Absorption Center of Photophoresis within Micro-Sized and Spheroidal Particles in a Gaseous Medium

Wen-Ken Li, Pei-Yuan Tzeng, Chyi-Yeou Soong, and Chung-Ho Liu

Abstract—The present study is concerned with the absorption center of photophoresis within a micro-sized and spheroidal particle in a gaseous medium. A particle subjected to an intense light beam can absorb electromagnetic energy within the particle unevenly, which results in photophoretic force to drive the particle in motion. By evaluating the energy distribution systematically at various conditions, the study focuses on the effects of governing parameters, such as particle aspect ratio, size parameter, refractivity, and absorptivity, on the heat source function within the particle and their potential influences to the photophoresis.

Keywords—photophoresis, spheroidal particle, aspect ratio, refractivity, absorptivity, heat source function

I. INTRODUCTION

In optical radiation fields, it has been well known that the particles suspended in a fluid medium will experience a net force to cause the so-called photophoresis phenomenon observed first by Ehrenhaft in 1917 [1]. A particle in a gaseous medium subjected to an intensive light beam absorbs and scatters light and then turns the absorbed electromagnetic energy into thermal energy within the particle [2, 3]. Asymmetric distribution of the heat energy then becomes driving force of the particle photophoretic motion. The photophoretic force is related to what is called heat source function (HSF) indicating the temperature distribution within the particle, and the strength and direction of the force is characterized by an index named asymmetry factor defined by integrating the HSF [4].

One area of increasing interest is the photophoresis of non-spherical particles, since most particles have irregular and complex structures in a wide variety of application areas. Non-spherical particles are abundant in natural and artificial environments. Furthermore, our knowledge and understanding of how non-spherical particles scatter and absorb electromagnetic energy remains incomplete and in some respects unsatisfactory [5]. From an extensive literature survey on light scattering theory, however, it is found that only a few studies on spheroidal particles [6-9]. In fact, suspensions of non-spherical particles, such as spores and dust grains, are seem to be rather well approximated by spheroids [10]. Spheroidal particles have the important applications in various fields of science, such as the spheroid describe efficient the shape of raining drops [11] and many bacteria and micro-weeds have the spheroidal shapes [12]. Ou and Keh [13] studied photophoretic motion of spheroidal particles in gaseous media, but the physical mechanisms were not addressed thoroughly.

To solve photophoresis problem it is necessary to decide the direction and the magnitude of the photophoretic force depending on the HSF within the particle. A number of studies have been performed on calculating the internal electric field within the particle for spheres [4, 14-20] and cylinders [21-25]. Relatively, less attention has been focused on spheroidal particles in the literature [26, 27].

From the above literature review, it was found that the effects of optical properties of particles on the radiant-absorption in spheroidal particle photophoresis were not dealt with in details. The purpose of the research presented in this article is to discuss the calculation of the absorption center of photophoresis within radiation-absorbing spheroids in gaseous media. Then, a detailed parametric analysis is performed for further understanding of the underlying physical mechanisms. The results reveal that the increases in either particle size or absorptivity enhances the energy absorbed on the illuminated (leading) side and tends to generate positive photophoresis; while an increase in the particle refractivity enhances the HSF peak intensity. Moreover, by appropriately varying the governing parameters such as particle size and optical properties of particle, an enhancement in energy absorption of the leading surface and/or degradation of energy peak intensity on the trailing side, which tends to lead the particle motion from negative to positive photophoresis.
II. THEORETICAL BACKGROUND

As shown in Fig. 1, an isotropic homogenous and prolate spheroid with a complex refractive index $m_p$ suspended in a gaseous fluid with the refractive index $m_e$. The spheroid is irradiated by a linearly polarized plane electromagnetic wave incident normal to the z axis and an implicit time dependence of $\exp(-i\omega t)$ is assumed. The particle size parameter $\alpha$ is defined by $\alpha = 2\pi a/\lambda$, where $a$ is the semimajor axis of the ellipse, and $\lambda$ is the wavelength of the incident wave. The shape of the spheroid is specified by the aspect ratio $a/b$ of the semimajor axis $a$ to the semiminor axis $b$.

![Fig. 1. Physical model of a prolate spheroid photophoresis](Image)

The temperature distribution $T_p$ inside the radiation-absorbing particle is governed by the Poisson-Boltzmann equation,

$$\nabla^2 T_p = -\frac{Q}{k_p}$$

(1)

where $k_p$ is the thermal conductivity of the particle, and $Q$ the radiant-absorption heat generation function which can be related to the electric field $E$ within the particle. For a plane monochromatic incident light, $Q$ has the form \[14, 28,\]

$$Q = \frac{4\pi n_p \kappa_p I}{\lambda} \left| E_0 \right|^2$$

(2)

where $n_p$ and $\kappa_p$, respectively, are the refractivity and absorptivity in the refractive index of particle, $m_p = n_p + \kappa_p i$, $I$ the intensity of the incident light, and $E_0$ the incident electric field strength.

