In this paper, a new method of information fusion – DSmT (Dezert and Smarandache Theory) is introduced to apply to managing and dealing with the uncertain information from robot map building. Here we build grid map form sonar sensors and laser range finder (LRF). Aiming to the uncertainty in static environment, we propose Classic DSm (DSmC) model for sonar sensors and laser range finder, and construct the general basic belief assignment function (gbbaf) respectively. Generally speaking, the evidence sources are unreliable in physical system, so we must consider the discounting theory before we apply DSmT. At last, Pioneer II mobile robot serves as a simulation experimental platform. We build 3D grid map of belief layout, then mainly compare the effect of building map using DSmT and DST. Through this simulation experiment, it proves that DSmT is very successful and valid, especially in dealing with highly conflicting information. In short, this study not only finds a new method for building map under static environment, but also supplies with a theory foundation for us to further apply Hybrid DSmT (DSmH) to dynamic unknown environment and multi-robots’ building map together.

**Abstract**—In this paper, a new method of information fusion – DSmT (Dezert and Smarandache Theory) is introduced to apply to managing and dealing with the uncertain information from robot map building. Here we build grid map form sonar sensors and laser range finder (LRF). Aiming to the uncertainty in static environment, we propose Classic DSm (DSmC) model for sonar sensors and laser range finder, and construct the general basic belief assignment function (gbbaf) respectively. Generally speaking, the evidence sources are unreliable in physical system, so we must consider the discounting theory before we apply DSmT. At last, Pioneer II mobile robot serves as a simulation experimental platform. We build 3D grid map of belief layout, then mainly compare the effect of building map using DSmT and DST. Through this simulation experiment, it proves that DSmT is very successful and valid, especially in dealing with highly conflicting information. In short, this study not only finds a new method for building map under static environment, but also supplies with a theory foundation for us to further apply Hybrid DSmT (DSmH) to dynamic unknown environment and multi-robots’ building map together.

**Keywords**—Map building, DSmT, DST, Uncertainty, Information fusion.

**I. INTRODUCTION**

The study on exploration of entirely unknown environment for intelligent mobile robots has been a popular and difficult subject for experts in robot community for a long time. Because of the contradiction between self-localization and map building, some one compares it as a chicken and egg puzzle[1],[2]. To solve this puzzle, some experts in this field have proposed many methods in self-localization and map building respectively, for example, EM[3], Markov[4] or Monte Carlo [5] self-localization, grid map[6],[7], geometrical feature or topological map, etc. Of course, only SLAM (simultaneous localization and mapping) [1] can solve really this puzzle. Presently many experts and scholars in this field are doing some research on it. Because map building is an important loop of SLAM, here we apply DSmT and DST (Dezert-Smarandache Theory) to grid map building, which is one of the most successful expression methods in building map. DST proposed by Dempster and Shafer[8] since 1976 has been widely applied since Professor Smets proposed a model of TBM (Transfer Belief Model)[9], which offered an explanation on it. Due to information acquired in grid map building presents characteristics of uncertainty, imprecision and even high conflict. It is very difficult for DST to deal with highly conflicting information, because its conflict factor can’t be one. DSmT proposed by Jean Dezert (French) and Florentin Smarandache (American) based on Bayesian theory and DS theory come forth since 2003, which is a general, flexible and valid arithmetic of fusion [10] – [12].

In this paper, we mainly introduce Classic DSmT based on discounting theory to fuse the information from different reliable evidence resource within the static environment, (e.g. here referring to sonar and laser). Of course, Aiming to the dynamic environment, for example, walking person, moving objects, and so on, we may consider the Hybrid DSmT (DSmH) model [10]. And even we can apply DSmH to multi-robot’s building map together.

**II. INTEGRATING UNRELIABLE SOURCES WITH DSmT**

DSmT is a new, general and flexible arithmetic of fusion, which can solve the fusion problem of different tiers including data-tier, feature-tier and decision-tier, and even not only can dispose the static problem of fusion, but also can dispose the dynamic one. Especially, it has a prominent merit that it can deal with the uncertain and highly conflicting information [10] – [12].

