T-DOF PID Controller Design using Characteristic Ratio Assignment Method for Quadruple Tank Process

Tianchai Suksri, U-thai Sritheeravirojana, Arjin Numsonman, Viriya Kongrattana, and Thongchai Werataweemart

Abstract—A control system design with Characteristic Ratio Assignment (CRA) is proven to be effective for SISO control design. But the control system design for MIMO via CRA is not a concrete procedure. In this paper, the control system design method for quadruple-tank process via CRA is presented. By using the decentralized method for both minimum phase and non-minimum phase, the results from PI and PID controller design via CRA can be illustrated the validity of our approach by MATLAB.

Keywords—CRA, Quadruple-Tank.

I. INTRODUCTION

GENERALLY, the performance analysis of control system focuses on time domain response such as percent overshoot, rise time, setting time, and steady state error. Although there are many methods to design the controller, it is a few approaches that can achieve the desired response. The characteristic ratio assignment (CRA) is one technique based on the definition of characteristic equation that is a famous method at present [1] [2].

To design controller by CRA method [3], adjustment of speed response and the damping ratio can be done by only one parameter. Therefore, this technique is convenient and suitable for tuning controller under the requirement of the system.

In this paper, the quadruple-tank process, the interactive process is controlled by PI controller for case of minimum phase and PID controller for case of non-minimum phase based on CRA method. The coefficient of characteristic ($a_i$) and time constant ($\tau_i$) are the parameters to determine the characteristic equation of the control system that is necessary to design the time domain system.

II. QUADRUPLE-TANK PROCESS

Consider the quadruple-tank-process shown in Fig. 1. This laboratory process has been used to illustrate many issues in multivariable control [4]. The target is to control the level in the lower two tanks with two pumps. The process inputs are $u_i$ and $u_i$ (input voltages to the pumps) and the outputs are $y_i$ and $y_i$ (voltage from level measurement devices). The linearised dynamics for the process is given as:

$$G(s) = \begin{bmatrix} \gamma_i c_i / (1 + s\tau_i) & (1 - \gamma_i) c_i / (1 + s\tau_i) \\ (1 - \gamma_i) c_i / (1 + s\tau_i) & \gamma_i c_i / (1 + s\tau_i) \end{bmatrix}$$

$$T_i = \frac{A_i}{\beta_i a_i \sqrt{\frac{2h_i}{g}}} , \ i = 1, \ldots, 4$$

and $c_i = T_i k_i / A_i$, $\beta_i = T_i k_i / A_i$. Here $A_i$ is the cross-sectional area of tank $i$, $a_i$ is the cross-sectional area of the outlet of tank $i$, $\beta_i$ is the flow ratio at the outlet of tank $i$, $h_i$ is the steady-state water level, $k_i$ is the gain of the pump $i$, $\gamma_i$ is the measurement gain, and $g$ is the acceleration of gravity. The parameters $\gamma_i, \gamma_i \in (0,1)$ are determined from how the valves are prior set to an experiment; the flow to tank 1 is proportional to $\gamma_1$, the flow to tank 4 is proportional to $(1 - \gamma_1)$, and similarly for $\gamma_2$ with respect to tank 2 and tank 3.

Since

$$\det G(s) = \frac{C_1 C_2}{\gamma_1 \gamma_2} \prod_{i=1}^{4} \frac{(1 + s\tau_i) (1 + s\tau_i) - (1 - \gamma_i)(1 - \gamma_i)}{\gamma_i \gamma_i}$$

Fig. 1 Schematic diagram of the quadruple-tank process
and transfer matrix $G$ has two finite zeros for $\gamma_1, \gamma_2 \in (0, 1)$.

The system is non-minimum phase for $0 < \gamma_1 + \gamma_2 < 1$, and minimum phase for $1 < \gamma_1 + \gamma_2 < 2$. Hence, by changing a single valve we can make the multivariable level control problem more or less difficult.

