Abstract—All practical real-time scheduling algorithms in multiprocessor systems present a trade-off between their computational complexity and performance. In real-time systems, tasks have to be performed correctly and timely. Finding minimal schedule in multiprocessor systems with real-time constraints is shown to be NP-hard. Although some optimal algorithms have been employed in uni-processor systems, they fail when they are applied in multiprocessor systems. The practical scheduling algorithms in real-time systems have not deterministic response time. Deterministic timing behavior is an important parameter for system robustness analysis. The intrinsic uncertainty in dynamic real-time systems increases the difficulties of scheduling problem. To alleviate these difficulties, we have proposed a fuzzy scheduling approach to arrange real-time periodic and non-periodic tasks in multiprocessor systems. Static and dynamic optimal scheduling algorithms fail with non-critical overload. In contrast, our approach balances task loads of the processors successfully while consider starvation prevention and fairness which cause higher priority tasks have higher running probability. A simulation is conducted to evaluate the performance of the proposed approach. Experimental results have shown that the proposed fuzzy scheduler creates feasible schedules for homogeneous and heterogeneous tasks. It also and considers tasks priorities which cause higher system utilization and lowers deadline miss time. According to the results, it performs very close to optimal schedule of uni-processor systems.

Keywords—Computational complexity, Deadline, Feasible scheduling, Fuzzy scheduling, Priority, Real-time multiprocessor systems, Robustness, System utilization.

I. INTRODUCTION

Many applications namely avionics, traffic control, automated factory, and military systems require real-time communication and computation. In real-time systems, all tasks have specific parameters such as deadline, priority, etc. Modern embedded computing systems are becoming increasingly complex [1]. Meanwhile, the traditional notions of best-effort and real-time processing have fractured into a spectrum of processing classes with different timeliness requirements including desktop multimedia, soft real-time, firm real-time, adaptive soft real-time, rate-based, and traditional hard real-time [2-5]. Many real-time systems are hard and missing deadline is catastrophic [5-8], whereas in soft real-time system occasional violation of deadline constraints may not result in a useless execution of the application or calamitous consequences, but decreases utilization[9]. A schedule which is executing all real-time tasks within their deadlines and all the other constraints are met, is called a feasible schedule [10]. Real-time scheduling can be classified in two categories, static [7] and dynamic [11] scheduling. A static real-time scheduling algorithm such as Rate Monotonic schedules all real-time tasks off-line using static parameters and requires complete knowledge about tasks and system parameters [12], while dynamic task scheduler calculates the feasible schedule on-line and allows tasks to be invoked dynamically. These algorithms use dynamic parameters such as deadline and laxity[2, 3, 10, 11, 13-16]. Scheduling in real-time system involves allocation of CPU and other resources to run corresponding tasks to meet certain timing constraints [13]. Nonetheless, scheduling is more significant in real-time systems than non-real-time systems[1, 9, 13, 15-20]. In real-time systems, tasks have to be performed correctly and in a timely fashion as well [21]. Tasks are classified as periodic and non-periodic [22, 23]. The execution requests of a periodic task repeatedly occur at regular intervals. On the contrary, execution requests of a non-periodic task are unpredictable.

Nowadays, using of real-time multiprocessor systems is dramatically increasing. Unfortunately, less is known about how to schedule multiprocessor-based real-time systems than that for uni-processors [14]. Optimal scheduling of real-time tasks on multiprocessor systems is known to be computationally intractable for large task sets [15]. Any practical scheduling algorithm in multiprocessor systems presents a trade-off between performance and computational complexity. Having more computational complexity in practical algorithm cause wide range of algorithm’s response time hence, deterministic timing behavior is the most important parameter for system’s robustness especially in hard real-time system[2, 3, 24-26]. This behavior cause decrease in utilization of the system when unpredictable conditions happened. In heterogeneous systems which tasks have different time constraints algorithm have to avoid starvation[26].

The performance of a scheduling algorithm is measured in terms of additional processor required to be added at a schedule without deadline violations as compared to optimal algorithm [15]. In [18] it has been proved that finding a minimal schedule for a set of real-time tasks in multiprocessor system is NP-hard.

In this paper, we focus on a real-time multiprocessor system with heterogeneous periodic and non-periodic tasks and compare performance and complexity of our proposed
fuzzy scheduler with other algorithms using computer simulation.

The rest of this paper is organized as follows. Section II describes scheduling algorithms and task model. Section III describes fuzzy inference engine. Section IV introduces the proposed fuzzy real-time scheduler. Experimental results are presented in section V.