**A. Simple Review of DSmT**

1) Let $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$, here $\Theta$ is the frame of discernment, which includes $n$ finite focal elements $\theta_i (i = 1, \cdots, n)$. Because the focal element is not precisely defined and separated, so that no refinement of $\Theta$ in a new larger set $\Theta_{ref}$ of disjoint elementary hypotheses is possible.
2) The hyper-power set $D^\Theta$ is defined as the set of all compositions built from elements of $\Theta$ with $\bigcup$ and $\cap$ ($\Theta$ generates $D^\Theta$ under operators $\bigcup$ and $\cap$) operators such that

a) $\emptyset, \theta_1, \theta_2, \ldots, \theta_n \in D^\Theta$

b) If $A,B \in D^\Theta$, then $A \cap B \in D^\Theta$ and $A \cup B \in D^\Theta$

c) No other elements belong to $D^\Theta$, except those obtained by using rules a) or b).

3) General belief function and Plausibility function

Let $\Theta = \{\theta_1, \theta_2, \ldots, \theta_n\}$ is the general frame of discernment. For every evidential source $S$, let us define a set of map of $m(\cdot)$: $D^\Theta \rightarrow [0,1]$ associated to it (abandoning Shafer’s model) by assuming here that the fuzzy/vague/relative nature of elements $\theta_i, (i = 1,2,3\ldots n)$ can be non-exclusive, as well as no refinement of $\Theta$ into a new finer exclusive frame of discernment $\Theta_{ref}$ is possible. The mapping $m(\cdot)$ is called a generalized basic belief assignment function (gbbaf), if it satisfies[10] $m(\emptyset) = 0$ and $\sum_{A \in D^\Theta} m(A) = 1$. $m(A)$ is called A’s generalized basic belief assignment function (gbbaf). The general belief function and plausibility function are defined respectively in almost the same manner as within the DST, i.e.

$$\text{Bel}(A) = \sum_{B \subset A, B \in D^\Theta} m(B)$$

$$\text{Pl}(A) = \sum_{B \cap A \neq \emptyset, B \in D^\Theta} m(B)$$

4) Classic (free) DSmT rule of combination

Let $M^f(\Theta)$ is a free model of DSmT, and then the Classic (free) DSmT rule of combination for $k \geq 2$ sources is given as follows:

$$m_{M^f(\Theta)}(A) = \sum_{X_1 \cap \ldots \cap X_k \in D^\Theta} \prod_{i=1}^{k} m_i(X_i)$$

### III. MODELING ON SENSORS’ UNCERTAINTY

There are lots of sensors on Pioneer II mobile robot shown in Fig. 1, e.g., interoceptive sensors (odometer, electronic compass) and exteroceptive sensors (sonar sensor, laser range finder and visual sensor, etc.). Map building mainly relies on exteroceptive sensors. Here we build grid map with sonar sensors and laser range finder.

#### A. Analysis on Uncertainty from Sensors

Uncertain information mainly comes from sonar sensors. Of course, there is still some uncertain information coming from laser range finder. We analyze the uncertainty as follows:

1. Uncertainty from Sonar Sensors
Sonar sensors’ working principle (as shown in Fig. 2) is: producing sheaves of cone-shaped wave, and detecting the objects by receiving the reflected wave. Due to the restriction of sonar physical characteristic, metrical data behaves out uncertainty as follows:

a) Beside its own error of making, the influence of external environment is also very great, for example, temperature, humidity, atmospheric pressure and so on.

b) Because the sound wave spreads outwards in the form of loudspeaker, and there exists a cone-shaped angle, we cannot know the true position of object detected among the fan-shaped area, with the enlargement of distance between sonar and it.

c) The use of lots of sonar sensors will result in interference with each other. For example, when the i-th sonar gives out detecting wave towards an object of irregular shape, if the angle of incidence is too large, the sonar wave might be reflected out of the receiving range of the i-th sonar sensor or also might be received by other sonar sensors.

d) Because sonar sensors utilize the reflection principle of sound wave, if the object absorbs very heavy sound wave, the sonar sensor might be invalid.

