World Academy of Science, Engineering and Technology
International Journal of Electronics and Communication Engineering
Vol:6, No:2, 2012

A Propagator Method like Algorithm for Estimation of Multiple Real-Valued Sinusoidal Signal Frequencies

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***Abstract*—In this paper a novel method for multiple one dimensional real valued sinusoidal signal frequency estimation in the presence of additive Gaussian noise is postulated. A computationally simple frequency estimation method with efficient statistical performance is attractive in many array signal processing applications. The prime focus of this paper is to combine the subspace-based technique and a simple peak search approach. This paper presents a variant of the Propagator Method (PM), where a collaborative approach of SUMWE and Propagator method is applied in order to estimate the multiple real valued sine wave frequencies. A new data model is proposed, which gives the dimension of the signal subspace is equal to the number of frequencies present in the observation. But, the signal subspace dimension is twice the number of frequencies in the conventional MUSIC method for estimating frequencies of real-valued sinusoidal signal. The statistical analysis of the proposed method is studied, and the explicit expression of asymptotic (large-sample) mean-squared-error (MSE) or variance of the estimation error is derived. The performance of the method is demonstrated, and the theoretical analysis is substantiated through numerical examples. The proposed method can achieve sustainable high estimation accuracy and frequency resolution at a lower SNR, which is verified by simulation by comparing with conventional MUSIC, ESPRIT and Propagator Method.**

***Keywords*—Frequency estimation, peak search, subspace-based method without eigen decomposition, quadratic convex function.**

I. INTRODUCTION

ESTIMATION of the frequencies of multiple sinusoidal components from a finite noisy discrete time measurements has numerous application in many areas such as communications, radar, sonar and geophysical seismology, doppler measurements. For all the applications it is essential to obtain an accurate estimate of the frequencies. Although there are numerous algorithms for frequency estimation of sinusoidal signals in the literature such as maximum likelihood [1], nonlinear least squares [2] and sub-space methods [3]–[5], little attention has been paid to the special case of harmonic In fact, real valued harmonic frequency estimation has important applications in speech signal processing [6]–[8], automotive control systems [9] as well as instrumentation and measurement [10]. A good number of algorithms for complex valued sinusoids are very well detailed in the literature [11]-[13] and for real-valued [14]-[16]. The data model used for the real-valued frequency estimation in [14] gives the dimension of signal subspace is two times the number of frequencies present in the observation. A direct method for sinusoid parameter estimation is also proposed in [17] based on the finite Fourier integral of the differential equation. A linear prediction based approach is discussed in [18], where the initial fundamental frequency estimate is first obtained by solving a standard least-squares equation with exploitation of the harmonic structure of the sinusoidal signal or by using the MUSIC approach. A propagator method based algorithm is discussed in [19] for complex valued sinusoids using Hankel matrix-Hankel block approach, in [20] an ESPRIT-Like Estimation of Real-Valued Sinusoidal Frequencies is proposed.

In this proposed algorithm first we form a data model from the given observation using linear prediction properties of the real harmonic signal and forward and backward averaging proposed by P.Palanisamy and S.P Kar in [21]. After formation of the improvised data model from the given data two new enhanced data model is formed with some novel mathematical approach which is discussed detail in the following sections. Then by using the new data models the algorithm is proposed.

The major contributions of this paper can be summarized as i) An efficient frequency estimators for multiple real sinusoids are devised and ii) a proof for their optimality in the case of a multiple real tone embedded in white Gaussian noise is provided, in comparison with conventional MUSIC,ESPRIT and Propagator Method (PM) based algorithms. The rest of the paper is organized as follows, in Section II problem of multiple real valued sinusoidal signal frequency is formulated and the improvised data model as proposed in [21] is postulated and also the two new enhanced data model is formulated and then in the Section III the proposed technique based on Propagator combined with SUMWE [22] is formed to develop the estimator. In Section IV the comparative studies had done with the existing methods and the robustness is verified by computer simulation followed by conclusion in Section V. For typographic convention, throughout the paper matrices are denoted by uppercase bold letters and vectors by lowercase bold letters. The superscript T denotes transposition of a matrix.

