΔP equals to zero. This is because even the system voltage stability is affected by both active and reactive power but when only the relation between the small-signal reactive power and voltage magnitude is desired, $\Delta P = 0$, the assumption is applicable [5]. Thus the Reduced Jacobian, J_R, can be written as

$$\Delta Q = (Q_V \; - \; Q_\alpha P_\alpha^{-1} P_V)\Delta V \stackrel{\text{def}}{=} J_R \Delta V \tag{5}$$

As an illustration, consider the singular value decomposition is given by [3]. Jacobian (or the Reduced Jacobian) matrix can be decomposed into its singular value equivalent as

$$J(\alpha, V) = \underline{U} \Sigma \underline{V}^T \tag{6}$$

where $\underline{U}$ and $\underline{V}$ designate unitary matrices of the eigenvectors of JJ^T and $J^T J$, respectively and Σ is a diagonal matrix with

$$\Sigma(J) = \textit{diag}\ [\sigma_i(J)] \qquad\qquad i = 1, 2, \ldots, n \tag{7}$$

where $\sigma_i \geqslant 0$ for all i and the diagonal elements of Σ are ordered non-increasing where the other entries are zero. In addition, singular value are the positive square roots of eigenvalues of $J^T J$ (or JJ^T).

$$\sigma_i(J) = \sqrt{\lambda_i(J^T J)} \tag{8}$$

Thus, at this stage one can conclude that singular values have similar property as eigenvalues but being represented only by real number.

The effect on the $[\alpha^T, V^T]^T$ vector of small change in P and Q injection, according to decomposition value, can be computed as

$$\begin{bmatrix} \Delta \alpha \\ \Delta V \end{bmatrix} = \underline{V} \Sigma^{-1} \underline{U}^T \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \tag{9}$$

By virtual comparison of Eq. (6) through Eq. (9), thus

$$\underline{V}\Sigma^{-1}\underline{U}^T = \sum_{i=1}^{n} \sigma_i^{-1}\underline{v}_i\underline{u}_i^T \tag{10}$$

The column vectors of $\underline{V}$, denoted $\underline{v}_i$, are called right or input singular vectors and the column vectors of $\underline{U}$, denoted $\underline{u}_i$, are called left or output singular vectors.

The inverse of the singular value, σ_i, interprets to be the incremental change in the state variables if σ_i is small enough. According to [6], σ_i of the Jacobian matrix becomes close to zero, which is called the minimum singular value (σ_n). Then, the matrix is singular which can be implied that the power flow solution cannot be obtained.

The maximum change in state variables occurs at σ_n. And from Eq. (9) and (10), it can be mapped that

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \underline{u}_n \tag{11}$$

where $\underline{u}_n$ represents the last column of $\underline{U}$, also

$$\begin{bmatrix} \Delta \alpha \\ \Delta V \end{bmatrix} = \sigma_n^{-1}\underline{v}_n \tag{12}$$

where $\underline{v}_n$ represents the last column of $\underline{V}$

By the interpretations in [7], the minimum singular value, σ_n, and the corresponding left, $\underline{u}_n$, and right, $\underline{v}_n$, singular vectors are defined as:

σ_n : Indicator of the proximity to steady-state stability limit.

$\underline{v}_n$: Sensitive voltages (and angles).

$\underline{u}_n$: The most sensitive direction for changes of active and reactive power injections.

Furthermore, the technique of modal analysis in [8] is adapted to find participation factor, which can be obtained from the multiplication of right and left corresponding eigen-vectors, have been applied to those singular vectors. In this paper, to avoid confusion, this bus participation factor is called participation vector and defined as

$$\underline{P}_n = \underline{u}_n \cdot \underline{v}_n \tag{13}$$

The interpretation is that the highest magnitude in participation vector represents the most sensitive in both voltage magnitude and direction for reactive power injection. It means that load bus has the largest involvement in the voltage instability. Consequently, it is also the most effective system location for implementing remedial measures. [5]

The proposed indices are validated their features on the Swedish system which the physical structure of the system is briefly described in Section IV.

IV. Test System

The simulation has been carried out on test system called Nordic 32 system. The system has been proposed by CIGRE Task Force 38.02.08 [9] and the single-line diagram of Nordic 32 system is depicted as shown in Fig. 2.

The Nordic 32 test system consists of four major regions; North, Central, External, and Southwest. The voltage levels in the system are 400, 130 and 220 kV. There are 23 generators and 32 high voltage buses in the system, nineteen of them are 400 kV buses, eleven are 130 kV buses, and two are 220 kV. The generating units in the system are located throughout the area but mainly in the North area. The generation and load in the system is distributed in a way that the power flows from North to Central. In addition, the Southwest region is weakly connected to the system.