2. Uncertainty from Laser Range Finder

There is a laser range finder installed to Pioneer II mobile robot, which only scans the horizontal plane. The resolution used is 1.0 degree per point (i.e., 181 measurements for 180 degrees). The accuracy of each measurement is ±5cm. In fact, its accuracy is very high. However, some uncertainty still appears, which is analyzed as follows:

a) If the objects are very small, and just is in the middle of the interval of twice measurements for every scan, then laser range finder won’t find it. Maybe it is detected by another scans. Aiming to the grid occupied by the small object, when we integrate this information from different scans, uncertainty and high conflict will happen.

b) If there are some transparent objects (i.e., glass) in the unknown environment, which can’t reflect light, or there are mirrors or very smooth objects, which can make laser produce surface-reflecting phenomenon. Under this condition, Laser range finder can’t detect them.

c) When laser range finder is scanning the environment, the mobile robot runs very fast, or it can’t give precise location and even gets lost, then uncertainty will happen.

d) Though here we don’t consider the dynamic environment, in fact, walking man or moving objects will also influence the measurement and produce uncertainty.

B. Modeling for Sonar Information based on DSmT

Pointing to the characteristics of sonar’s measurement, we construct a model of uncertain information acquired from grid map using sonar based on DSmT. Here we suppose there are two focal elements in system, that is, \( \Theta = \{ \theta_1, \theta_2 \} \), here \( \theta_1 \) means grid is empty, \( \theta_2 \) means occupied, and then we can get its hyper-power set \( D^\Theta = \{ \emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2 \} \). Every grid in environment is scanned \( k \geq 2 \) times, each of which is viewed as source of evidence. Then we may define a set of map aiming to every source of evidence and construct the general basic belief assignment functions (gbbaf) as follows: \( m(\theta_1) \) is defined as the gbbaf for grid-unoccupied (empty); \( m(\theta_2) \) is defined as the gbbaf for grid-occupied; \( m(\theta_1 \cap \theta_2) \) is defined as the gbbaf for holding grid-unoccupied and occupied simultaneous (conflict). \( m(\theta_1 \cup \theta_2) \) is defined as the gbbaf for grid-ignorance due to the restriction of knowledge and present experience (here referring to the gbbaf for these grids still not scanned presently ), it reflects the degree of ignorance of grid-unoccupied or occupied.

The gbbaf of a set of map \( m(\cdot): D^\Theta \rightarrow [0,1] \) is constructed by authors such as the formulae (5)–(8) according to sonar physical characteristics.

\[
m(\theta_1) = E(\rho)E(\theta) = \begin{cases} \|1-(\rho/R)^2\|^{\lambda} & \text{if } R_{\text{min}} \leq \rho \leq R_{\text{max}}, \\ 0 & \text{otherwise} \end{cases}
\]

\[
m(\theta_2) = O(\rho)O(\theta) = \begin{cases} \exp\left(-3\lambda(\rho-R)^2\right)^{\lambda} & \text{if } R_{\text{min}} \leq \rho \leq R_{\text{max}} \\ 0 & \text{otherwise} \end{cases}
\]

\[
m(\theta_1 \cap \theta_2) = \begin{cases} \tanh(2(\rho-R))^{\lambda} & \text{if } R \leq \rho \leq R_{\text{max}} \\ 0 & \text{otherwise} \end{cases}
\]

where \( \lambda = E(\theta) = O(\theta) \) is given by (see[6] for justification)

\[
\lambda = E(\theta) = O(\theta) = \begin{cases} \|1-2\theta\alpha^2\|^{\lambda} & \text{if } 0 \leq \theta \leq \alpha/2 \\ 0 & \text{otherwise} \end{cases}
\]

Where, \( \rho \) in formula (6) is defined as environment adjusting variable, that is, the fewer the object is in environment, the greater the variable \( \rho \), is, and makes the function of \( m(\theta_2) \) more sensitive. Here let \( \rho \) be one. \( E(\cdot), O(\cdot) \) is expressed as the Effect Function of \( \rho, \theta \) to grid’s empty or occupancy. In order to assure the sum of all masses to be one, we must renormalize it. The analysis on the characteristics of gbbaf are shown as Fig3–Fig7, when \( R=1.5m \).