II. PROBLEM FORMULATION AND DATA MODELS

A. Modified Data model

The problem of the multiple real valued sinusoidal signal frequency estimation is a problem with vivid applications in array signal processing [25], so the received noisy signal can be formulated as per the following model.

$$r(n)=x(n)+v(n), \qquad \text{for } n=0,1,2\ldots N-1 \tag{1}$$

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where $x(n)=\sum_{k=1}^{D} a_k \cos(\omega_k n+\varphi_k)$, N denotes available observations of a data set comprising of D real valued sinusoidal signal in presence of noise, a_k denotes the amplitude, ω_k=(0, π) and φ_k =[0, 2π) denotes the frequency and phase of the kth real- valued sinusoid respectively, while v(n) is an additive white noise with zero mean and unknown variance σ_v^2 . It can also be notified that the unknown phase component φ_k for k=1,2,...D, which is uniformly distributed over [0,2π) are independent of v(n). Furthermore, it is assumed that the number of sinusoids D is known *a priori.* The task here is to estimate ω_k for k=1,2…D, based on the observations of x(n) , n=0,1,2,…,N. Let us define following M×1 snapshot vector, without loss of generality we can assume that M>D,

$$\mathbf{y}(n)=\frac{1}{2}[\mathbf{y}_1(n)+\mathbf{y}_2(n)], \quad \text{for } n=0,1,2...N\text{-}1 \tag{2}$$

where

$$\mathbf{y}_1(n)\triangleq[r(n)\ r(n+1)...r(n+M-1)]^T \tag{3-a}$$

$$\mathbf{y}_2(n)\triangleq[r(n)\ r(n-1)..\ r(n-M+1)]^T \tag{3-b}$$

with the help of equation of (1) applying (3) in (2) we can devise the following vector equation

$$\mathbf{y}(n)=\mathbf{A}\mathbf{s}(n)+\mathbf{q}(n) \qquad \text{for } n=0,1,2,...,N\text{-}1 \tag{4}$$

where $\mathbf{A}\triangleq[\boldsymbol{\gamma}(\omega_1)\ \boldsymbol{\gamma}(\omega_2)\ \boldsymbol{\gamma}(\omega_3)...\boldsymbol{\gamma}(\omega_D)]$ is an M×D matrix, and $\mathbf{s}(n)=[a_1\cos(\omega_1 n+\varphi_1)...a_D\cos(\omega_D n+\varphi_D)\ ]^T$ is a D×1 signal vector, $\mathbf{q}(n)\triangleq[q_1(n)\ q_2(n)\ ...q_M(n)]^T$ is an M×1 noise vector, where $q_j(n)=\frac{1}{2}[v(n+j-1)+v(n-j+1)\ ]$ for j=1,2,...,M and $\boldsymbol{\gamma}(\omega_i)=[1\ \cos\omega_i\quad \cos 2\omega_i\ ...\ \cos{(M\text{-}1)\omega_i}\ ]^T$ is an M×1 vector which constitute the columns of the matrix **A.** The matrix **A** is a full rank matrix because all the columns are linearly independent to each other..

B. Enhanced Data Models

Before advancing into the proposed algorithm we define two new signal models by implementing the following M×1 vectors based on the observations of (1),

$$\mathbf{z}_1(n)\triangleq[r(n+1)\ \ r(n)\ r(n-1)\ ...\ r(n-M+2)]^T \tag{5-a}$$

$$\mathbf{z}_2(n)\triangleq[r(n-1)\ \ r(n)\ r(n+1)...\ r(n+M-2)]^T \tag{5-b}$$

$$\mathbf{z}_3(n)\triangleq[r(n-1)\ \ r(n-2)\ r(n-3)\ ...\ r(n-M)]^T \tag{5-c}$$

$$\mathbf{z}_4(n)\triangleq[r(n+1)\ \ r(n+2)\ r(n+3)\ ...\ r(n+M)]^T \tag{5-d}$$

let us define another M×1 snapshot vector **z**(n) which is defined as follows,

$$\mathbf{z}(n)=\frac{1}{4}\sum_{j=1}^{4}\mathbf{z}_j(n) \quad \text{for } n=0,1,2,3,\ldots,N\text{-}1 \tag{6}$$

by substituting (5-a) to (5-d) in (6) we obtained a precise expression for **z**(n) which can readily be shown as follows,

