Periodic Control of a Wastewater Treatment Process to Improve Productivity

Muhammad Rizwan Azhar and Emadadeen Ali

Abstract—In this paper, periodic force operation of a wastewater treatment process has been studied for the improved process performance. A previously developed dynamic model for the process is used to conduct the performance analysis. The static version of the model was utilized first to determine the optimal productivity conditions for the process. Then, feed flow rate in terms of dilution rate i.e. \( \frac{Q}{Q_{in}} \) is transformed into sinusoidal function. Nonlinear model predictive control algorithm is utilized to regulate the amplitude and period of the sinusoidal function. The parameters of the feed cyclic functions are determined which resulted in improved productivity than the optimal productivity under steady state conditions. The improvement in productivity is found to be marginal and is satisfactory in substrate conversion compared to that of the optimal condition and to the steady state condition, which corresponds to the average value of the periodic function. Successful results were also obtained in the presence of modeling errors and external disturbances.

Keywords—Dilution rate, nonlinear model predictive control, sinusoidal function, wastewater treatment.

I. INTRODUCTION

A large number of technologies are present to treat different kinds of waste in the water to appropriate level, but conventional techniques which are being used in the industrialized countries are not only expensive to build but also require high cost for operation and maintenance. That is why a lot of research is being carried out to develop cost effective treatment technologies which can be appropriate in rural, semi urban, isolated communities and a variety of industrial simulation [1]. Generally wastewater treatment plants (WWTP) are combinations of complex nonlinear systems, subject to large disturbances in which different physical (such as settling) and biological phenomena are taking place. A number of control strategies have been proposed in the literature for wastewater treatment plants but their comparison and evaluation are not easy. This is somewhat due to the variability of the influent, to the physical and biochemical phenomena and to the large range of time constants (from few minutes to several days) inherent in the activated sludge process [2]. Recent work on WWTP involves using genetic algorithms to minimize effluent concentration and operating cost [3]. Menzi and Steiner [4] reported their effort with controlling nitrogen-eliminating WWTP using a model based multivariable control based on model predictive control concept and H∞ theory. An Internal Model Control (IMC) was also examined for the control of the effluent concentrations of a bioreactor used for ethanol production from glucose [5]. The model predictive control (MPC) algorithms are different from the other advanced classes of controllers in a way that a dynamic problem is solved on-line each control execution [6]. Due to some distinguished features such as constraint handling and superiority for processes having a large number of manipulated and controlled variables, it became the most widely used control system in chemical industries [7-8]. Ali and Ali [9] utilized NLMPC to control the molecular weight distribution of polyethylene product. Due to the predictive nature and dynamic optimization of NLMPC, the controller was able to recognize the need to operate the process in a periodic fashion in order to achieve the desired objectives. Ali et al studied the periodic operation of a reverse osmosis desalination process using nonlinear model predictive control, resulted in enhanced process operation [10]. O’ Brien et al applied MPC for the activated sludge in Lancaster, North England in real time and ended up with the results that up to 25% cost for aeration has been reduced using MPC over the previous existing control system. These findings are valid for low BOD load [11]. Due to success of MPC on improving the performance of chemical processes as mentioned above, the technique will be adopted for WWTP. The objective is not to make the output follow periodic trajectories but to force the process to operate in periodic fashion to improve the overall performance.

II. PROCESS MODEL

A. Reactor without Recycle

We will take the case of the bioreactor shown in Fig. 1, but without recycle and clarifier. The dynamic model for this process is taken from Zhao and Skogestad [12] using the following assumptions:

- Reactor influent contains no biomass.
- The reactor is operated under aerobic condition which means the quantity of oxygen to carry out the reaction is adequate.
- The kinetic model does not comprise cell maintenance and cell death.

The dynamic equations are given as follows:

\[
\dot{X} = rX - DX \\
\dot{S} = D(S_i - S) - \frac{r}{r_i} X
\]
where the specific growth rate ($r$) also known as Monod reaction rate is defined as follows:

$$r = \frac{\mu S}{K + S}$$  \hspace{1cm} (3)

The nominal plant steady state operating condition is given in Table I.

<table>
<thead>
<tr>
<th>D (l/h)</th>
<th>S_i (g/l)</th>
<th>X (g/l)</th>
<th>S (g/l)</th>
<th>$\mu$ (l/h)</th>
<th>K (g/l)</th>
<th>Y (g/g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.17</td>
<td>1.0</td>
<td>0.38</td>
<td>0.05</td>
<td>0.55</td>
<td>0.09</td>
<td>0.3</td>
</tr>
</tbody>
</table>

**B. Reactor with Recycle**

Now we consider the bioreactor with recycle as shown in Fig. 1.

