The Banzhaf-Owen value for fuzzy games with a coalition structure

Fan-Yong Meng

Abstract—In this paper, a generalized form of the Banzhaf-Owen value for cooperative fuzzy games with a coalition structure is proposed. Its axiomatic system is given by extending crisp case. In order to better understand the Banzhaf-Owen value for fuzzy games with a coalition structure, we briefly introduce the Banzhaf-Owen value for two special kinds of fuzzy games with a coalition structure, and give their explicit forms.

Keywords—cooperative fuzzy game, Banzhaf-Owen value, multilinear extension, Choquet integral

I. INTRODUCTION

As an important kind of games, games with coalition structures proposed by Owen [1] have been researched by many scholars and experts. The payoff indices for games with coalition structures are researched by Owen [1, 2] and Alonso-Mejide and Fiestras-Janeiro [3]. The axiomatic systems of the payoff indices for games with coalition structures are considered in [1-8]. But all above are only introduced for crisp games.

As Aubin [9] pointed, there are some situations where players do not fully participate in a coalition, but take action only according to the rate of participation. Generally speaking, it is difficult to identify a characteristic function of fuzzy games in practice. Hence, the characteristic function of fuzzy games is often constructed on the basis of the characteristic function of the original crisp games. Owen [10] defined a kind of fuzzy games, which is called fuzzy games with multilinear extension form. Tsurumi et al. [11] defined a kind of fuzzy games with proportional value, and Tsurumi et al. [11]. A crisp coalition structure in N is denoted by (N, Γ). For any S0, S0 ∈ P(N, Γ) is called a feasible coalition, where P(N, Γ) denotes the set of all feasible coalitions in (N, Γ).

Example 1: Let N = {1, 2, 3, 4} and Γ = {B1, B2}, where B1 = {1, 2} and B2 = {3, 4}, then P(N, Γ) = {Φ, {1}, {2}, {3, 4}, {1, 2, 3, 4}, {1, 3, 4}, {2, 3, 4}, N}.

Owen [10] proposed the Banzhaf-Owen value for crisp games with a coalition structure, which can be expressed by

$$\varphi_i(N, v_0, \Gamma) = \sum_{R \subseteq \Gamma \setminus B_0^k \setminus T \subseteq P^k \setminus B_0} \frac{1}{2^{m-1}} \frac{1}{2^{n-1}} (v_0(Q_0 \cup T_0 \cup i) - v_0(Q_0 \cup T_0)) \quad \forall i \in N,$$

where Q0 = \cup_{Q \subseteq \Gamma \setminus B_0} B_0 \subseteq \Gamma is an union such that i \in B_0^k.

III. THE BANZHAF-OWEN VALUE FOR FUZZY GAMES WITH A COALITION STRUCTURE

Let L(N) denote the set of all fuzzy coalitions in N. The coalitions in L(N) are denoted by S, T, . . . . For a fuzzy coalition S and player i, S(i) indicates the membership grade of i in S, i.e., the rate of the ith player’s participation in S. For any S ∈ L(N), the support is denoted by SuppS = \{i ∈ N | S(i) > 0\}, and the cardinality is denoted by |SuppS|. We use the notation S ⊆ T if and only if S(i) = T(i) or S(i) = 0 for any i ∈ N. A function v : L(N) → \mathbb{R}_+.
satisfying \( v(\emptyset) = 0 \), is called a fuzzy game. Let \( G(N) \) denote the set of all fuzzy games in \( L(N) \). For any \( S \in L(N) \), we will use \( S = \{S(i)\}_{i \in N} \) to denote it, and we will omit braces for singletons, e.g., by writing \( v(S), S \vee (\wedge T), S \setminus T, U(i) \) instead of \( v(S), \{S\} \vee (\wedge \{T\}), \{S\} \setminus \{T\}, \{U(i)\} \) for any \( v \in G(N) \) and any \( \{U(i)\}, \{S\}, \{T\} \in L(N) \). In this paper, union and intersection of two fuzzy coalitions are defined as usual, i.e., \( (S \vee T)(i) = S(i) \vee T(i) \) and \( (S \wedge T)(i) = S(i) \wedge T(i) \).

Similar to crisp case, a fuzzy coalition structure \( \Gamma_F = \{B_1, B_2, \ldots, B_m\} \) in is a partition of \( U \), i.e., \( U \in L(U, \Gamma_F) \), and \( U \in SuppB_k \) if and only if \( \{i,j\} \in \Gamma_F \) for any \( i \neq j \in M = \{1, 2, \ldots, m\} \), where \( B_i \subseteq U \). The fuzzy coalition structure \( \Gamma_F \) in \( U \) is denoted by \( (U, \Gamma_F) \). For any \( S \in L(U, \Gamma_F) \), \( S \) is called a feasible fuzzy coalition, where \( L(U, \Gamma_F) \) denotes the set of all feasible fuzzy coalitions in \( (U, \Gamma_F) \).