Seen from Fig. 3, \( m(\theta_1) \) has a falling tendency with the addition of distance between grid and sonar, and has the maximum at \( R_{\text{max}} \) and zero at \( R \). From the point of view of the working principle of sonar, the more the distance between them approaches the measured value, the more that grid might be occupied. Thus the probability that grid indicated is empty is very low, of course the gbbaf of grid–unoccupied is given low value.

\( m(\theta_2) \) takes on the distribution of gaussian function with respect to the addition of distance between them, has the maximum at \( R \), which answers for the characteristic of sonar acquiring information.
IV. Ego-Motion Estimation with the LRF Data

To assure the correctness of map building, Self-localization for the mobile robot is very necessary, especially in the large, complex, and even loop environment. Because the mobile robot might have no precise location or lose itself in the environment due to using odometry-based dead reckoning, which might cause the accumulated error as the robot moves. Under this condition, ego-motion estimation using external sensors (vision, LRF or sonar sensors) is very necessary. Here we adopt the LRF data for ego-motion estimation thanks to its reliability and accuracy, as long as the number of features in the environment is enough to be extracted and match.

Because in this paper we mainly discuss map building, we only give a simple introduction to Ego-motion as follows:

\[ m(\theta_1 \cap \theta_2) \] takes on the distribution of parabola function with respect to the addition of distance between them. In fact, when \( m(\theta_1) \) equals \( m(\theta_2) \), \( m(\theta_1 \cap \theta_2) \) has the maximum there. But it is very difficult and unnecessary to find the point of intersection of the two functions. Generally, we let the position of R-2 replace the point of intersection. Experience indicates that its approximate value is more rational.

\[ m(\theta_1 \cup \theta_2) \] takes on the distribution of hyperbola function with respect to the addition of distance between them, and zero at R. This function reflects well the ignorance of grid information at \( R \leq \rho \leq R_{\max} \).

The relation between \( \theta \) and \( \lambda \) is reflected in Fig 4, where the more the position of grid approaches central axis, the greater \( \lambda \) becomes, that is, the greater the contribution to belief assignment is. Otherwise, the lower it is.

In short, the general basic belief assignment functions (gbbaf) entirely fit with the characteristic of sonar acquiring information. This supply with a theoretic foundation for dealing with uncertain information in grid map building.

C. Modeling for Laser Information

The measurement of the LRF is fairly reliable and the reliability does not basically depend on the distance to objects or obstacles. we suppose that the accuracy of each measurement is \( \pm 5 \text{cm} \), that is, we take the grids in the close interval \([R-5, R+5]\) as occupied, and take the grids in the open interval \((0, R-5)\) as free. Other is taken as unknown. Though, we know that, the measurement’s accuracy of LRF accords with normal school. To simply calculate, we may define the basic belief assignment as follows:

\[
m_{\text{EgoUT}}(\theta_1) = \begin{cases} 
0.9 & 0 < \rho < R - 5 \\
0.05 & R - 5 \leq \rho \leq R + 5 
\end{cases}
\]

\[
m_{\text{DisUT}}(\theta_2) = \begin{cases} 
0.05 & 0 < \rho < R - 5 \\
0.9 & R - 5 \leq \rho \leq R + 5 
\end{cases}
\]

\[
m_{\text{DisUT}}(\theta_1 \cup \theta_2) = \begin{cases} 
0.05 & 0 < \rho < R - 5 \\
0.05 & R - 5 \leq \rho \leq R + 5 
\end{cases}
\]

Known from Fig. 5, the ego-motion \((\Delta x, \Delta y, \Delta \theta)\) between two consecutive frames is estimated as follows. Let \((X_i, Y_i)\) and \((X'_i, Y'_i)\) be the \(i\)th matched pair of features \((i=1,2,\ldots,N)\). Supposed there doesn’t exit error, then the following equation must hold.

\[
\begin{bmatrix}
\cos \Delta \theta & \sin \Delta \theta & X' \\
-\sin \Delta \theta & \cos \Delta \theta & Y'
\end{bmatrix}
= 
\begin{bmatrix}
X_i \\
Y_i
\end{bmatrix}
\]