The relative gain (RGA) was introduced by Bristol [5] as a measure of interaction in multivariable control systems. The RGA $\Lambda$ is defined as $\Lambda = G(0)^T G(0)$, where the asterisk denotes the schur product (element-by-element matrix multiplication) and $-T$ inverse transpose. It is possible to show that the elements of each row and column of $\Lambda$ sum up to one, so for a $2 \times 2$ system the RGA is determined by the scalar $\gamma = \Lambda_{11}$. The RGA is used as a tool mainly in the process industry to decide on control structure issues such as input-output pairing for decentralized controllers [6]. The RGA of the Quadruple-Tank Process is given by the simple expression

$$\gamma = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 - 1}, \quad \text{where} \quad \Lambda = \begin{bmatrix} \lambda & 1 - \lambda \\ 1 - \lambda & \lambda \end{bmatrix}$$

III. STRUCTURE OF THE CONTROL SYSTEM WITH PI AND PID CONTROLLER

From RGA analysis suggests that input-output pairing for decentralized control be chosen. In case of minimum phase system, transfer function $G_{11}$ and $G_{22}$ are used for design the controller, but the case of non-minimum phase system the transfer function $G_{12}$ and $G_{21}$ are instead used. The structure of MIMO control system using PI and PID controller for minimum phase and non-minimum phase are shown in Fig. 2 and Fig. 3 respectively, for case of minimum phase

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} = \begin{bmatrix} B_{11}(s)B_{21}(s) & B_{12}(s)B_{22}(s) \\ A_{11}(s)A_{21}(s) + B_{11}(s)B_{21}(s) & A_{12}(s)A_{22}(s) + B_{12}(s)B_{22}(s) \end{bmatrix}$$

The transfer functions used for design are:

A. Case of Minimum Phase

Loop 1 (Y1-R1),

$$Y_1(s) = \frac{B_{11}(s)B_{21}(s)}{A_{11}(s)A_{21}(s) + B_{11}(s)B_{21}(s)}.$$  \hspace{1cm} (4)

Characteristic equation is

$$P_{11} = A_{11}(s)A_{21}(s) + B_{11}(s)B_{21}(s) = a_1s^3 + (a_2 + K_{11}K_{12})s + K_{11}K_{11}.$$  \hspace{1cm} (5)

Loop 2 (Y2-R2),

$$Y_2(s) = \frac{B_{12}(s)B_{22}(s)}{A_{12}(s)A_{22}(s) + B_{12}(s)B_{22}(s)}.$$  \hspace{1cm} (6)

Characteristic equation is

$$P_{22} = A_{22}(s)A_{21}(s) + B_{12}(s)B_{22}(s) = d_1s^3 + (d_2 + K_{21}K_{22})s + K_{21}K_{21}.$$  \hspace{1cm} (7)

B. Case of Non-Minimum Phase

Loop 1 (Y1-R2),

$$Y_1(s) = \frac{B_{21}(s)B_{22}(s)}{A_{11}(s)A_{22}(s) + B_{21}(s)B_{22}(s)}.$$  \hspace{1cm} (8)

Characteristic equation is

$$P_{12} = A_{12}(s)A_{22}(s) + B_{12}(s)B_{22}(s) = b_1s^3 + (b_2 + K_{12}K_{22})s^2 + (b_3 + K_{12}K_{12})s + K_{12}K_{12}.$$  \hspace{1cm} (9)

Loop 2 (Y2-R1),
The characteristic equation is
\[ P(s) = A(s)B(s) + B(s)A(s) = \cdots + c_1s + c_0 \]
where \( A(s) \) and \( B(s) \) are polynomials of \( s \).

The CRA method is designed for tuning only one parameter that follows the characteristic ratio and the inverse form of characteristic equation
\[ G(s) = \frac{a_0}{s^{n+1} + \cdots + a_{n-1}s + a_n} \]

Where \( a_i \geq 1.2374a_i' \) and \( a_i', a_i < 1 \), to obtain the system stability, which is given by
\[ \sqrt{a_i a_i'} < 1.4656, i = 1, \ldots, n \]

The CRA method can be tuned by changing a value of time constant as shown in the next equation. Assume, the transfer function is given as
\[ G(s) = \frac{a_0}{a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1s + a_0} \]