$$\mathbf{z}(n)=\mathbf{A\Phi s}(n)+\mathbf{q}_r(n) \tag{7}$$

where $\boldsymbol{\Phi}$=diag{$\cos\omega_1\ \cos\omega_2 ...\cos\omega_D$} is a D×D diagonal matrix and $\mathbf{q}_r$(n) is a M×1 noise vector defined as $\mathbf{q}_r(n)=[q_{r1}(n)\ q_{r2}(n)...q_{rM}(n)]^T$, and where $q_{ri}(n)=\frac{1}{4}[v(n-i+2)+v(n+i-2)+v(n+i)+v(n-i)]$, i=1,2,…M. Similarly like (5-a) to (5-d) we can construct another signal model from the following M×1 vectors based on the observations of (1)

$$\mathbf{g}_1(n)\triangleq[r(n-M+2)\ ...\ r(n-1)\ r(n)\ \ r(n+1)]^T \tag{8-a}$$

$$\mathbf{g}_2(n)\triangleq[r(n+M-2)\ ...\ r(n+1)\ \ r(n)\ r(n-1)]^T \tag{8-b}$$

$$\mathbf{g}_3(n)\triangleq[r(n-M)\ ...\ r(n-3)\ \ r(n-2)\ r(n-1)]^T \tag{8-c}$$

$$\mathbf{g}_4(n)\triangleq[r(n+M)...r(n+3)\ \ r(n+2)\ r(n+1)]^T \tag{8-d}$$

from the above four M×1 observation vectors we can define another new data model as ,

$$\mathbf{g}(n)=\frac{1}{4}\sum_{j=1}^{4}\mathbf{g}_j(n) \quad \text{for } n=0,\ 1,\ 2\ldots,N\text{-}1 \tag{9}$$

by substituting (8-a) to (8-d) in (9) we obtained a precise expression for **g**(n) which can readily shown as follows.

$$\mathbf{g}(n)=\mathbf{J}\mathbf{A}\ \boldsymbol{\Phi}\mathbf{s}(n)+\mathbf{q}_r(n) \tag{10}$$

J is defined as M×M counter identity matrix in which 1s present in the principal anti-diagonal. Hence in this section the two enhanced data models are developed. Development of the proposed estimator is detailed in the following section.

III. Development of Proposed Algorithm

In this section we develop our proposed algorithm taking consideration of the derived data models in the previous sections. Based on the assumption of the data models, from (3), (7) the cross correlation matrix $\mathbf{R_{yz}}$ between **y**(n), **z**(n) is obtained as ,

$$R_{yz}=E\ [y(n)z^T(n)]\ =AR_{ss}\ \Phi\ A^T \tag{11}$$

where $\mathbf{R_{ss}}$ is nothing but source signal covariance matrix defined by $\mathbf{R_{ss}}=E[\mathbf{s}(n)\ \mathbf{s}^T(n)]$, Similarly from (3) and (10) we can get another cross correlation matrix $\mathbf{R_{yg}}$ which can be readily shown as follows,

$$\mathbf{R_{yg}}=E[\mathbf{y}(n)\mathbf{g}^T(n)]=\mathbf{AR_{ss}}\ \mathbf{J\Phi}\ \mathbf{A^T} \tag{12}$$

In noise free case $\mathbf{R_{yg}}=\mathbf{JR_{yz}}$ but in noise corrupted case $\mathbf{R_{yg}}\cong\mathbf{JR_{yz}}$ so from the above equations and the assumptions we can formulate an extended cross correlation matrix of size M×2M as ,

$$R_y=[R_{yz}, R_{yg}]=[R_{yz}\ ,JR_{yz}]\ =A[R_{ss}\ \Phi\ A^T, JR_{ss}\ \Phi\ A^T] \tag{13}$$