II. SCHEDULING ALGORITHMS AND TASK MODEL

A. Task Model

A task is a complete sequence of instructions. Task execution starts when a task is selected by task dispatcher and one of the system's processors starts to run task's instructions. Tasks are classified according to their deadline, priority, arrival characteristic, and computation cycles requests.

B. Scheduling Algorithms

First-Come-First-Served (FCFS) algorithm [20] selects the task with the earliest arrival time. If system contains periodic tasks, their release time will be considered. This algorithm makes no effort to consider a task's deadline.

Earliest Deadline First (EDF) algorithm [15, 20] always chooses the task with the earliest deadline. It has been proved that this algorithm is optimal in a uni-processor system. Since it cannot consider priority and therefore cannot analyze it, this algorithm fails under overloading conditions.

Least Laxity First (LLF) algorithm [13] selects the task that has the lowest laxity among all the ready ones whenever a processor becomes idle, and executes it to completion. This algorithm is non-preemptive and avoids the well-known problem of its preemptive counterpart that sometimes degenerates to a processor-sharing policy.

Robust Earliest Deadline (RED) algorithm proposed in [3, 14, 15] calculates residual time and workload of tasks as their schedulability. It has some task rejection mechanism to handle system load when there is no feasible schedule [19].

Lee et al. [21] present a fuzzy scheduling algorithm. Their proposed algorithm uses task laxity and task criticality as system parameters. Their simulation model contains small number of tasks on a uni-processor system and they did not consider system overloads. All the tasks in a system are seen as real-time and fairness is not considered.

Thai [15] proposed a real-time scheduling algorithm for multi-processor distributed systems. In their approach, the task with higher computation time is assigned to bottleneck processor and system's worst case processing time is computed. However it is not clear how this task is detected. Their algorithm needs communication time between processors and assume tasks processing times are different but real-time. They do not consider heterogeneous tasks and fairness. The proposed algorithm has acceptable resistance to system overload especially when number of processors is increased.

The model described in [27] uses fuzzy inference for scheduling non-preemptive periodic tasks in soft real-time multiprocessing systems. They use priority and deadline as tasks' parameters and use a fuzzy inference engine to compute each task's priority and select the task with maximum priority to process. Although they wish to use TSK inference engine in their model, their rules are Mamdani. They assume all task are periodic and it is not clear that their processor on system is homogeneous or heterogeneous. The proposed model does not consider task's processing time. Therefore results are more similar to EDF and not suitable for multiprocessing systems.

Chen et al. [28] proposed a scheduling model and a related algorithm that is suitable for both uni-processor and multiprocessor systems. They provide a method to detect work overloading and try to balance load with task dispatching.

Dynamic integrated scheduling of hard real-time, soft real-time, and none real-time tasks are discussed in [29]. They can generate feasible schedules but their model is restricted to periodic tasks and change the tasks’ periods dynamically when overloading occurs.

III. FUZZY INFERENCE ENGINE

Fuzzy logic [30, 31] is a superset of conventional Boolean logic and extends it to deal with new aspects such as partial truth and uncertainty.

Fuzzy inference is the process of formulating the mapping from a given input set to an output using fuzzy logic. The basic elements of fuzzy logic are linguistic variables, fuzzy sets, and fuzzy rules [32]. The linguistic variables’ values are words, specifically adjectives like “small,” “little,” “medium,” “high,” and so on. A fuzzy set is a collection of couples of elements. It generalizes the concept of a classical set, allowing its elements to have a partial membership. The degree to which the generic element “x” belongs to the fuzzy set A (expressed by the linguistic statement x is A) is characterized by a membership function (MF), \( \mu_A(x) \). The membership function of a fuzzy set corresponds to the indicator function of the classical sets. It can be expressed in the form of a curve that defines how each point in the input space is mapped to a membership value or a degree of truth between 0 and 1. The most common shape of a membership function is triangular, although trapezoidal and bell curves are also used. This operation normalizes all inputs to the same range and has a direct effect on system performance and accuracy.

A fuzzy set A is defined within a finite interval called universe of discourse U as follows:

\[
A = \{(x, f_A(x)), f_A(x) : U \rightarrow [0,1]\}
\]
U is the whole input range allowed for a given fuzzy linguistic variable. All fuzzy sets related to a given variable make up the term set, the set of labels within the linguistic variable described or, more properly, granulated. Fuzzy rules form the basis of fuzzy reasoning. They describe relationships among imprecise, qualitative, linguistic expressions of the system’s input and output. Generally, these rules are natural language representations of human or expert knowledge and provide an easily understood knowledge representation scheme. A typical conditional fuzzy rule assumes a form such as

**IF Speed is “Low” AND Race is “Dry” THEN Braking is “Soft”**.

Speed is Low AND Race is Dry is the rule’s premise; while Braking is Soft is the consequent. The premise predicate might not be completely true or false, and its degree of truth ranges from 0 to 1. We compute this value by applying the membership functions of the fuzzy sets labeled “Low” and “Dry” to the actual value of the input variables Speed and Race. After that, fuzzification is applied to the conclusion; the way in which this happens depends on the inference model.