Where \( a_i \geq 1.2374a_i' \) and \( a_i', a_i < 1 \), to obtain the system stability, which is given by
\[ \sqrt{a_i a_i'} < 1.4656, i = 1, \ldots, n \]

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\[ \sqrt{a_i a_i'} < 1.4656, i = 1, \ldots, n \]
then $0 < k < 1$; damping ratio will be decreased.

$$ G_k(s) = \frac{k^{\frac{1}{2}}}{a_0 s^{\frac{1}{2}} + k^{\frac{1}{2}} a_1 s + \ldots + k^{\frac{1}{2}} a_n s^{\frac{1}{2}} + a_0} $$

(31)

V. THE SIMULATION RESULTS

In this paper, the simulation results of the quadruple-tank process for case of minimum and non-minimum phase, the interactive process is given by MATLAB. The experiment of PI and PID control based on CRA illustrates the adjustment of speed response and damping ration. Parameters and operating point show detail in Table I and Table II.

### Table I

<table>
<thead>
<tr>
<th>Parameters of Quadruple-Tank Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1, A_2, A_3, A_4, \text{cm}^2$</td>
</tr>
<tr>
<td>69.3978</td>
</tr>
</tbody>
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### Table II

<table>
<thead>
<tr>
<th>Operating Point</th>
<th>Minimum Phase</th>
<th>Non-Minimum Phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(h_1^<em>, h_2^</em>) \text{cm}$</td>
<td>(11.1, 11.6)</td>
<td>(10.8, 11.5)</td>
</tr>
<tr>
<td>$(h_3^<em>, h_4^</em>) \text{cm}$</td>
<td>(0.68, 0.39)</td>
<td>(7.33, 3.99)</td>
</tr>
<tr>
<td>$(a_1^<em>, a_2^</em>) kV$</td>
<td>(5, 5)</td>
<td>(5, 5)</td>
</tr>
<tr>
<td>$(k_1, k_2)$ cm$^2$/V$\cdot$s</td>
<td>(2.6972, 2.3949)</td>
<td>(2.7887, 2.3950)</td>
</tr>
<tr>
<td>$(\gamma_1, \gamma_2)$</td>
<td>(0.791, 0.772)</td>
<td>(0.328, 0.237)</td>
</tr>
<tr>
<td>$(\beta_1, \beta_2)$</td>
<td>(0.4759, 0.4124)</td>
<td>(0.4759, 0.4124)</td>
</tr>
<tr>
<td>$(\beta_3, \beta_4)$</td>
<td>(0.3837, 0.5388)</td>
<td>(0.3837, 0.5388)</td>
</tr>
</tbody>
</table>

V. I Case of Minimum Phase

A. Step Response of Nominal Parameter

Loop 1 (Y1-R1)

$$ G_{y1}(s) = \frac{3.4347}{111.7228 s + 1} $$

According to equation (32), design of PI controller based on CRA, is assigned parameter follow as $\alpha = 2$ and $\tau = 20$

$$ P_{y1}(s) = 111.7228 s^2 + 11.1722 s + 0.5586 $$

and the parameter of PI controller is given by $K_p = 2.9616$, $K_i = 0.1626$

Loop 2 (Y2-R2)

$$ G_{y2}(s) = \frac{3.5113}{131.7973 s + 1} $$

In the same manner, the $P_{y2}(s)$ is

$$ P_{y2}(s) = 131.7973 s^2 + 13.1797 s + 0.659 $$

and the parameter of PID controller is given by $K_p = 3.4687$, $K_i = 0.1877$.

B. The Adjustment Speed of Response

By using CRA method, the adjustment of speed response can be produced by adjusting value of $k$ according to equation (25).

Loop 1 (Y1-R1)

$$ G_k(s) = \frac{k^{\frac{1}{2}} 0.5586}{111.7228 s^2 + k^{\frac{1}{2}} 11.1722 s + k^{\frac{1}{2}} 0.5586} $$

Loop 2 (Y2-R2)

$$ G_k(s) = \frac{k^{\frac{1}{2}} 0.659}{131.7973 s^2 + k^{\frac{1}{2}} 13.1797 s + k^{\frac{1}{2}} 0.659} $$

When $k$ is adjust from 1.3 to 2 respectively, settling time is decrease from 64 sec to 42 sec for tank 1 and tank 2, while interaction for tank 1 is decrease from 4.3 percent to 3 percent and interaction for tank 2 is decrease from 5.5 percent to 4 percent, therefore the system response is faster and interaction is decrease.
C. The Adjustment of Damping Ratio

By using CRA method, the adjustment of damping ratio can be produced by adjusting value of $k$ according to equation (31).

