Calculation of Voided Slabs Rigidities

Gee-Cheol Kim, Joo-Won Kang

Abstract—A theoretical study of the rigidities of slabs with circular voids oriented in the longitudinal and in the transverse direction is discussed. Equations are presented for predicting the bending and torsional rigidities of the voided slabs. This paper summarizes the results of an extensive literature search and initial review of the current methods of analyzing voided slabs. The various methods of calculating the equivalent plate parameters, which are necessary for two-dimensional analysis, are also reviewed. Static deflections on voided slabs are shown to be in good agreement with proposed equation.

Keywords—voided slab, bending rigidity, torsional rigidity, orthotropic plate.

I. INTRODUCTION

An orthotropic plate is defined as one which has different rigidities in two orthogonal directions. In general two forms of orthotropic plate are identified, namely, material orthotropic plate and shape orthotropic plate. Most of the actual orthotropic plates are of the latter type, as like ribbed slabs and voided slabs. As voided slabs, voids running in the longitudinal direction are frequently in traduced into concrete slabs in order to reduce their self weight. Such voids are often of circular shape because they are simple then to fabricate, and it is relatively easy to ensure that compaction of the concrete under the void takes place during casting. Circular voided slabs of this nature are used both for floor slabs and for medium span slab bridge.

An approach commonly used is to assume the concrete is uncracked and linearly elastic, and thus ignore the reinforcement. This approach has the advantage of simplicity and of closely modeling the behavior of a slab. And the concept of converting an actual slab into an equivalent orthotropic plate for the purpose of determining the distributions of stresses is well established. Numerous investigators have suggested expressions of the determination of these bending rigidities but few have compared them. In this paper, the bending rigidities are derived from finite element analyses of cross sections of voided slabs, with the voids symmetrical with respect to the slab middle surface. And this paper summarizes the results of literature research and the review process, giving details of the various methods of analysis and calculating the properties of simplified mathematical models.

II. GOVERNING EQUATION OF PLATE

The governing equation of the plate ignoring the extensibility of the middle surface is given by

$$D_y\frac{\partial^4w}{\partial x^4} + (D_{xy} + D_{yy} + D_x + D_z)\frac{\partial^4w}{\partial x^2\partial y^2} + D_z\frac{\partial^4w}{\partial y^4} = p(x,y)$$  (1)

In which $D_x$ and $D_y$ denote the bending rigidities, $D_{xy}$ and $D_{yz}$ are the torsional rigidities and $D_t$ and $D_z$ are the coupling rigidities. The values of solid slab are defined as follows;

Bending rigidity is

$$D = \frac{E t^3}{12(1-\nu^2)}$$  (2)

Torsional rigidity is

$$D_{xy} = \frac{G t^3}{12}$$  (3)

The elasticity modulus, shear elasticity modulus and Poisson’s ratio are denoted by $E$, $G$ and $\nu$ respectively.

It becomes necessary to calculate the equivalent parameters for a two-dimensional analysis. The various parameters may be required for all two-dimensional analyses of voided slab. The orthotropic plate rigidities are required for the orthotropic plate and two dimensional finite element analyses.

The cross section and the notation of voided slab are as shown in Fig. 1.

III. BENDING AND TORSIONAL RIGIDITIES

Bending rigidities($D_x$ and $D_y$) in the longitudinal direction and in the transverse direction and torsional rigidities($D_{xy}$ and $D_{yz}$) can be obtained by treating the structure as equivalent stiffnesses.

For uncracked concrete voided slabs, [1] proposed the following equation for calculation of $D_x$ and $D_y$.

$$D_t = E \left[ \frac{t}{12} - \frac{\pi d^4}{64P_y} \right]$$  (4)

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Bending rigidities in the longitudinal direction is specified in the Ontario Highway Bridge Design Code (1975).

Elliott (1975) concludes that the spacing of voids has little effect on $D_x$ and $D_y$. These rigidities can be obtained by the following equation without incurring any significant error.

$$D_x = \frac{E I}{12} \left[ 1 - 0.59 \left( \frac{d}{t} \right)^3 \left( \frac{d}{P} \right)^2 \right]$$

(5)

$$D_y = \frac{E I}{12 \left( 1 + \nu \right)} \left[ 3n \left( \frac{d}{d_v} \right)^2 \left( \frac{1}{1 - \nu^2} \right) \left( \frac{1}{1 - \nu^2} \right) \right]$$

(13)

where $n$ is the number of holes.