There are two types of fuzzy inference models:
1. Mamdani [33],
2. TSK or Sugeno [34].

Interpreting an if-then rule involves two distinct parts: first evaluating the antecedent and then applying results to the consequent (known as implication) [35, 36]. In the case of two-valued or binary logic, if-then rules do not present much difficulty. If the premise is true, then the conclusion is true, whereas with fuzzy approach, if the antecedent is true to some degree of membership, then the consequent is also true to that same degree.

Mamdani-type [33] inference expects the output membership functions to be fuzzy sets. After the aggregation process, there is a fuzzy set for each output variable that needs defuzzification. It is possible, and in many cases much more efficient, to use a single spike as the output’s membership function rather than a distributed fuzzy set. This is sometimes known as a singleton output membership function, and it can be thought of as a pre-defuzzified fuzzy set. It enhances the efficiency of the defuzzification process because it greatly simplifies the computation required by the more general Mamdani method, which finds the centroid of a two-dimensional function. Rather than integrating across the two-dimensional function to find the centroid, Sugeno-type systems use weighted sum of a few data points. In general, Sugeno-type systems can be used to model any inference system in which the output membership functions are either linear or constant.

**IV. PROPOSED MODEL**

As shown in Fig. 1, the major factors considered in our approach to determine the scheduling are task priority, deadline, required computation time, and used CPU time. The notion of laxity is used in the proposed approach to facilitate the computation. Laxity is the maximum time that a task can wait before being executed (i.e., laxity = deadline - computation time).

A task’s priority shows the importance of the task. The inputs of these parameters are justified and represented as linguistic variables and fuzzy rules are then applied to those linguistic variables to compute the level value for deciding which task to select to schedule next.

CPU time is another parameter which could guarantee scheduling fairness. We considered 5 trapezoid membership functions for task’s priority. “Very high”, “High”, “Medium”, “Low” and “Very low” are these membership functions. This number and naming of membership is same for task’s laxity however CPU time membership function considered 3 and also trapezoid. “High”, “Medium” and low are the name of these functions. For the \( f(x) \) as the membership function, a large class of functions can be taken such as triangular, trapezoidal, Gaussian and bell function however we selected trapezoidal for its usability in fuzzy dedicated hardware [35-37]. The used membership functions for this model illustrated in Fig. 2 and 3.

In our proposed algorithm as shown in Fig. 4, a newly arrived task will be added to the input queue. This queue consists of the remaining tasks from last cycle that has not yet been assigned.

1. **For each task of input queue**
   a. Feeds task’s run-time priority using fuzzy inference engine

2. **While system has a free processor**
   a. assign the task with highest run-time priority to the processor

3. **Loop forever**
   a. If processor event occurs
      i. Go to 2.
   b. If scheduling event occurs
      i. Update tasks parameters.
      ii. Go to 1.

**Fig. 4 Proposed algorithm**

Fuzzy scheduler processes each task separately and computes its run-time priority and sends it to task
dispatcher’s priority queue. In a multiprocessor system, this queue offers tasks to dispatcher by their run-time priority order. Dispatcher offers a new task whenever one of the processors of the system finishes its task.

![Fig. 5 System view of soft-real-time fuzzy scheduler](image)

Periodic tasks which their execution requests occur repeatedly will remain in the system queue while non-periodic tasks will be finished and their next request starts with task initialization. In this model all kinds of tasks are considered (Fig. 5).

In firm real-time systems with more real-time constraints, our model can be adapted with multiple input queues with multiple schedulers where task’s priorities differ. With this technique, when the number of system tasks is very high, scheduler can select most important tasks and send them to dispatcher queue while with last model processing all tasks parameters could be time consuming and waste system time.

Scheduler and dispatcher are independent components and they are connected with a queue; consequently our proposed scheduler is extendable.

Due to the model extendibility and adaptability, this model can be used in a variety of systems with multi-criteria constraints.