Loop 1 (Y1-R1)

$$G_1(s) = \frac{k_0.5586}{111.7228s^2 + k111.1722s + k0.5586}$$

Loop 2 (Y2-R2)

$$G_2(s) = \frac{k_0.659}{131.7973s^2 + k13.1797s + k0.659}$$

When $k$ is changed from 1.2 to 1.4 respectively then the damping ratio is also increased but the overshoot is decreased from 1.7 percent to 0.5 percent and interaction is decreased from 4.5 percent to 4 percent for tank 1 and decreased form 1.5 percent to 0.6 percent and interaction is decreased from 5.5 percent to 5 percent for tank 2.

V.II Case of Non-Minimum Phase

Loop 1 (Y1-R2)

$$G_{1,1}(s) = \frac{2.9019}{12409.705s^2 + 228.8107s + 1}$$

According to equation (34), design of PID controller based on CRA, is assigned parameter follow as $\alpha_1 = 2.5, \alpha_2 = 2$ and $\tau = 200$

$$P_{1,1}(s) = 12409.705s^3 + 310.2417s^2 + 3.878s + 0.0194$$

and the parameter of PID controller is given by $K_{1,1} = 30.1292, \ K_{1,2} = 0.9918, \ K_{1,3} = 0.0067$

Loop 2 (Y2-R1)

$$G_{1,2}(s) = \frac{3.5437}{7760.889s^2 + 190.3685s + 1}$$

In the same manner, the $P_{1,2}(s)$ is

$$P_{1,2}(s) = 7760.889s^3 + 194.0221s^2 + 2.4253s + 0.0121$$

and the parameter of PID controller is given by $K_{2,1} = 1.031, \ K_{2,2} = 0.4022, \ K_{2,3} = 0.0034$. 
The step response in Fig. 4(a) is for tank 1 and Fig. 4(b) is for tank 2. For tank 1 the overshoot is 7.5 percent, settling time is 880 sec and interaction is 36 percent. For tank 2 the overshoot is 9.5 percent, settling time is 920 sec and interaction is 44 percent.

B. The Adjustment Speed of Response
By using CRA method, the adjustment of speed response can be produced by adjusting value of $k$ according to equation (25).

\[
G_1(s) = \frac{k^30.0194}{12409.705s^3 + k^3310.2417s^2 + k^33.878s + k^30.0194}
\]

Loop 2 (Y2-R1)
\[
G_2(s) = \frac{k^30.0121}{7760.889s^3 + k^3194.0221s^2 + k^32.4253s + k^30.0121}
\]

C. The Adjustment of Damping Ratio
By using CRA method, the adjustment of damping ration can be produced by adjusting value of $k$ according to equation (31).

Loop 1 (Y1-R2)
\[
G_1(s) = \frac{k^30.0194}{12409.705s^3 + k^3310.2417s^2 + k^33.878s + k^30.0194}
\]

Loop 2 (Y2-R1)
\[
G_2(s) = \frac{k^30.0121}{7760.889s^3 + k^3194.0221s^2 + k^32.4253s + k^30.0121}
\]

When $k$ is changed from 1.1 to t.2 respectively then the damping ration is also increased but the overshoot is decreased from 2.5 percent to 0 percent and interaction is decreased from 31 percent to 27.5 percent for tank 1 and decreased from 3.5 percent to 0 percent and interaction is decreased from 38 percent to 33 percent for tank 2.

VI. CONCLUSION
In this paper, the design of PI and PID controller using CRA for quadruple-tank is presented; the simulation results
from MATLAB for both minimum phase and non-minimum phase are able to illustrate the advantage which only one parameter is to be adjusted. Thus, our scheme is convenient and suitable for designing and tuning the controller.

REFERENCES