[5] have recommended formulate which give good comparisons with numerical and experimental studies. The equations for stiffness, $D_x$, $D_y$, $D_{xy}$, and $D_{yx}$, are given by

$$D_x = \frac{E I}{P_y \left( 1 - \nu^2 \right)}$$

(14)

$$D_y = \frac{E I}{12 \left( 1 - \nu^2 \right)} \left[ 1 - \left( \frac{d}{t} \right)^4 \right]$$

(15)

$$D_{xy} = 12(1 + \nu) \left[ 1 - 0.85 \left( \frac{d}{t} \right)^4 \right]$$

(16)

[6] proposed the following equation for calculation of $D_x$ and $D_y$ for orthotropic plate with rib.

$$D_x = \frac{E I_x}{P_x \left( t - t_i \right)}$$

(17)

$$D_y = \frac{E I_y}{12 \left( P_y - t_i \right) \left( t / (t - d) / 2 \right)}$$

(18)

Kim(2002) proposed the following equation for calculation of $D_x$ and $D_y$. These rigidities can be obtained by the moment of inertia of voided section.

$$D_x = \frac{E I_x}{P_x \left( t - t_i \right)}$$

(19)

$$D_y = \frac{E I_y}{P_y \left( t - t_i \right)}$$

(20)

Park(2011) proposed the following equation for calculation of $D_x$ and $D_y$. These rigidities can be obtained by the ratio of deflection.

$$D = \frac{E_n I}{12 \left( 1 - \nu^2 \right)}$$

(21)

where $E_n$ is the deflection ratio of voided slab with solid slab.

Table I presents the rigidities of voided slab that have suggested by literature research.
IV. EVALUATION OF RIGIDITIES

Static deflections are compared in order to verify the rigidities proposed by numerous investigators. Numerical example voided slab is shown in Fig. 2.

![Fig. 2 Numerical example voided slab](image)

The dimension of numerical example voided slab is 3000×4000×210 and the properties are shown in Table II. With the slab fixed supported at four edges, the concentrated load is applied at center point of example slab.

<table>
<thead>
<tr>
<th>TABLE I RIGIDITIES OF VOIDED SLAB</th>
<th>Unit: kN/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aster</td>
<td>Elliott</td>
</tr>
<tr>
<td>(D_x)</td>
<td>1.637E+07</td>
</tr>
<tr>
<td>(D_y)</td>
<td>1.636E+07</td>
</tr>
<tr>
<td>Elliott &amp; Clark</td>
<td>Ugural</td>
</tr>
<tr>
<td>(D_x)</td>
<td>1.684E+07</td>
</tr>
<tr>
<td>(D_y)</td>
<td>1.605E+07</td>
</tr>
</tbody>
</table>

![Fig. 3 Static deflection on the longitudinal line-self weight](image)

![Fig. 4 Static deflection on the transverse line-self weight](image)

The results are presented in the form of deflection as shown in Fig. 3 and Fig. 4 which are the deflection on the longitudinal and transverse center line. By comparisons with the deflection, the validity of the proposed method for rigidities calculation is demonstrated.

The methods appear to be the most appropriate for calculating the various equivalent plate parameters. From this study the following conclusions can be drawn concerning the choice of elastic rigidities of circularly voided slabs for use in orthotropic plate theory. With this suggested expressions of bending, coupling and torsional rigidities, the deflections and stress resultants in circularly voided slabs predicted by orthotropic plate theory show good agreement with results obtained by using 3D solid element.

V. CONCLUSIONS

As there is an infinite number of possible void shape and sizes, a comprehensive tabulation of rigidities is not possible. To determine rigidities parameters of a general cross-section, the simplified method can be used. The various methods that have been used during so years for analysis of voided slabs. The equivalent plate parameters (\(D_x\), \(D_y\) and \(xyD\)) could have significant effects on the accuracy of the analysis. These parameters should, therefore, model the actual structures as closely as possible without their requiring complex calculations. The methods appear to be the most appropriate for calculating the various equivalent plate parameters. From this study the following conclusions can be drawn concerning the choice of elastic rigidities of circularly voided slabs for use in orthotropic plate theory. With this suggested expressions of bending, coupling and torsional rigidities, the deflections and stress resultants in circularly voided slabs predicted by orthotropic plate theory show good agreement with results obtained by using 3D solid element.

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