The dynamic model for the complete process is taken from Sundstrom et al. [13] with the following assumptions:

- There is no reaction in the settler so that the substrate concentration in the recycle flow is equal to that in the reactor effluent.
- The dynamic of clarifier is neglected.

The surface area of the clarifier is so large such that the biomass concentration leaving the settler is zero.

The assumptions used for no recycle case are also applied in this case. However, cell maintenance is corrected here. The resulted differential equations are given as follows:

$$\dot{X} = DU X_r - D(1 + U)X + rX - k_d X$$  \hspace{1cm} (4)
$$\dot{S} = D(S_i - S) - \frac{rX}{Y}$$  \hspace{1cm} (5)
$$X_r = \frac{X(1+U)}{(D+UW)}$$  \hspace{1cm} (6)

The endogenous decay constant, $k_d$, is taken equal to 0.005 l/h in this study.

**III. OPTIMAL PLANT OPERATING CONDITIONS**

Using this type of model encounters specific phenomena related to the process conditions; with very low feed rate, the cell will die out of starvation because no enough nutrition is provided rapidly to maintain the cell metabolism and, when the feed is too high, the residence time decreases to a level that there will be no sufficient time for the cell (biomass) to grow resulting in any reaction, i.e., no conversion of the substrate. This is known as ‘washout’. This occurs only when no cell recycle is used. In that case, avoiding washout impose an upper bound on the feed flow rate as follows [12]:

$$D < \mu \frac{(1 + \overline{S}_i)/(\overline{S}_i - \beta(1 + \overline{S}_i))}{D_c}$$  \hspace{1cm} (7)

where $\beta = k_d/\mu$ and $\overline{S}_i = S/K$.

The reaction is autocatalytic so recycle can improve the conversion by increase in cell concentration but it may deteriorate performance. This is because recycle dilute the substrate and lowers the residence time. According to Sundstrom et al. [12], if $D > D_c$, the fractional conversion of input substrate increases monotonically with both increasing recycle ratio ($U$) and recycled cell concentration ($X_r$). For this case, Sundstrom et al. [12] illustrated that conversion increases with recycle ratio only if $X_r$ exceeds certain critical value ($X_{rc}$) which is given as follows:

$$X_r > \frac{\overline{S}_i}{\overline{S}_i + \beta} = \frac{1}{\gamma - 1 - \beta} = X_{rc}$$  \hspace{1cm} (8)

where $\overline{S}_i$ and $\beta$ are defined as before, $\gamma = \mu/D$ and $\overline{X}_r = X_r/KY$. For our case $X_{rc}$ is found to be 0.2286 g/l and $D_c$ to be 0.56 l/hr. Thus, based on the above situations, one can obtain the optimum operating condition of the reactor that maximize the substrate conversion and avoid washout by solving the following optimization problem:

$$\min_{x, S, U, D, X_r} \frac{X_r - \overline{S}_i}{\overline{S}_i} + D$$  \hspace{1cm} (9)

subject to:

$$\dot{X} = 0$$
$$\dot{S} = 0$$
$$0 \leq D < D_c$$

Fig. 1 Schematic diagram of activated sludge process
The maximum conversion occurs at high throughput so the second term is added in the objective function to ensure maximum conversion. The optimum conditions are by solving (9) in table II using MATLAB software and listed in table II. It is more interesting to demonstrate the effect of varying $U$ and $X_r$ on the substrate conversion while fixing $D$ at its optimal value listed in table II. The result is shown in Fig. 2.

![Conversion plot](image)

**Fig. 2 Conversion of substrate; dotted curve: $X_r = 10$, dashed curve: $X_r = 5$, dash-and-dot curve $X_r = 1$, solid curve: $X_r = 0.2286$, dash-and-double dots curve: $X_r = 0.1**

**TABLE II

<table>
<thead>
<tr>
<th>Optimum Operating Point</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_r$ (g/l)</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>0.315</td>
</tr>
</tbody>
</table>

