Abstract—Over the past years, the EMCCD has had a profound influence on photon starved imaging applications relying on its unique multiplication register based on the impact ionization effect in the silicon. High signal-to-noise ratio (SNR) means high image quality. Thus, SNR improvement is important for the EMCCD. This work analyzes the SNR performance of an EMCCD with gain off and on. In each mode, simplified SNR models are established for different integration times. The SNR curves are divided into readout noise (or CIC) region and shot noise region by integration time. Theoretical SNR values comparing long frame integration and frame adding in each region are presented and discussed to figure out which method is more effective. In order to further improve the SNR performance, pixel binning is introduced into the EMCCD. The results show that pixel binning does obviously improve the SNR performance, but at the expensive of the spatial resolution.

Keywords—EMCCD, SNR improvement, pixel binning

I. INTRODUCTION

EMCCDs have revolutionized the world of low light imaging by bringing improved detection limits along with high readout rates [1]. Their unique feature is that they amplify the signal above the readout noise floor of the CCD [2]. This is achieved by an additional multiplication register inserted between the readout register and the output amplifier, as shown in Fig. 1 [3]. When signal electrons are transferred through the multiplication register, amplification occurs by harnessing a process known as impact ionization. The signal is multiplied in the charge domain and the readout noise is effectively reduced to subelectron level, which allows the EMCCD to run at much faster readout speeds while retaining an excellent detection limit. This pushes the boundary of conventional CCDs and makes EMCCDs sensitive to single electrons.

SNR PERFORMANCE AND SIMPLIFIED MODELS

For an EMCCD, the SNR can be defined as [9]:

\[
SNR = \frac{S}{\sqrt{F^2G^2(N_p^2 + N_d^2 + N_c^2 + N_r^2)}}
\]

where \(S\) is detected signal, \(N_{tot}\) is total noise, \(F\) is noise factor, \(G\) is multiplication gain, \(N_p\) is photon shot noise, \(N_d\) is dark current noise, \(N_c\) is clock induced charge noise and \(N_r\) is readout noise.

Here the integration time is introduced as the independent variable, and Equation (1) can be rewritten in...
\[ SNR = \frac{P \eta t}{\sqrt{F^2(P \eta t + I_d t + C) + \left(\frac{N_r}{G}\right)^2}} \]  

where \( P \) is mean incident photon flux, \( \eta \) is quantum efficiency of the EMCCD, \( t \) is integration time, \( I_d \) is dark current, \( C \) is clock induced charge (CIC).

Then we analyze the SNR in two operation modes: Gain Off and Gain on.

A. Gain Off

If the gain is off, an EMCCD is equal to a conventional CCD. CIC is completely buried in the readout noise and can be ignored. Noise factor is unity. Equation (2) is written as:

\[ SNR_{Gain\ Off} = \frac{P \eta t}{\sqrt{P \eta t + I_d t}} \]  

Photon shot noise and dark current noise are proportional to integration time. While integration time is very short, they are neglected and readout noise is dominated. Equation (3) can be simplified as:

\[ SNR_r = \frac{P \eta t}{N_r} \]  

As integration time is increased, photons detected by the EMCCD grow and dark current is also accumulated within the device. Photon shot noise and dark current noise exceeds readout noise, and Equation (3) can be simplified as:

\[ SNR_s = \frac{P \eta t}{\sqrt{P \eta + I_d}} \]  

The condition for which \( SNR_r \) and \( SNR_s \) give the same performance is given by:

\[ \frac{P \eta t_0}{N_r} = \frac{P \eta}{\sqrt{P \eta + I_d}} \sqrt{t_0} \]  

where \( t_0 \) is the integration time. Solving for \( t_0 \) then gives:

\[ t_0 = \frac{N_r^2}{P \eta + I_d} \]  

Fig.2 shows \( SNR_{Gain\ Off} \), \( SNR_r \) and \( SNR_s \) variation as a function of integration time for an cooled EMCCD with the following parameters: \( P = 100 \) photons/pixel/s, \( \eta = 50\% \), \( C = 0.1e/\)pixel, \( I_d = 0.05 e/\)pixel/s, \( N_r = 20 e \).

B. Gain On

Followed by discussion of the case \( G = 1000 \). For a gain greater than \( \approx 10 \), \( F \) approaches \( \sqrt{2} \). Effective readout noise \( \frac{N_r}{G} << 1 \) and can be ignored. Equation (6) is written as:

\[ SNR_{G=1000} = \frac{P \eta t}{\sqrt{2(P \eta + I_d t)}} \]  

For short integration time, CIC becomes the main noise and Equation (8) can be simplified as:

\[ SNR_c = \frac{P \eta t}{\sqrt{2C}} \]  

For long integration time, photon shot noise and dark current noise are main noise sources. Equation (8) can be simplified as:

\[ SNR_s = \frac{P \eta}{\sqrt{2(P \eta + I_d)}} \]  

The condition for which \( SNR_c \) and \( SNR_s \) give the same performance is given by:

\[ \frac{P \eta t_0}{\sqrt{2C}} = \frac{P \eta}{\sqrt{2(P \eta + I_d)}} \sqrt{t_0} \]  

where \( t_0 \) is the integration time. Solving for \( t_0 \) then gives:

\[ t_0 = \frac{C}{P \eta + I_d} \]
Fig. 3 shows $SNR_{G=1000}$, $SNR_c$ and $SNR_s$ variation as a function of integration time for an cooled EMCCD with the same parameters as Fig.2.