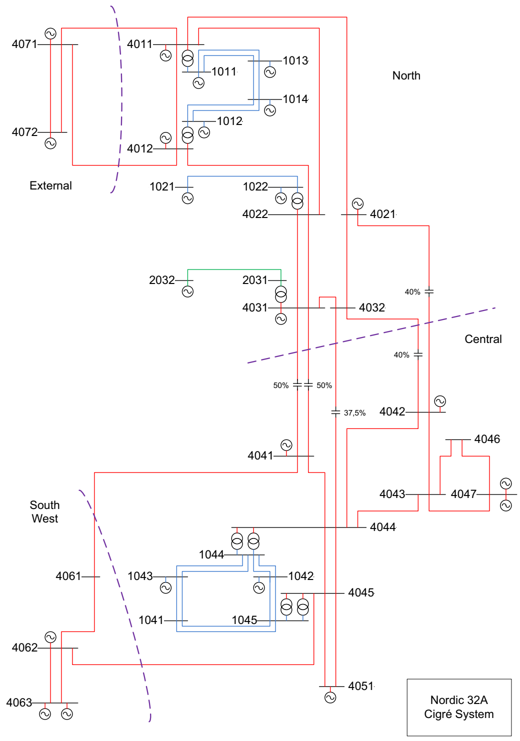

Fig. 2. Single-line diagram of the Nordic 32 test system [9]

V. Simulation

In this Section, the voltage stability indices which are introduced earlier have been applied to capture the slow voltage instability. To use voltage stability indices, scenario is created as when reactive power consumption increases, then voltage magnitudes drop from their nominal values. In addition, the study in [4] confirm that voltage stability problems will become more dominating over angle stability when the loading is increases.

Such a scenario can be created by modeling the load increment as typical constant reactive power models and is assumed to change according to

$$Q_L = Q_{Lo}(1 + \lambda) \tag{14}$$

where Q_{Lo} is the initial base reactive power levels of all nodal load and λ is the varying parameter representing the loading factor. In this paper, λ is increment in step of 0.005 p.u.

The calculation of Jacobian and Reduced Jacobian matrices are introduced as mentioned in Section III. Fig. 3 shows minimum singular value of J and J_R versus the reactive power consumption where ΔQ is the reactive load increment, for the Nordic 32 test system.

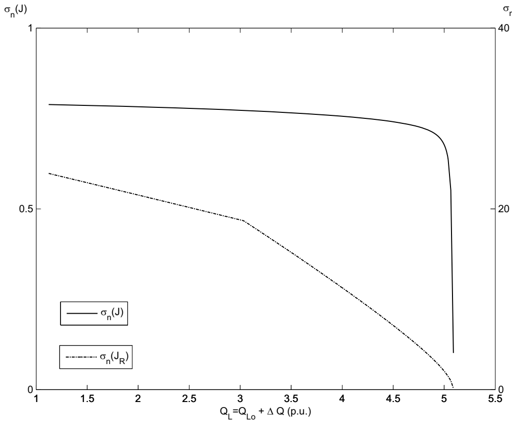

Fig. 3. Rate of change of $\sigma_n(J)$ and $\sigma_n(J_R)$

It can be seen from Fig. 3 that when loading increases, the value of minimum singular value is decreased for both $\sigma_n(J)$ and $\sigma_n(J_R)$. Moreover, it can be concluded that $\sigma_n(J_R)$ is the better stability index compared to $\sigma_n(J)$ because $\sigma_n(J_R)$ experiences larger change in each increment. This implies that J_R is capable of providing a better *early warning* before system confronts the contingency. It is worth noting that the computing $\sigma_n(J_R)$ is fast computation since a simple inverse iteration technique can be readily applied to J_R to determine this value in a few iterations. This is also confirmed in [6]. In addition, there is no significant information in the numerical solution. The trajectory of minimum singular value is of most interest in itself.

According to Fig. 3, $\sigma_n(J_R)$ is selected to measure voltage stability limit. Then, the system security, ϵ, is set differently on different networks. Security margin acts as the safety accounts for keeping system operates at a reasonably high level of reliability. This margin often determined by experience, running simulations of selected contingencies [10]. More effective method for extracting voltage security can be found in [11]. For this paper ϵ is set to be equal to 10% of $\sigma_n(J_R)$ at the steady state condition. The corrective action is applied for aiding the system if the margin is violated.

It is highly desirable to prevent voltage reduction. There are several corrective actions such as adding massive reactive compensation or shedding at a certain amount of load. But load shedding offers more favorable balance between cost and reliability [12]. It is always questionable which bus should be shed. Table I depicts top 5 largest reactive loading buses of the Nordic 32 system.