IV. THE CONTROL OBJECTIVE AND IMPLEMENTATION

There is limited number of control objectives in biological reactors. Traditionally, the control outputs for the process are dissolved oxygen concentration in the aerator, biomass and substrate concentrations in the reactor effluent and the liquid level in the settler. The only controlled variable in our case is the productivity of the process i.e. $P_r/D(S_r-S)$; we have two variables in one single controlled variable so it is difficult to control such a process. The steady state analysis revealed the maximum productivity is 0.296 g/h which occurs at $D=0.39$/h. Conceptually, this maximum cannot be exceeded at steady state. Therefore, we seek operating the process in periodic fashion such that the average value of the productivity would exceed the maximum value. The idea is to make advantage of the variable feed rate which is very common in waste treatment processes. In addition, periodic stepping of the feed creates sudden transition from one steady state to another. Therefore, these sudden changes may result in variable values for the productivity leading to an average value beyond the expected maximum. The common disturbances to the system are feed flow rate, and inlet substrate and biomass concentrations. The only manipulated variable is the dilution rate, $D$. In this paper the objective is to operate the process cyclically above the optimum value of productivity in case of steady state plant operation at maximum point.

V. NON-LINEAR MPC ALGORITHM

In this work, the structure of the MPC version developed by Ali and Zafiriou [6] that utilizes directly the nonlinear model for output prediction is used. A usual MPC formulation solves the following on-line optimization problem:

$$\min_{\Delta u(t_k)...\Delta u(t_{k+M-1})} \sum_{i=1}^{M} ||y(t_{k+i}) - R(t_{k+i})||^2 + \sum_{i=1}^{M} ||A\Delta u(t_{k+i-1})||^2$$

subject to

$$A^T\Delta u(t_k) \leq b$$

For nonlinear MPC, the predicted output, $y$ over the prediction horizon $P$ is obtained by the numerical integration of:

$$\frac{dx}{dt} = f(x,u,t)$$

$$y = g(x)$$

from $t_k$ up to $t_{k+P}$ where $x$ and $y$ represent the states and the output of the model, respectively. The symbols $\| \cdot \|$ denotes the Euclidean norm, $k$ is the sampling instant,$\Lambda$ and $\Lambda$ are diagonal weight matrices and $R = [r(k+1) \cdots r(k+P)]^T$ is a vector of the desired output trajectory. $\Delta u(t_k) = [\Delta u(t_k) \cdots \Delta u(t_{k+P-1})]$ is a vector of $M$ future changes of the manipulated variable vector $u$ that are to be determined by the on-line...
optimization. The control horizon \((M)\) and the prediction horizon \((P)\) are used to adjust the speed of the response and hence to stabilize the feedback behavior. The parameters \(\Gamma\) and \(A\) are weights on outputs and inputs. The objective function (Eq. 10) is solved on-line to determine the optimum and hence to stabilize the feedback behavior. The parameters amplitude and period of oscillation. The controlled output cycle period \(p\) as follows:

\[ \Delta U(t_k) = \text{implementation on the plant. At the next sampling instant, the whole procedure is repeated. In order to compensate for modeling errors and eliminate steady state offset, a regular feedback is incorporated on the output predictions, } y(t_{k+1}) \text{ through an additive disturbance term. Therefore, the output prediction is corrected by adding to it the disturbance estimates. The latter is set equal to the difference between plant and model outputs at present time } k \text{ as follows:} \]

\[ d(k) = y_p(k) - y(k) \quad (14) \]

The disturbance estimate, \(d\) is assumed constant over the prediction horizon due to the lack of explicit means for predicting the disturbance. The simulation results will be presented by transforming dilution rate \(D\) into sinusoidal function as follows for NLMPC in discrete time fashion

\[ D(t_k) = D_{ss} + A_m \sin(\beta) \quad (15) \]

where \( A_m \) is the period amplitude, \( t_k \) is the sampling instant and \( \beta \) is the argument of the sin function that includes the cycle period \( p \) as follows:

\[ \beta = \frac{2\pi t_k}{p} \quad (16) \]

The, NLMPC will manipulate the feed flow rate in terms of \( D \) indirectly through regulating its input characteristics, i.e. the amplitude and period of oscillation. The controlled output embedded in (10) includes time average value of productivity. This output is defined as ratio to its corresponding steady state value as follows

\[ y(t_k) = \frac{\int_0^p Pr(c)dt}{\int_0^p Pr_{cst}(c)dt} \quad (17) \]

In discrete time formulation, the numerical integration can be approximated by summation over the predefined simulation time. For future prediction, the model equations can be numerically integrated over the future \( p \) horizon from \( t = 0 \) to \( t = t_{k+p} \) to estimate the average value for the controlled outputs at \( t_{k+p} \).