Example 2: Let \( N = \{1, 2, 3, 4\} \), and \( U = \{U(i)\}_{i \in N} \) be a fuzzy coalition in \( L(N) \) such that \( U(i) > 0 \) for any \( i \in N \). \( \Gamma = \{B_1, B_2\} \) is a fuzzy coalition structure in \( U \), where \( B_1 = \{U(1), U(2)\} \) and \( B_2 = \{U(3), U(4)\} \), then \( L(U, \Gamma_F) = \{\emptyset, U(i)\}_{i \in N}, \{U(1), U(2)\}, \{U(3), U(4)\}, \{U(1), U(2), U(3), U(4)\} \).

Remark 1: In this paper, if there is no special explanation \( \Gamma_F = \{B_1, B_2, \ldots, B_m\} \) and \( M = \{1, 2, \ldots, m\} \) is assumed. Moreover, we use \( \{N, v_0, \Gamma\} \) and \( (U, v, \Gamma_F) \) to denote crisp game \( v_0 \in G_0(N) \) in coalition structure (\( N, \Gamma \)) and fuzzy game \( v \in G(N) \) in coalition structure \( (U, \Gamma_F) \), respectively.

Definition 1: Let \( v \in G(N) \), \( v^\beta \) is called a quotient game on \( \Gamma_F \), if it satisfies \( v^\beta(R) = v((\bigvee_{i \in R} B_i)) \) for all \( R \subseteq U \).

Similar to Lehrer [23], we give the concept of the “reduced” fuzzy game as follows:

For any \( U(i), U(j) \in \Gamma_F \), put \( U(p) = \{U(i), U(j)\} \) and consider the “reduced” fuzzy game \( v_p \) (with \( (U \setminus U(p)) \vee (U(p)) \) as the set of players) defined by \( v_p(S) = v(S) \) and \( v_p(S \setminus U(p)) = v(S \setminus U(p)) \) for any \( S \subseteq U \setminus U(p) \).

From above, we know the “reduced” fuzzy game \( v_p \) has index \( \text{Supp}(U_p) = 1 \).

Furthermore, we give the concept of the “reduced” game for quotient game \( v^\beta \) on \( (U, \Gamma_F) \). For any different indices \( k, q \in M \), let \( g = \{k, q\} \) and consider the “reduced” game \( v^\beta_g \) (with \( (M \setminus g) \cup \{g\} \)) as the set of players) defined by \( v^\beta_g(R) = v^\beta(R) \) and \( v^\beta_g(R \cup \{g\}) = v^\beta(R \cup g) \) for any \( R \in M \setminus g \).

From above, we know the “reduced” quotient game has index \( m - 1 \).

Similar to Alonso-Mejídie et al. [3], we give an axiomatic definition of the Banzhaf-Owen value for fuzzy games with a coalition structure as follows:

Definition 2: Let \( v \in G(N) \). The function \( f : (U, v, \Gamma_F) \rightarrow \mathbb{R}_{\geq 0}^{\Gamma_F(U)} \) is called a Banzhaf-Owen value in \( (U, \Gamma_F) \) if it satisfies

\[ f_i(U, v, \Gamma_F) = 0; \]

\[ f_i(U, v, \Gamma_F) = \frac{1}{2^{|\text{Supp}(B_i)|}} \left( v(S \vee U(i)) - v(S \vee Q) \right); \]

\[ f_i(U, v, \Gamma_F) = \frac{1}{2^{|\text{Supp}(B_i)|}} \left( v(S \vee Q \cap U(i)) - v(S \vee Q) \right), \]

\[ f_i(U, v, \Gamma_F) = \frac{1}{2^{|\text{Supp}(B_i)|}} \left( v(S \vee Q \cup U(i)) - v(S \vee Q) \right). \]
Axiom 4: From Eq.(2), we have
\[
\beta_i(U, v, \Gamma_F) + \beta_j(U, v, \Gamma_F) = \sum_{R \subseteq M \land k \subseteq B_k \setminus \{U(i)\}} \frac{1}{2^{\text{Supp}B_k - 1}} (v(S \cup v(U(i))) - v(S \cup Q)) + \sum_{R \subseteq M \land k \subseteq B_k \setminus \{U(j)\}} \frac{1}{2^{\text{Supp}B_k - 1}} (v(S \cup Q) - v(S \cup Q)).
\]

Uniqueness. Since, uniquely expressed by
\[
\bigcup_{l \in H} B_l = \{1, 2, \ldots, k - 1, k + 1, \ldots, m\},
\]
we have
\[
\beta_i(U, v, \Gamma_F) = \beta_i(U, v, \Gamma_F).
\]