we considered the fact that the number of finite snapshots (M) is greater than the number of source signals that is M>D, hence the matrix **A** can be portioned into two sub matrices as $\mathbf{A}\triangleq[\mathbf{A_2}^T\quad \mathbf{A_2}^T]^T$ where $\mathbf{A}_1$ and $\mathbf{A}_2$ are the D ×D and (M-D) ×D sub matrices consisting of the first D rows and last (M-D) rows of the matrix **A,** since **A** is a full rank matrix, clearly $\mathbf{A_1}$ is also a full rank matrix and hence the rows of the matrix $\mathbf{A_2}$ can be expressed as linear combination of the rows of the sub matrix $\mathbf{A_1}$ so equivalently there is a D×M-D linear operator **P** between $\mathbf{A_1}$ and $\mathbf{A_2}$ as in [24] , such that $\mathbf{A_2}=\mathbf{P^T A_1}$**.** By using the above assumptions we can segregate (13) into the following two matrices,

$$R_y=\begin{bmatrix}A_1\\A_2\end{bmatrix}[R_{ss}\ \Phi\ A^T, JR_{ss}\ \Phi\ A^T] \tag{14}$$

$$=\begin{bmatrix}A_1\\P^TA_1\end{bmatrix}[R_{ss}\ \Phi\ A^T, JR_{ss}\ \Phi\ A^T]=\begin{bmatrix}R_{y1}\\R_{y2}\end{bmatrix}$$

where $\mathbf{R}_{y1}$ and $R_{y2}=P^T R_{y1}$ consist of first D and last (M-D) rows of the extended cross correlation matrix $\mathbf{R}_y$ respectively. Hence the linear operator **P** can be found from $\mathbf{R}_{y1}$ and $\mathbf{R}_{y2}$ as [23] however, a least-squares solution for the entries of the propagator matrix **P** satisfying the relation $\mathbf{R}_{y2} = \mathbf{P}^T \mathbf{R}_{y1}$ may be obtained by minimizing the cost function described as follows,

$$\xi(P)=\| R_{y2}- R_{y1}P\|_F^2 \tag{15}$$

where $||.||_F^2$ denotes the Frobenius norm. The cost function $\xi(\mathbf{P})$ is a quadratic (convex) function of **P**, which can be minimized to give the unique least-square solution for **P** , that can be evidently shown as,

$$\mathbf{P} = (\mathbf{R}_{y1}{}^T \mathbf{R}_{y1})^{-1} \mathbf{R}_{y1}{}^T \mathbf{R}_{y2} \tag{16}$$

further by defining the matrix $\mathbf{Q} = [\mathbf{P}^T \ - \mathbf{I}_{M-D}]^T$ **,** which can be used to estimate the real valued harmonic frequencies as like [22]. SUMWE is an effective low complex algorithm to estimate the bearings of direction of arrival, which we used here effectively to estimate the frequencies of the multiple real valued sinusoidal signal frequencies. In noise free case the matrix $\mathbf{QA=0_{M\text{-}D\times D}}$, while in noisy environment this condition does not hold true so we have defined a projection matrix in order to estimate the parameters. Let us define it as **E,** so when the number of snapshots is finite we can estimate the frequencies by minimizing the following cost function**,** $\hat{f}(\omega)=\mathbf{a}^T(\omega)\ \hat{E}\ a(\omega)$ where $a(\omega) =[1 \cos\omega \ldots \cos\big(M\text{-}1\big)\,\omega]^T$, $\hat{E}=\hat{Q}\ (\hat{Q}^T\ \hat{Q})^{-1}\ \hat{Q}^T$ while $\hat{Q}=\big[\hat{P}^T\ \text{-}I_{M\text{-}D}\big]^T$. The orthonormality of matrix $\hat{Q}$ is used in order to improve the estimation performance while **E** is calculated implicitly using matrix inversion lemma as [22] and $\hat{E}$ and $\hat{Q}$ are the estimates of **E** and **Q**. The estimated cost function is minimized, **MUSIC** like fashion as follows,

$$p(\omega)=\frac{1}{\hat{f}(\omega)}=\frac{1}{\mathbf{a}^T(\omega)\ \hat{E}\ a(\omega)}$$

The proposed method has two notable advantages over the conventional MUSIC algorithm [14], that is computational simplicity and less restrictive noise model ,though it required peak search but there is no eigen value decomposition (SVD or EVD) involved in the proposed algorithm unlikely MUSIC, where the EVD of the auto covariance matrix is needed. It also provide quite efficient estimate of the frequencies .Also its found out that the estimation accuracy probability is correct up to 0.0025π which is very much close proximity to the estimation accuracy of the MUSIC based estimators, and also it provides a very good performance in low SNR range. The RMSE in low SNR range is very much similar to a MUSIC estimator. The probability of estimation and bias of the estimation is also quite attractive compared to the conventional methods. The statistical performance is discussed in detail in the following Section.