Here we take the line segments, which come from flat walls, regular smooth objects, and so on. So we may estimate the function, and make it minimize.

\[
S = \sum_{i=1}^{N} \left[ \left( X_i - \cos \Delta \theta Y_i + \sin \Delta \theta + \Delta x \right)^2 + \left( Y_i - \left( -X_i \sin \Delta \theta + Y'_i \cos \Delta \theta + \Delta y \right) \right] \right.
\]
Here $S$ expresses the sum of some matched pair of features on the line segments. To minimize the function $S$, the ego-motion must satisfy the following equations:

$$\frac{\partial S}{\partial \Delta x} = 0, \quad \frac{\partial S}{\partial \Delta y} = 0, \quad \frac{\partial S}{\partial \Delta \theta} = 0$$  \hspace{1cm} (15)

So $\Delta x$, $\Delta y$, $\Delta \theta$ are obtained respectively through the above equation (15).

V. ANALYSIS ON THE SIMULATION EXPERIMENTAL RESULT

In this study, we develop a visual interface as a software platform for experiment by ourselves with Visual C++ 6.0 and OpenGL as shown in Fig. 6.

A. The fusion of Sensors Information

Pioneer II mobile robot has 16 sonar sensors. Seen from Fig. 7, there are just 8 front sonar sensors shown, their distribution is asymmetrical. LRF is mentioned in the part B of section 3. Flow chart of procedure of robot sonar map building based on DSmT is shown in Fig. 8.

1) At the beginning of procedure, we proposed that all grids are fully occupied, that is, $Bel(\theta_1) = 1$, and mobile robot begins from certain location in the environment to move, and explore the unknown indoor room. Therefore, we may need some tactics to plan the exploring route of the mobile robot. Here, we can adopt the tactics of “Z”. Of course, mobile robot’ path planning and avoiding obstacles must be considered, but it is not an important point herein. We are more concerned about the building map. Robot may get the information of all sonar sensors and LRF at spot ($i,j$). For simplified calculation, we apply the arithmetic of restricted spreading, which only computes the grid information in the fan-shaped area that each sonar or LRF can scan (shown in Fig 2) [15]. At the same time we also suppose that each sonar sensor and LRF have different reliable degree, i.e. the $i$-th sonar’s one is $p_i$ ($i=1$~$16$) and LRF’s one is $1$. Here $p_i$ and $p_i$ are acquired by experiment, that is, by judging its occurrence of malfunction. Of course, there is a rule: if the information from the different sonar is fused, the corresponding reliable degree ought to renormalize.

2) Utilizing the model of uncertainty belief established in the section 4, through the formulas (5)~(9), sonar sensors’ $g_{bbaf}$ (such as $m(\theta_1) \cdot m(\theta_2) \cdot m(\theta_1 \cap \theta_2)$ and $m(\theta_1 \cup \theta_2)$) is respectively computed. However, for LRF, we can get its $g_{bbaf}$ from the formulas (11)~(13). If sum of masses is not one, then we should renormalize it.

3) Judge whether the information of every grid scanned by all sonar sensors or LRF is new or not. If yes, then goto step 1. Otherwise, goto next step.

4) Judge whether the grid is scanned repeatedly or not. If No, save the information of this grid. If Yes, goto step 5

5) Go on judging whether the fusion times is more than two times or not. If yes, then stop fusing it. Otherwise, goto step 6.

6) Go on fusing it and at the same time, judge further. Whether the grid’s information is fused for the first time or not. If it is not the first time, then compute and update the Bel at last. Then goto step 7. if it is the first time, goto step 8.

7) Check whether Bel of all grids to needs be updated or not. If Yes, then goto step 9. Otherwise, goto step 1.

8) Update the grid’s original mass with the new mass after fusion. Then goto step 1.

9) Stop and Exit the procedure at last.

B. Result

Here we design an original environment as shown Fig.9. Of course, we build map of the environment with the grid method. The global environment is the environment (size: 20m × 20m) partitioned in 40000 discrete even rectangular grids (200 × 200).
information from sonar sensors and LRF, in order to build the
grid map for the mobile robot. Here we give the 3D grid map of
belief layout based on DSmT as shown in Fig. 9. To compare
more clearly the effects between them, we can transform the 3D
grid map into 2D grid one in Fig. 10 and Fig. 11, where the
difference marked by the authors is very distinct.