Axiom 5: From Eq.(2), we get
\[
\beta_i(U, v, \Gamma_F) = \sum_{R \subseteq M \land k \subseteq B_k \setminus \{U(i)\}} \frac{1}{2^{\text{Supp}B_k - 1}} (v(S \cup Q \cup U(i))) - v(S \cup Q).
\]

Axiom 6: From Eq.(2), we obtain
\[
\beta_i(U, v, \Gamma_F) = \sum_{R \subseteq M \land k \subseteq B_k \setminus \{U(i)\}} \frac{1}{2^{\text{Supp}B_k - 1}} (v(S \cup Q \cup U(i))) - v(S \cup Q).
\]

Let \(\Gamma_j = \{B_1', B_2', \ldots, B_{m+1}'\}\), where \(B_{m+1}' = \{U(j)\}\), \(B_k' \cup U(j)\) and \(B_l = B_l\) for any \(l \in \{1, 2, \ldots, k - 1, k + 1, \ldots, m\}\). Thus,
\[
\beta_i(U, v, \Gamma_F) = \sum_{R \subseteq M \land k \subseteq B_k \setminus \{U(i)\}} \frac{1}{2^{\text{Supp}B_k - 1}} (v(S \cup Q \cup U(i))) - v(S \cup Q).
\]

Uniqueness. Since, any \(v \in G(N)\) limited in \((U, \Gamma_F)\) can be uniquely expressed by
\[
\bigcup_{A \subseteq S} \bigcup_{H \subseteq \text{Supp}A} \bigcup_{k \in H} (-1)^{r-k} (\sum_{A \subseteq S} (-1)^{r-k} (\text{Supp}A - \text{Supp}U) v(A \cup Q)),
\]
h denote the cardinalities of \(R\) and \(H\), respectively. \(Q = \bigcup_{l \in H} B_l\), and \(T = \bigcup_{l \in H} B_l\), such that \(\emptyset \neq S \subseteq B_k\).

\(u_T(R) = \begin{cases} 1 & T \subseteq R \\ 0 & \text{otherwise} \end{cases}\)

From Axiom 2, we only need to show the uniqueness of Eq.(2) on the unanimity games. For any \(\emptyset \neq T \subseteq L(U, \Gamma_F)\), let
\[
M' = \{k \in M : \text{Supp}B_k, \text{Supp}T \neq \emptyset\}
\]
and
\[
\text{Supp}B_k = \text{Supp}B_k, \text{Supp}T \quad \forall k \in M.
\]

Define the unanimity quotient game as follows:
\[
u_T^R(R) = \begin{cases} 1 & M' \subseteq R \\ 0 & M' \not\subseteq R \end{cases}
\]

where \(R \subseteq M\).

Let \(f\) be a solution in \((U, u_T, \Gamma_F)\) that satisfies the mentioned axioms in Definition 2. From Axiom 1, we have \(f(U, u_T, \Gamma_F) = 0\) for any \(i \not\in \text{Supp}T\).

Case (i): When \(m' = 1\), where \(m'\) denote the cardinality of \(M'\). From Axioms 1, 2, 3, 4, we get
\[
f_i(U, u_T, \Gamma_F) = \frac{1}{2^{\text{Supp}B_k - 1}} (u_B(R \cup k) - u_B(R))
\]

From Eq.(2), we have \(\beta(U, v, \Gamma_F) = f(U, u_T, \Gamma_F)\).

Case (ii): When \(m' > 1\), and \(\text{Supp}B_k = \{i\}\). From Axioms 5, we get
\[
f_i(U, u_T, \Gamma_F) = \frac{1}{2^{\text{Supp}B_k - 1}} (u_B(R \cup k) - u_B(R))
\]

On the other hand, from Eq.(2), we have
\[
\beta_i(U, u_T, \Gamma_F) = \sum_{R \subseteq M \land k \subseteq B_k \setminus \{U(i)\}} \frac{1}{2^{\text{Supp}B_k - 1}} (u_T(S \cup Q \cup U(i))) - u_T(S \cup Q)
\]

Thus, we have \(\beta(U, u_T, \Gamma_F) = f(U, u_T, \Gamma_F)\).