IV. Simulation Results

Computer simulations had been carried out to evaluate the performance of the proposed algorithms using MATLAB 7.10.0.0499(R2010a) software to demonstrate the performance of the proposed algorithm in estimating frequencies of multiple real-valued sinusoidal signals in presence of zero-mean additive white Gaussian noise with unit variance. Basically we had done mainly four type of analysis in order to verify the robustness of our proposed estimator. 1) Determining the frequency spectra at a constant SNR 2) Statistical performance analysis, by determining RMSE against varied SNR and number of finite snapshots (3) To determine the probability of accurate estimation of harmonics with a predefined resolution against varying SNR 4) Bias of estimation against varying SNR.

A. Determining The Frequency Spectra At A Constant SNR

Four sinusoidal signal with identical amplitude and different frequencies, 0.16π, 0.24 π, 0.3π and 0.38π are considered at a constant SNR of -5dB, at a finite number of snapshot N=100 and dimension of the snapshot vector M=20 are considered and the spectrum of the given signal is obtained where the peaks of the spectrum obtained gives the corresponding frequency components, which is graphically shown in Fig.1, and in TABLE-I the detection capability of the estimator is shown.

B. Statistical Performance Analysis, By Determining RMSE Against Varied SNR And Number Of Finite Snapshots (N)

To verify the efficiency of the estimator we analyzed the RMSE of the estimated frequency ($\hat{\omega}$) with respect to the transmitted signal frequency (ω) at varied SNR and number of snapshots (N). All the experimental results based on 500 independent trails. For the first experiment the N is fixed at 100, M=12 and the SNR is varied from -15dB to30dB and the RMSE is measured in dB. The RMSE is calculated as [26] using the following relation

$$\text{RMSE}=\sqrt{\frac{1}{L}\sum_{l=1}^{L}(\omega\text{-}\hat{\omega}_l)^2}$$

where L is number of independent trials, ω is actual frequency and $\hat{\omega}_l$ is the frequency estimate of the *l*-th trial. The RMSE values versus SNR for various methods are plotted and compared in Fig.2. From the analysis, it is evident that the proposed method shows much behavior like a MUSIC estimator in low SNR region and having far greater performance compared to the conventional propagator method (PM) and ESPRIT proposed in [21].Similarly another experiment is carried out by taking the SNR constant at 0dB and varying the number of snapshots (N) from 100 to 350 and the performance comparisons are graphically interpreted in Fig.3 which also shows that the proposed estimator behaves very much like a MUSIC estimator as the number snapshot increases, and hence robustness of the proposed algorithm in comparison with the conventional estimators.

C. To Determine The Probability Of Accurate Estimation Of Harmonics With A Predefined Resolution Against Varying SNR.

This experiment is carried out in order to provide the estimation accuracy results for the proposed estimator. Here the experiment is carried out by keeping the N constant at 200 the M=12 and the frequency taken is 0.25π rad/s and 500 independent trials had been done in order to determine the probability of the detection at a resolution of 0.0025π.in a SNR range of -20dB to 40dB And its plotted in Fig.4. From the experiment it is evident that the proposed method performs far superior in comparison with the conventional propagator method (PM) and ESPRIT [5] and having very close performance compare to conventional MUSIC estimator [14].

D. Bias Of Estimation Against Varying SNR.

This experiment is carried out in order to provide the bias of estimation results for the proposed estimator. Here the experiment is carried out by keeping the N constant at 200 the M=12 and the frequency taken is 0.25π rad/s and 500 independent trials had been done in order to determine the bias of estimation in a SNR range of -20dB to 10dB, and it is plotted in Fig.5. From the experiment it is evident that the proposed method performs far superior in comparison with the Conventional propagator method (PM) and ESPRIT [21] and having very close performance compare to conventional MUSIC estimator as well as at very low SNR the proposed estimator provides better bias ability compared to the conventional MUSIC estimator [14].