In Fig.3, $t_0 = 2\text{ms}$. For integration time less than $t_0$, CIC is leading noise and $SNR_c$ curve agrees with $SNR_{G=1000}$ well, whilst for integration time in excess of $t_0$, shot noise becomes dominant and $SNR_s$ is consistent with $SNR_{G=1000}$.

C. Long Frame Integration vs. Frame Adding

From the above analysis, the SNR performance of an EMCCD can be divided into two regions: readout noise (or CIC) region and shot noise region. We compare the SNR differences between long frame integration and frame adding in each region.

First, in readout noise (or CIC) region, long frame integration time is $1\text{s}$ with gain off and $10^{-3}\text{s}$ with $G=1000$. For frame adding, integration time is $0.1\text{s}$ with gain off and $10^{-2}\text{s}$ with $G=1000$. Secondly, in shot noise region, long frame integration time is $100\text{s}$ with gain off and $10^{-1}\text{s}$ with $G=1000$. For frame adding, integration time is $10\text{s}$ with gain off and $10^{-2}\text{s}$ with $G=1000$. The number of unit frames that are added together is $N$ ($N=10$). Frame adding multiplies the SNR by a factor of $\sqrt{N}$. The results are displayed in Tables 1 and 2.

### TABLE I

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<th>10</th>
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<td>——</td>
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<tr>
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<td>——</td>
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<tr>
<td>$\sqrt{N} SNR_s$</td>
<td>——</td>
<td>——</td>
<td>70.6753</td>
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</table>

Tables 1 and 2 show that long frame integration provides substantially better SNR values as compared to frame adding in readout noise (or CIC) region. In shot noise region, both results are the same. However, frame adding can reduce the random noise and it is preferable in shot noise region.

### III. PIXEL BINNING

Pixel binning is an image readout scheme, which combines adjacent pixel charges in both the horizontal and vertical direction into a single larger charge or “super pixel”. Binning of $1\times1$ means that the individual pixel is used as is. A binning of $2\times2$ means that 4 adjacent pixels have been combined into one larger pixel, and so on. Pixel binning effectively increase the pixel size while also increasing sensitivity [10]. The SNR of an EMCCD using pixel binning is given by:

$$SNR_{\text{Binning}} = \frac{MP\eta}{\sqrt{F^2 M (P\eta + I_d t + C) + \left(\frac{N_x}{G}\right)^2}}$$

(13)

where $M$ is the binning factor, which represents the number of pixels that are combined to form a larger pixel.

If the gain is off, Equation (13) is written as:

$$SNR_{\text{Binning,G=Off}} = \frac{MP\eta}{\sqrt{M(P\eta + I_d t + C) + N_x^2}}$$

(14)

In the case of $G=1000$, Equation (13) is written as:

$$SNR_{\text{Binning,G=1000}} = \frac{MP\eta}{\sqrt{2(P\eta t + I_d t + C)}}$$

(15)

Fig. 4 and 5 show SNR variation as a function of integration time for an cooled EMCCD with different binning factors. The parameters are the same as Fig. 2.
In Fig. 4 and 5, pixel binning does improve the SNR of the EMCCD. The larger the binning factor, the higher the SNR. Comparing Fig. 4 and 5, the latter provides better SNR for short integration time but loses advantage for long integration time. The reason is that gain effectively reduces readout noise, which is the dominate noise for short integration time in Fig. 4. As the integration time extends, shot noise is the main noise, and the noise factor introduced by gain degrades the SNR in Fig. 5.

As illustration of the above theoretical work, a set of pictures (Fig. 6) show the SNR improvements provided by pixel binning in an EMCCD system. The pictures are captured by the Luca Camera in a darkroom. An integrating sphere is coupled to the EMCCD as a light source with the illumination of 10^{-3}lx. The integration time is 20ms. From the top to the bottom, the SNR improves when binning more and more pixels. From the left to the right, gain provides better SNR performance when the binning factor is the same. However, comparing (b1) with (b3), we find that the drawback of pixel binning is a reduction in image resolution. To some extent, it limits the binning numbers.

IV. CONCLUSION

In this paper, the SNR performance of the EMCCD are divided into readout noise (or CIC) region and shot noise region by integration time. In readout noise (or CIC) region, long frame integration provides better SNR than frame adding. In contrast, frame adding outperforms.

Pixel binning offer benefits in faster readout speeds and improved SNR. In theory, the binning factor is only subject to the total pixel elements of the EMCCD. In fact, it is limited by full well and the spatial resolution. Therefore, we need to select an appropriate binning factor to achieve the balance between SNR and spatial resolution.

REFERENCES


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