The T-matrix method is ideally suited for the computation of electromagnetic scattering by non-spherical particles \[29-31\]. The T-matrix solution begins with an expansion of the incident, scattered, and internal electrical fields in terms of the vector spherical harmonic functions basis $N^{(1)}$, $M^{(1)}$, $N^{(3)}$, and $M^{(3)}$ \[32\]. The HSF can be characterized by the magnitude of $\left| E \right|^2$, and the internal electrical field is expanded as \[33\]:

$$E^{\text{Int}}(m_a) = E_0 \sum_{l=0}^{\infty} \sum_{n=-l}^{l} \left[ c_n M^{(1)}_{n,l}(m_a) + d_n N^{(1)}_{n,l}(m_a) \right]$$

(3)

The internal field expansion coefficients $c_n$ and $d_n$ in the above equations are given by \[34\],

$$\begin{bmatrix} K + mJ & L + mL \end{bmatrix} \begin{bmatrix} c_n \\ I + mL \end{bmatrix} = \begin{bmatrix} -iA^{(n+1)} \frac{P_n^{(1)}(\cos \theta)}{\sin \theta} \\ -iA^{(n)} \frac{d}{d \theta} P_n^{(1)}(\cos \theta) \end{bmatrix}$$

(4)

where $I, J, K, L$ are the integrals which must be numerically evaluated over the surface of the particle and are defined as \[35\]

$$I = \frac{k^2}{\pi} \int_S \left[ \pi N^{(3)}(k r) \times M^{(3)}(k r) \right] dS$$

(5a)

$$J = \frac{k^2}{\pi} \int_S \left[ \pi N^{(3)}(k r) \times N^{(1)}(k r) \right] dS$$

(5b)

$$K = \frac{k^2}{\pi} \int_S \left[ \pi N^{(3)}(k r) \times M^{(1)}(k r) \right] dS$$

(5c)

$$L = \frac{k^2}{\pi} \int_S \left[ \pi N^{(3)}(k r) \times N^{(1)}(k r) \right] dS$$

(5d)

where $k = 2\pi/\lambda$, $\pi$ is the unit vector normal to the surface, and $r$ is the position vector from an internal origin to the particle surface $S$.

The vector components of the internal electric field within the particle and unit incident electric field amplitude can be express as \[34\]

$$E_r = \frac{n(n+1)}{m\alpha} J_{m\alpha}(m\alpha) \cos(l\phi) P_n^{(1)}(\cos \theta) d_a$$

(6)

$$E_\theta = J_{m\alpha}(m\alpha) \cos(l\phi) \frac{P_n^{(1)}(\cos \theta)}{\sin \theta} c_n + \left[ J_{m\alpha-1}(m\alpha) - \frac{n J_{m\alpha}(m\alpha)}{m\alpha} \right] \frac{\cos(l\phi)}{\sin \theta} d_a$$

(7)

$$E_\phi = -J_{m\alpha}(m\alpha) \sin(l\phi) \left[ \frac{ncos^2 \theta P_n^{(1)}(\cos \theta)}{\sin \theta} - (n+l) \frac{P_{n+1}^{(1)}(\cos \theta)}{\sin \theta} \right] c_n$$

$$\left[ J_{m\alpha-1}(m\alpha) - \frac{n J_{m\alpha}(m\alpha)}{m\alpha} \right] \frac{\sin(l\phi)}{\sin \theta} P_n^{(1)}(\cos \theta) d_a$$

(8)

where $m$ is the refractive index of the spherical particle relative to the gaseous medium, $J_{m\alpha}(m\alpha)$ is the spherical Bessel function of the first kind, and $P_{n+1}^{(1)}(\cos \theta)$ is the associated Legendre function.

III. RESULTS AND DISCUSSION

The photophoretic force stems from the internal energy absorbed by the particle, and thus the HSF within the particle plays a key role. The present calculation of HSF distribution within a spheroidal particle in gaseous media has been successfully verified as comparing with previous results for spherical particles assuming the aspect ratio $a/b = 1$.

A. Effects of Particle Shape and Size

To address the physical mechanisms involved in the energy absorption and the photophoresis, the HSF characterizing heat...
energy distribution is studied. Figure 2 presents the HSF distributions with changes in the shape of weakly spheroids by aspect ratios at the condition of size parameter \( \alpha = 10 \) and \( m_p = 1.5 + 0.05i \). In the case of \( a/b = 1 \) shown in Fig. 2(a), the particle acts as a microlens and light is refracted on the shaded side of the particle. There is an evident absorption peak close to the back surface, which is beneficial to the occurrence of negative photophoresis. Increasing the particle aspect ratio to \( a/b = 1.5 \) and 2, the HSF peak intensity is reduced distinctly as presented in Figs. 2(b) and (c). Further increasing the particle aspect ratio up to \( a/b = 3 \), in Fig. 2(d), the elongated shape degrades the intensity of the energy peak due to the weaker light refraction. A comparison of these energy distributions inside spheroids discloses the profound differences between these geometries.