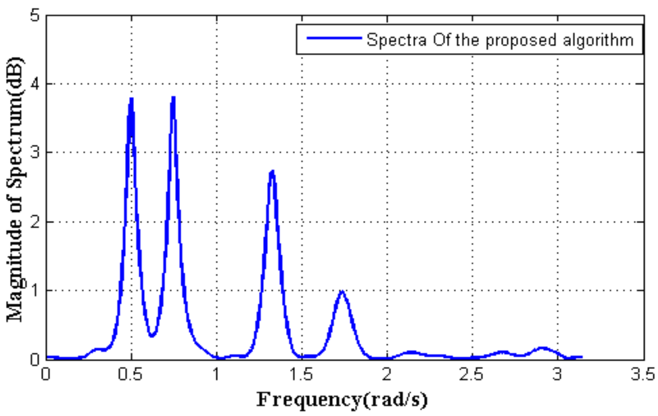

Fig. 1 Spectrum of the estimator

TABLE I
DETECTION CAPABILITY OF THE ESTIMATOR

Frequencies (rad/s)	Original frequency	Estimated frequency
ω_1	0.16π	$0.1607\ \pi$
ω_2	$0.24\ \pi$	$0.2407\ \pi$
ω_3	0.30π	$0.3026\ \pi$
ω_4	0.38π	$0.3788\ \pi$

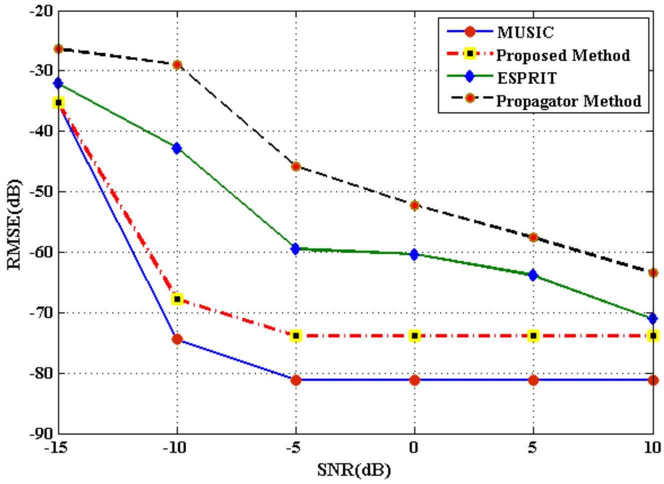

Fig. 2 RMSE vs SNR (dB) comparison

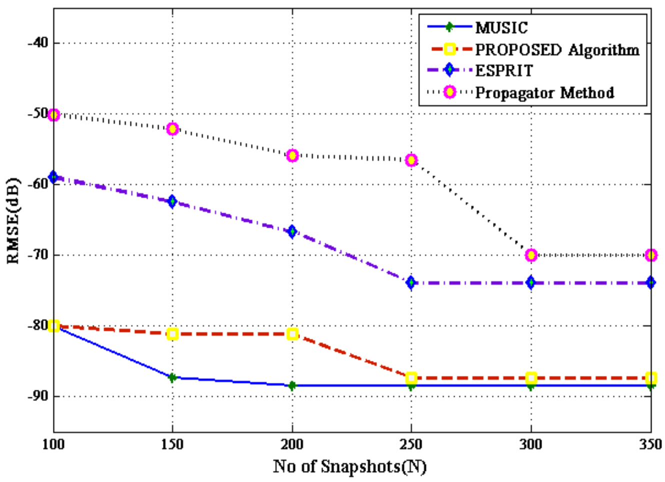

Fig. 3 RMSE vs Number Of snapshots comparison

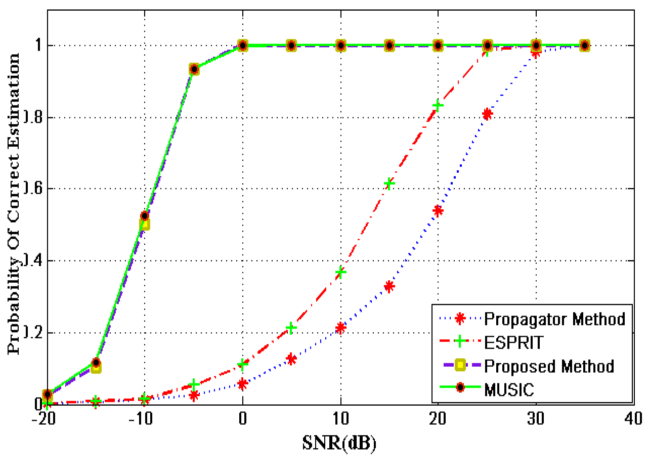

Fig. 4 Probability of estimation vs SNR (dB)