TABLE I
SOME BUS DATA OF NORDIC 32 SYSTEM

Bus	P_{Load} (MW)	Q_{Load} (MVAR)
43	900	238.83
51	800	253.22
63	590	256.19
1044	800	300.00
1045	700	250.00

Scheme 1: Shedding top 5 largest reactive load bus.
Nearly three hundred iterations of load increment, $\sigma_n(J_R)$ is less than ϵ setting. Thereby, the load shedding scheme is applied to the system. The selected criterion is to shed five largest reactive loads and the minimum singular value of the system is as shown in Fig 4.

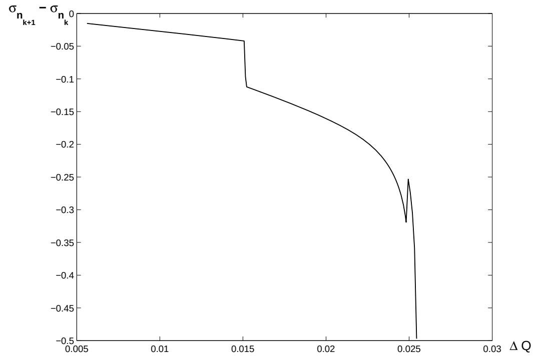

Fig. 4. $\Delta\sigma_n(J_R)$ when shedding largest reactive load

According to Fig 4, shedding the five largest loading does not *buy* more time before $\Delta\sigma_n(J_R)$ declines to zero or in other words, power flow cannot be converged. This implies that system will become unstable at the same rate of change in minimum singular value, $\sigma_{\mathbf{n_{k+1}}} - \sigma_{\mathbf{n_k}}$, as without the load shedding.

Scheme 2: Shedding the largest magnitude of $\underline{P}_n$ load.
As described in Section III, $\underline{v}_n$ and $\underline{u}_n$ corresponding to $\sigma_n(J_R)$, could be used to find the most sensitive voltage node of the system and the most sensitive direction in reactive power injection. Fig 5 illustrates the plot of $\underline{v}_n$ and $\underline{u}_n$.

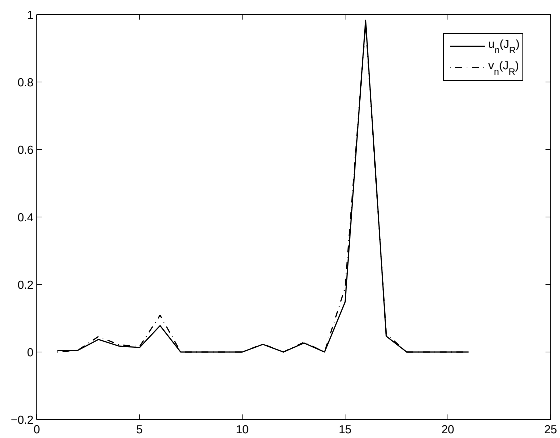

Fig. 5. Right and Left singular vector of Nordic 32 system represent the load increment

The plots of magnitude of the corresponding vectors, as seen in Fig 5, are specific to the load increment case. Both of them indicate only one bus that is sensitive voltage bus of the system. Also, the plot of $\underline{v}_n$ and $\underline{u}_n$ almost coincide. This implies that the more of change in reactive power of that node, the more affect to the voltage stability indices.

Therefore, shedding the load of this node would have the most beneficial effect on system. The result has been confirmed by the magnitude of participation vector (see Table II).

TABLE II
PARTICIPATION VALUE OF SOME LOAD BUSES

Bus	$\underline{P}_n(J_R)$
1041	0.9555
1045	0.0283
4045	0.0085
1044	0.0025

Fig 6 depicts the Bus 1041, which has P_{Load} and Q_{Load} equal to 600 MW and 200 MVAR, respectively, is shed from the system. This plot illustrates that system has some time allowance to solve voltage instability problem before system collapse which also implied that load shedding by considering $\underline{P}_n$ is more appropriate than shedding several largest consumption load buses.

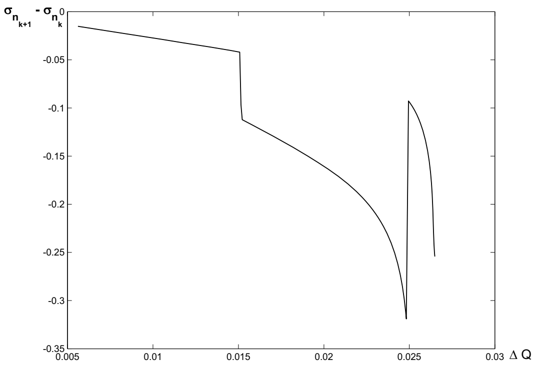

Fig. 6. $\Delta\sigma_n(J_R)$ when shedding largest magnitude of $\underline{P}_n$ load

As seen in Fig. 3, the $\sigma_n(J_R)$ plot corresponds the similar behavior as the upper (stable) part of a nose curve [13]. The nose curve can be described by a polynomial function defined as

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 \quad (15)$$

The proposed technique for predicting instability, referred to as Reactive Power critical point (Q_{LCr}), consists of mapping the $\sigma_n(J_R)$ plot with the polynomial expression.