The effects of particle size on the heat source function characteristics for a spheroidal particle of aspect ratio \( a/b = 2 \) and \( m_p = 1.5 + 0.05i \) are shown in Fig. 3. At very small particle sizes, \( \alpha = 0.5 \) in Fig. 3(a), the HSF within the spheroid seems a uniform distribution. With the particle size a little increases, as the case of \( \alpha = 5 \) in Fig. 3(b), the particle acts as a microlens and light is focused on a region close to the shaded (trailing) surface, where a peak of the HSF appears leading to negative photophoresis. At increased size, \( \alpha = 10 \), both illuminated and shaded sides contain the peaks of absorption tend to be balanced as shown in Fig. 3(c). Finally, for a large particle size, \( \alpha = 20 \) in Fig. 3(d), the energy absorbed by the large area of the leading surface becomes remarkable and the HSF peak on the trailing side is negligible. In this situation, the heat distribution turns into positive photophoresis with the dominant light absorption of the particle dominant.

The effects of particle refractivity on the heat source function for a spheroidal particle of aspect ratio \( a/b = 2 \), size parameter \( \alpha = 10 \) and \( m_p = n_p + 0.1i \) with (a) \( n_p = 1.1 \); (b) \( n_p = 2.0 \); (c) \( n_p = 3.0 \); and (d) \( n_p = 4.0 \)

B. Effects of Particle Refractivity

Figure 4 reveals the effects of particle refractivity on the HSF distribution for a spheroidal particle of aspect ratio \( a/b = 2 \) and size parameter \( \alpha = 10 \). At low particle refractivity, \( n_p = 1.1 \), the HSF is of low level and a little uniform for the weak absorption and refraction in Fig. 4(a). Enhancing particle refraction with \( n_p = 1.5 \), as shown in Fig. 4(b), an apparent HSF peak appears at the trailing side. Further increasing the particle refraction, i.e. \( n_p = 2.0 \) and 3.0 in Figs. 4(c) and 4(d), the HSF peak becomes more remarkable as well as moves toward the center of the particle. The movement of the peak location is a consequence of the increase in light refraction, which is defined as the ratio of the \( \sin(\text{angle of incidence}) \) to \( \sin(\text{angle of refraction}) \) by Snell's law. As \( n_p \) increases, the refracted ray approaches the center of the particle and the absorption peak also shifts to the center [25].

C. Effects of Particle Absorptivity

Figure 5 illustrates the effects of particle absorptivity on HSF contours for spheroids of aspect ratio \( a/b = 2 \) and size parameter \( \alpha = 10 \). In the case of non-absorbing particles \( \kappa_p = 0.0 \), the HSF peak stemmed from the light refraction onto the trailing part of the spheroid as shown in Fig. 5(a), which leads to a negative
photophoresis. Increasing the spheroid absorptivity to \( \kappa_p = 0.1 \), as shown in Fig. 5(b), enhances energy absorption of the illuminated leading surface and diminishes HSF peak located near the trailing surface of the spheroidal particle. As the particle absorptivity increases to \( \kappa_p = 0.2 \), the strength of the refraction peak is weakened obviously and the energy absorbed into the leading part of the particle becomes relatively dominant in Fig. 5(c). It means near the critical condition for the transition of negative-positive photophoresis. At further higher absorptivity, \( \kappa_p = 0.4 \) in Fig. 5(d), the HSF distribution reveals the most energy is absorbed by the leading surface and the HSF peak becomes almost vanished, which enables premature positive photophoresis.

\[
\begin{align*}
\text{(a) } a/b = 2 & \quad m_p = 1.5 + 0.0i \\
\text{(b) } a/b = 2 & \quad m_p = 1.5 + 0.1i \\
\text{(c) } a/b = 2 & \quad m_p = 1.5 + 0.2i \\
\text{(d) } a/b = 2 & \quad m_p = 1.5 + 0.4i
\end{align*}
\]

Fig. 5. Effects of particle absorptivity on the heat source function for a spheroidal particle of aspect ratio \( a/b = 2 \), size parameter \( \alpha = 10 \) and \( m_p = 1.5 + \kappa_p i \) with (a) \( \kappa_p = 0.0 \); (b) \( \kappa_p = 0.1 \); (c) \( \kappa_p = 0.2 \); and (d) \( \kappa_p = 0.4 \).

**IV. CONCLUDING REMARKS**

In the present work, a parametric analysis of energy absorption concerned with photophoresis of a micro-sized and spheroidal particle in gaseous media has been performed systematically. We have calculated the HSF distribution with the particle at various conditions of particle shape, size and optical properties of the particle. The present analysis demonstrates that the effects of the governing parameters have strong influences on the absorbed energy distribution. Based on the present analysis, the following conclusions can be drawn.

1. For a weakly spheroid, the particle acts as a microlens, which is beneficial to the occurrence of negative photophoresis. Increasing spheroidal particle aspect ratio significantly degrades the level of light refraction by the particle.
2. Both increases in particle size and absorptivity enhance the energy absorbed on illuminated or leading side and positive photophoresis can be expected.
3. Increasing particle refractivity tends to enhance the ray refracted onto a certain spot area between the center and the trailing surface of the particle.

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**REFERENCES**


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