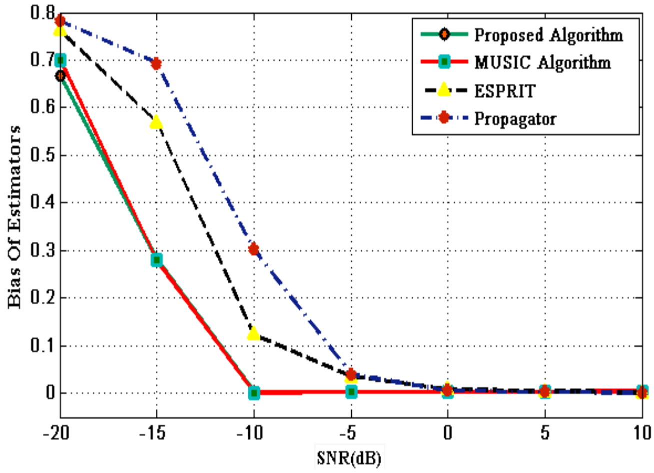

Fig. 4 Bias of estimators vs SNR (dB)

V. CONCLUSION

In many practical applications of array processing, a computationally simple frequency estimation method with good statistical performance is quite attractive. In this paper, we proposed a new computationally efficient subspace-based method for multiple real sinusoids in presence of white noise based on modified propagator method in collaboration with a simple peak search algorithm. The proposed method does not require the computationally demanding eigen decomposition, and the effect of additive noise is eliminated. The explicit expressions of asymptotic RMSE of the estimated frequencies were derived. The simulation results showed that the proposed method has less computational burden and superior estimation performance with high probability of successful detection of the real harmonic frequencies. The estimation performance of the proposed algorithm was demonstrated, and the theoretical analysis was substantiated through numerical examples. Although the method performs slightly worse than the subspace-based method based on peak search as well as eigen decomposition such as the (FBSS-based) MUSIC in general. But it mostly out performs the subspace-based methods without eigen decomposition such as the Propagator Method (PM), and also sub-space method with eigen decomposition but without peak search like ESPRIT and the simulation results showed that the algorithm has the advantages of reduced computational load and superior estimation performance in resolving closely spaced signal frequencies with a short length of data and at low SNR.

REFERENCES

[1] D. C. Rife and R. R. Boorstyn, "Multiple tone parameter estimation from discrete-time observations," *Bell Syst. Tech. J.*, pp. 1389–1410, Nov.1976.

[2] P. Stoica and A. Nehorai, "Statistical analysis of two nonlinear least-squares estimators of sine wave parameters in the colored noise case," in Proc. *Int. Conf. Acoustic., Speech, Signal Processing,* vol. 4, Apr. 1988, pp. 2408–2411.

[3] V. F. Pisarenko, "The retrieval of harmonics by linear prediction," *Geo-phys. J. R. Astron. Soc.*, vol. 33, pp. 347–366, 1973.

[4] R. O. Schmidt, "Multiple emitter location and signal parameter estimation*," IEEE Trans. Antennas Propagat.,* vol. 34, pp. 276–280, Mar. 1986.

[5] R. Roy and T. Kailath, "ESPRIT—Estimation of signal parameter via rotational invariance techniques," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, pp. 984–995, July 1989.

[6] R. J. Mcaulay and T. F. Quateri, "Speech analysis/synthesis based on a sinusoidal representation*," IEEE Trans. Acoust., Speech, Signal Processing*, vol. 34, pp. 744–754, Apr. 1986.

[7] T. Nakatani and M. Miyoshi, "Blind dereverberation of single channel speech signal based on harmonic structure," in Proc. *IEEE Int. Conf. Acoust. Speech, Signal Processing*, vol. 1, Hong Kong, China, Apr. 2003, pp. 92–95.