The same simulation is computed again. However instead of continuing the same procedure until power flow is no longer converted, simulation stops when $\sigma_n(J_R)$ reach 50% reduction of its steady state value. Next, three coordinate points of $\sigma_n(J_R)$ and Q_L are selected (Table III shows an example of the selected coordinate of $\sigma_n(J_R)$ and Q_L). The selected points are placed them in polynomial function, thus one will get

$$\sigma_{n1}(J_R) = a_2 Q_{L1}^2 + a_1 Q_{L1} + a_0$$
$$\sigma_{n2}(J_R) = a_2 Q_{L2}^2 + a_1 Q_{L2} + a_0$$
$$\sigma_{n3}(J_R) = a_2 Q_{L3}^2 + a_1 Q_{L3} + a_0$$

TABLE III
SELECTED $\sigma_n(J_R)$ AND Q_L

$\sigma_n(J_R)$	$Q_L(p.u.)$
19.1347	2.8684
15.5470	3.4497
12.1984	3.8884

These equations in matrix form become:

$$\begin{bmatrix} \sigma_{n1} \\ \sigma_{n2} \\ \sigma_{n3} \end{bmatrix} = \begin{bmatrix} 1 & Q_{L1} & Q_{L1}^2 \\ 1 & Q_{L2} & Q_{L2}^2 \\ 1 & Q_{L3} & Q_{L3}^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

Therefore,

$$[a] = [Q_L]^{-1}[\sigma_n]$$

Finally, the elements from $[a]$ are used and the critical reactive power can now be determined as following

$$\sigma_{nCr} = 0 = a_2 Q_{LCr}^2 + a_1 Q_{LCr} + a_0$$

$$Q_{LCr} = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2 a_0}}{2a_2}$$

Fig 7 shows the graphical comparison of minimum singular value between proposed instability prediction and the pre-simulated result.

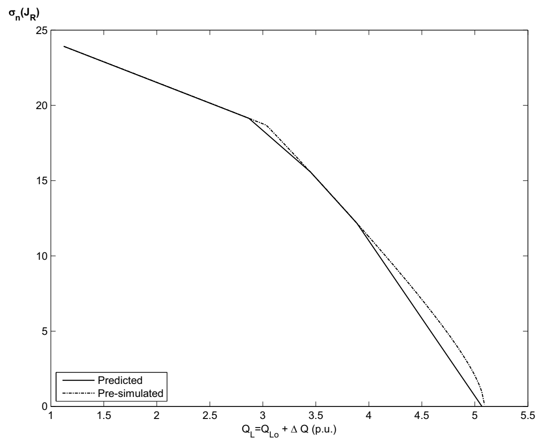

Fig. 7. Predicted technique VS Pre-simulation

The critical reactive power of pre-simulation is 5.0902 p.u. whereas Q_{nCr} is equal to 5.0670 p.u. (for a_2=21.1027, a_1=3.8575, and a_0= -1.5832) in case of proposed technique. Both methods generate very small difference. This implies that the predicted technique is not only valid but also gives a huge advantage to determine the stability limit of the system ahead of time before system collapse.

VI. CONCLUSIONS

This paper revises the use of Singular Value Decomposition in voltage stability index for the estimation of the voltage stability. The *early warning* signal obtained by using stability index is used to devise necessary actions to counteract or reduce risk of cascading failures induced by slow voltage instability. The proposed idea is validated in the Nordic 32 benchmark system. The obtained results indicate that the information extracted in the voltage stability indices can be sufficient to take preventive measures (such as load shedding) in order to reduce risk of encountering a cascading failure. In addition, the technique for predicting instability is proposed for aiding system operators to forecast stability limit when system is perturbed by disturbance.

VII. Future work

The optimal (minimize necessary) load shedding of the focal nodes based on the voltage stability indices seems to be an interesting area to study. Then, the validation of the proposed optimal shedding can be done in a larger scale detailed power system model and/or validate the results in a real time simulator. Moreover since load shedding conflicts to economic issue [14], this might draws an intention of using edged-technology electronic devices, e.g. HVDC to maximize systems capacity. Sequentially to regulate systems back to the operational and/or acceptable level (in engineering aspects) as before the occurrence of disturbance.

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