Satisfactory performance is achieved by using 39 Sugeno rules only. This number is obtained by simplifying 169 rules in different examples. Some of them are mentioned below:

- If (Laxity is “Very low”) and (Priority is “Very high”) then
  \[ R_{Priority} = 100 \times priority - 10 \times laxity. \]
- If (Laxity is “Low”) and (Priority is “Very high”) then
  \[ R_{Priority} = 50 \times priority - 20 \times laxity. \]
- If (Laxity is “Medium”) and (Priority is “Normal”) and (CPU time is “High”) then
  \[ R_{Priority} = 25 \times priority - 40 \times laxity - 50 \times CPU/time \]

Choosing number of rules and membership functions directly affects system accuracy while performance of the system increases with rule size decrease. There are some techniques for adjusting membership functions however; in this paper we did not consider these approaches.

![The corresponding decision surface to these rules and membership functions is illustrates in Fig. 6.](image)

V. EXPERIMENTAL RESULTS

We are simulated our algorithm using our custom-designed simulator implemented using Java. In our simulator we have 100 tasks, among which 10 has very high priority, 30 has high priority, 20 has medium and 20 has very low priority. We considered priority in 0-1000.

Each task’s deadline and required computation cycles considered in 0-1000 which means maximum allowed laxity is 1000. These parameters are generated or updated randomly when a new arrived task generated of its computation finished.

![Fig. 7 High priority task generation](image)

Simulation results show that by increasing the number of system’s processors, generation of high priority tasks increases until high priority task’s waiting times is reduced to an acceptable range (Fig. 7). By increasing processor, high priority tasks have higher probability of execution while their laxity would not decrease to critical region. This behavior results in more execution for low priority tasks in medium load cycles. Next, low priority tasks generation increases to handle low priority task’s waiting time.

Simulation results show that the model can feasibly schedule tasks when system load increases and keep system processors loads close to one even at crowded times. However, other algorithms like LLF and EDF break down when the system is overloaded. In this model, we did not consider scheduler processing time and this process is...
independent of the number of system’s processors. We note that processor’s load remain always below one because of dispatcher’s processing time. By analysis of task scheduler, periodic task’s period increases automatically by scheduler with consideration of their priority and CPU time. This behavior of the system is similar elastic scheduling proposed by [19].

While number of the system’s processors increases, our model balances the load between processors. This well balancing will causes efficient processing time in symmetric systems. The proposed scheduler’s average waiting time is close to LLF and EDF algorithms. Simulations demonstrate the algorithm is capable of task balancing when the number of processors increases. However in comparison to other algorithms high priority tasks have smaller waiting times. This implies a better response time for the system and it selects high priority tasks with higher probability.

Since the total system computation power is constant, so when some high priority tasks get a higher portion of system’s computation power, the other ones will receive lower attention. This causes a reduction in selection of low priority tasks (Fig. 8).

<table>
<thead>
<tr>
<th>HP to all ratio</th>
<th>Proposed</th>
<th>Priority based</th>
<th>EDF</th>
<th>LLF</th>
<th>FCFS</th>
</tr>
</thead>
<tbody>
<tr>
<td>20%</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>40%</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>80%</td>
<td>4</td>
<td>4</td>
<td>7</td>
<td>7</td>
<td>8</td>
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</table>

**TABLE I**

<table>
<thead>
<tr>
<th>LP to all ratio</th>
<th>Proposed</th>
<th>Priority based</th>
<th>EDF</th>
<th>LLF</th>
<th>FCFS</th>
</tr>
</thead>
<tbody>
<tr>
<td>20%</td>
<td>6</td>
<td>12</td>
<td>4</td>
<td>4</td>
<td>4</td>
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<tr>
<td>40%</td>
<td>4</td>
<td>8</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>80%</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
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</tbody>
</table>

**TABLE II**

As shown in Table I, by considering ratio of high priority task to all, our model select high priority tasks with higher probability which makes acceptable system’s waiting time and miss ratio for high priority tasks by use the lower number of processors. For applications which average waiting time for all tasks is an important parameter and designer have to care about low priority task to restrict average waiting time. Number of processor required for acceptable miss ratio is listed in Table II which its first column is the ratio of low priority tasks to all tasks. Our algorithm provides an average utilization similar to other algorithm. However, Fig. 8 demonstrate, our algorithm significantly performs better for high priority tasks in a real-time environment.

**VI. CONCLUSION AND FUTURE WORK**

The proposed scheduler which proposed in this paper has low complexity due to the simplicity of fuzzy inference engine. As a consequence, its computation complexity and response time is constant and by increasing the number of processors will not increase. This model is efficient when system has heterogeneous tasks with different constraints.

Our future work is to map this algorithm on our real-time fuzzy processor.

**REFERENCES**


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