[8] L. Y. Ngan, Y. Wu, H. C. So, P. C. Ching, and S. W. Lee, "Joint time delay and pitch estimation for speaker localization," in Proc. *IEEE Int. Symp. Circuits Systems,* vol. 3, Bangkok, Thailand, May 2003, pp. 722–725.

[9] U. Kiencke and L. Nielsen, Automotive Control Systems for Engine, Driveline, and Vehicle. Berlin, Germany: *Springer-Verlag*, 2000.

[10] G. Simon, R. Pintelon, L. Sujbert, and J. Schoukens, "An efficient non-linear least square multi sine fitting algorithm," *IEEE Trans. Instrum. Meas*., vol. 45, pp. 750–755, Aug. 2002

[11] B.D.Rao and K.V.S.Hari, "Weighted Sub-space methods and spatial smoothing: Analysis and comparison", *IEEE Trans. Signal processing*, vol. 41,nNo.2, pp.788-803, 1993.

[12] P.Stoica, A. Eriksson and T.Soderstrom, "Optimally Weighted MUSIC for frequency Estimation", *SIAm Jour. On Matrix Anal. and Appl*., vol.16, no.3, pp.811-827,1995.

[13] P.Stoica and T.Soderstrom, " Statistical analysis of MUSIC and subspace rotational estimates of sinusoidal frequencies*", IEEE Trans. Signal processing*, vol. 39, no.8, pp.1836-1847, 1991.

[14] P.Stoica and A.Eriksson, "MUSIC estimation of real-valued sine-wave frequencies", *Signal processing*, vol.42, no.2, pp.139-146, 1995.

[15] M.A.Altinkaya, H.Delic, B.Sankur and B.Anarim, "Subspace-based frequency estimation of sinusoidal signals in alpha-stable noise", *Signal processing*, vol.82, no.12, pp.1807-1827,2002.

[16] K.Mahata, "Subspace fitting Approaches for Frequency Estimation using Real-Valued data", *IEEE Trans. Signal processing*, vol.53 , no.8, pp.3099-3110, 2005.

[17] Shigeru Ando, Takaaki Nara ," An Exact Direct Method of Sinusoidal Parameter Estimation Derived From Finite Fourier Integral of Differential Equation" *IEEE transactions on signal processing*, vol. 57, no. 9, pp3317-3329.

[18] K.W.Chan and H.C.So, "Accurate Frequency Estimation for Real Harmonic Sinusoids*", IEEE signal processing Lett*., vol.11, no.7, pp.609-612, 2004.

[19] H. Shatnawi, Qasaymeh M. M, Gami Hiren, M.E. Sawan "An Improved Frequency Estimator" *Proceedings of the 5th Annual GRASP Symposium, Wichita State University, 2009.*

[20] Kaushik Mahata and Torsten Söderström, "ESPRIT-Like Estimation ofReal-Valued Sinusoidal Frequencies", *IEEE transactions on signal processing,* vol. 52, NO. 5, pp-1161-1170 May-2004.

[21] P.Palanisamy, Sambit Prasad Kar , "Estimation of Real-Valued Sinusoidal Signal Frequencies based on ESPRIT and Propagator methods", in proc *IEEE-International Conf. on Recent Trends in Information Technology, ICRTIT 2011.* pp.69-73 June-2011.

[22] Jingmin Xin, Akira Sano, "Computationally Efficient Subspace-Based Method for Direction-of-Arrival Estimation Without Eigen decomposition" *IEEE transactions on signal processing,* vol. 52, No. 4, , pp-876-893 ,April 2004.

[23] Guangmin Wang, Jingmin Xin, Nanning Zheng, Akira Sano, "Computationally Efficient Subspace-Based Method for Two-Dimensional Direction Estimation With L-Shaped Array*", ieee transactions on signal processing,* vol. 59, no. 7, , pp-3197-3212 July 2011

[24] S. Marcos, A. Marsal, and M. Benidir, "The propagator method for source bearing estimation," *Signal Process.* vol. 42, no. 2, pp. 121–138, 1995.

[25] H. L. Van Trees, Optimum Array Processing, Part IV of Detection, Estimation, and Modulation Theory. New York: Wiley, 2002.

[26] S. M. Kay, Modern Spectral Estimation. Upper Saddle River, NJ: Prentice-Hall, 1988