The analysis on the result of fusion:

a) Fig. 10 and Fig. 11 show the result of building map online
with DSmT based on discounting theory for Pioneer II mobile
robot. In Fig. 10, Z axis shows the belief of every grid
occupied, zero shows the grid is empty fully, however, one
shows this grid is fully occupied. “S” refers to sonar sensors,
and “L” refers to LRF in the caption of Fig. 10. Though the
discerning rate for only integrating the sonar information is
very high in small and simple environment, with the increment
of the complexity of the environment, the discerning rate might
decrease. Here due to integrating LRF and sonar information
together, the discerning rate is improved distinctly. This
facilitates very much the development of human-computer
interface of mobile robot exploring unknown, dangerous and
sightless area.

b) Low coupling. Though there are many objects in grid map,
but there occurs no phenomenon of the apparently severed, but
actually connected. Thus, it supplies with a powerful evidence
for self-localization, path planning and navigation of mobile
robot.

c) High validity of calculation. The discounting approach
processed for DSmT fusion rule considering the restrained
spreading arithmetic is adopted, and overcomes the
shortcoming that the global grids in map must be reckoned once
for sonar scanning every time, and improves the validity of
calculation.

d) In this paper, we just apply Classic model of DSmT to
static environment. While aiming to dynamic environment such
as moving object and walking person therein, we must consider
the hybrid model of DSmT.

C. Comparison from the fusion result between DSmT and
DST

1) Though both DSmT and DST can be applied to building
grid map from sonar sensors, DSmT can express more clearly
the grid information than DST, which can be seen easily from
the comparison between Fig. 11 and Fig. 12 in terms of the grid
map of original environment in Fig. 8. There are some
imperfect places marked by the authors with the ellipses of
broken line in the Fig. 12.

2) DSmT can deal with highly conflictive information, while
DST can’t do. So when we apply DST to grid map building, it
might be fail. Under this situation, we sometimes have to
abandon this evidence source, try another one. If this, the total
amount of computing will increase, the time to finish building
global grid map also increase, and even sometimes the global
grid map can’t be built or is a mistaken one.

3) In this study, though we define two focal elements in either
DSm model or D-S one, according to their combination rule,
DSmT should have greater amount of computing, however, in
fact it is inverse. We analyze, it must take a deal of time to deal
with high conflict with DST. If we suppose the discernment
frame might be refined further, in order to start with the same
information as DSmT, we must define three focal elements in
D-S model, and then the amount of computing is obviously
greater than that of DSmT[11].

VI. CONCLUSION

In this paper, we applied both DSmT and DST to mobile
robot’s map building in a static environment. Through the
simulation experiment, DSmT proved to be more valid than
DST finally. Especially, when there are desks and tables in very
large and complex, or irregular and even loop environment, because the sonar and LRF only acquire the information from a horizontal plane, we must integrating other sensors together (e.g. vision sensor, etc) with DSmT other than DST. In short, this study supplied with a shortcut for human-computer (e.g. vision sensor, etc) with DSmT other than DST. In short, we must integrating other sensors together because the sonar and LRF only acquire the information from a large and complex, or irregular and even loop environment, and established a firm foundation for us to apply Hybrid DSmT to dynamic unknown environment and multi-robots’ building map together.

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i=0, let Bel(θ_2)

i-th spot of acquiring information

Get sensors’ datum

Sonar data

LRF data

Robot’s location in global coordinate

Ego-motion estimation

Compute the gbba of every grid scanned by all sensors

i+1

Y

Is information of all grids new?

N

Save

Is the grid in the intersection where either sonar or LRF can scan?

Y

Is fusion times of this grid <= 2?

N

Stop fusing it

Y

Go on fusing it and check: Is the fusing the first times?

Compute Bel

Replace prior gbba of grid with mass after fusion

N

Is Bel of all grids computed?

Y

Exit

Fig. 8 Flowchart of procedure of sonar map building based on DSmT