VI. RESULTS AND DISCUSSION

The objective in this study is to maximize ratio of average value of productivity to the steady state value of productivity at the maximum point. The comparison between the two cases (non-periodic and periodic) feed conditions is necessary to clearly demonstrate the effectiveness of periodic operation. So first of all proposed NLMPC algorithm will be implemented for the case when no periodicity is introduced in the dilution rate. Implementation of the proposed NLMPC algorithm for servo problem is shown in Fig. 3 where the set point is fixed at 10% increase over the maximum productivity value. The steady state starting value for \( D \) is taken arbitrarily as 0.3 l/h, at which the corresponding value of productivity is 0.2636 g/h. The upper and lower limits on \( D \) are set to \( \pm 0.3 \) l/h. A sampling time of 0.5 h is used in the simulation. The NLMPC parameter values are \( M = 1, P = 1, A = [0 0] \) and \( \Gamma = [1] \). The closed loop response of the process is shown in the Fig. 3. It is clear from the Fig. 3 that controller even could operate the process to get productivity greater than 1 but could not reach to our objective which is 10% increase over the maximum. The increase in productivity is a result of, the ability of MPC to handle constraints on inputs, as in our case controller was allowed to operate in the range of input values which is \( D \) i.e. \( \pm 0.3 \) l/h. As clear from Fig. 3 that the controller forced the input greater than 0.3 l/h and as a result the ratio of average value of \( D \) to its steady state initial value is 1.61 which indicates that to get higher productivity than optimal 61% more feed rate is required which means high pumping cost.

The servo problem is tested again using the periodic control. The period per sampling time \( (P)\) is constrained between 3 and 10. Note that 3 is the minimum value that allows for complete periodic behaviour within the given sampling time and simulation interval. A sampling time of 0.5 h is used in the simulation. The NLMPC parameter values are \( M = 1, P = 2, A = [0 0] \) and \( \Gamma = [1] \). The starting value of \( D \) is 0.3 l/h, as the same which used in case of non-periodic study. The simulation outcome is shown in Fig. 4 which illustrated the ability of MPC to generate oscillatory response which resulted in a reasonable improvement of the process operation. For example 10% increase in the productivity over the maximum steady state value is observed. The interesting part is that the enhancement was achieved without additional increment in the feed conditions. In fact, the ratio of the time-average value of the dilution rate \( D \) to its corresponding steady state value is 0.86. This is the main goal after periodic operation, i.e. with less input e.g. 14% less than the steady state, 10% increase in the productivity is made possible in case of periodic operation.
The simulation is carried out in case of perfect model. Now the control objective is repeated in the presence of parametric errors in the model. As environmental conditions have significant influence on the quality of wastewater and so as on the modelling parameters. Temperature, PH, etc. due to uncertain environmental conditions can disturb modelling parameters. Specifically, 25% step change in Y which is yield coefficient, 10% step change in maximum growth constant \( \mu \) and 10% step change in saturation constant K will be introduced in the model for simulation study. The closed-loop response under these circumstances is depicted in Fig. 5.

The simulation time and NLMPC parameters values are the same which are used in case of periodic case as before. Despite the effect of the model-plant mismatch, NLMPC was able to move the process operation to an arbitrary point towards 10% increase in productivity. However, the obtained process response does not exactly match that resulted from utilizing perfect model. It is equally important to examine the effectiveness of the regulatory performance of the NLMPC for the WWT process. The feed load conditions are not always same in case of WWT process so now we will take the case when 10% step change in the substrate concentration is introduced in the influent. The NLMPC parameters are same as before for the periodic case, simulation is run for 200 h of plant operation. The simulation result is shown in Fig. 6. The controller performance was satisfactory in the presence of external disturbance. Although, the process output have some uneven behavior but still follow the desired trajectory, and operate the process towards 10% increase in productivity.

VII. CONCLUSION

The operation of a waste water treatment process under forced periodic inputs in order to improve its performance is investigated. A previously developed model for wastewater treatment plant, consisting of a bioreactor with recycle and a clarifier, is used for steady state and dynamic analysis of the process. The steady state model is used to determine the optimal operating conditions of the plant that maximize the substrate conversion and avoid washout situations. The dynamic model is used to investigate the performance of standard Non-linear Model Predictive Control algorithm for the single control objective of the process i.e. enhanced process productivity. The periodic forcing is imposed via feedback control. NLMPC regulates the dilution rate indirectly through manipulating its transformation parameters. The feedback simulation indicated the effectiveness of NLMPC to generate periodic input functions that helped in enhancing the time-averaged value of the productivity. Improvement of up to 10% increase in the process productivity is observed. The promising outcome is maintained even in the presence of model uncertainty and in the sudden injection of input disturbance.

REFERENCES