Case (iii): When \(m' > 1\), and \(\text{Supp}B_k = \{i\} \). Without loss of generality, suppose \(|\text{Supp}B_k| = h\), where \(\text{Supp}B_k = \{j_1, j_2, \ldots, j_h\}\). Let \(\Gamma_j = \{B_j, B_{j+1}, \ldots, B_{m+1}\}\), where \(B_{m+1} = \{U(j)\}\), \(B_k = B_k \cup U(j)\), and \(B_l = B_l\) for any \(l \in \{1, 2, \ldots, k - 1, k + 1, \ldots, m\}\). Let \(\Gamma_j = \{B_j, B_{j+1}, \ldots, B_{m+1}\}\), where \(B_{m+2} = \{U(j)\}\), \(B_k = B_k \cup U(j)\), and \(B_l = B_l\) for any \(l \in \{1, 2, \ldots, k - 1, k + 1, \ldots, m, m + 1\}\). From recursive
method and Axioms 5, 6, we get
\[
f_i(U, u_T, \Gamma_F) = \sum_{R \subseteq (M \cup H')^k} \left( \frac{1}{2^m+1} u^R(R \cup k) - u^R(R) \right) \\
= \frac{1}{2^m+1} \sum_{M \subseteq k, \Gamma \subseteq M} \sum_{k \in M} \left( \frac{1}{2^m+1} \sum_{k \in M} \sum_{k \in H'} \sum_{k \in M \cup H'} \right) \\
= \frac{1}{2^m+1} \sum_{M \subseteq k, \Gamma \subseteq M} \sum_{k \in M} \sum_{k \in H'} \sum_{k \in M \cup H'} \\
\text{where } H' = \{1, 2, ..., h-1\}. \text{ From Eq.(2), we obtain}
\[
\beta(U, u_T, \Gamma_F) = \frac{1}{2^{m+1}} \frac{1}{2^{h-1}} = \frac{1}{2^{m+h-2}}.
\]

Thus, Eq.(2) is unique on the unanimity games, and the proof is finished.

IV. THE BANZHAF-OWNEN VALUE FOR TWO SPECIAL KINDS OF FUZZY GAMES WITH A COALITION STRUCTURE

In this section, we shall discuss the Banzhaf-Ownen value for two special kinds of fuzzy games with a coalition structure, which are proposed by Owen [10] and Tsutsumi et al. [11]. The fuzzy coalition value for fuzzy games with "multilinear extension form" is expressed by (Owen [10])
\[
v_{G}(U) = \sum_{T \subseteq U} \Pi_{i \in \text{Supp}(T)}(U(i))\Pi_{i \in \text{Supp}(\Gamma \setminus T)}(1 - U(i)) \\
v_{G}(\text{Supp}(T)) \quad \forall U \in L(N).
\]
The fuzzy coalition value for fuzzy games with "Choquet integral form" is written as (Tsutsumi et al. [11])
\[
v_{C}(U) = \sum_{i=1}^{q(U)} v_{0}(\{U\}_{h_i})(h_i - h_{i-1}) \quad \forall U \in L(N).
\]
where \( |U|_{h_i} = \{i \in N | U(i) \geq h_i\} \) and \( Q(U) = U(i)U(i) > 0, i \in N \). \( q(U) \) denotes the cardinality of \( Q(U) \). The elements in \( Q(U) \) are written in the increasing order as 0 = \( h_0 \leq h_1 \leq \cdots \leq h_q(U) \).

Let \( G_{O}(U) \) and \( G_{C}(U) \) denote the set of all fuzzy games in \( L(N) \) given by Owen [10] and Tsutsumi et al. [11], respectively. For \( (U, v, \Gamma_F) \), when we restrict \( v \in G(N) \) in the setting of \( v_{O} \in G_{O}(U) \) and \( v_{C} \in G_{C}(U) \), then we get the coalition structure \((U, v_{O}, \Gamma_F)\) and \((U, v_{C}, \Gamma_F)\).

Theorem 2: Let \( v_{O} \in G_{O}(U) \), the function \( \beta_{O} : (U, v_{O}, \Gamma_F) \rightarrow \mathbb{R} \) is defined by
\[
\beta_{O}(U, v_{O}, \Gamma_F) = \sum_{R \subseteq (M \cup H')^k} \left( \frac{1}{2^m+1} \cdot 2^{\text{Supp}(R) - 1} \right) \\
\prod_{j \in \{T \subseteq \text{Supp}(\{V \subseteq W_{B_p}\}), U(j) = 0, j \in \text{Supp}(\{V \subseteq W_{B_p}\}) \}} \\
\prod_{j \in \{T \subseteq \text{Supp}(\{V \subseteq W_{B_p}\}), U(j) = 1, j \in \text{Supp}(\{V \subseteq W_{B_p}\}) \}} (1 - U(j)) \\
v_{0}(T_{0}) - \sum_{W \subseteq L(R)} \sum_{T_{0} \subseteq \text{Supp} \{S \subseteq \text{W}_{B_p}\}, U(j) = \sum_{j \in \text{Supp}(\{S \subseteq W_{B_p}\}) \}} (1 - U(j)) \\
v_{0}(T_{0}) \quad \forall i \in \text{Supp}(U), \quad (5)
\]

where \( L(R) \) denotes the power set of \( R \), \( \Gamma_{F}^{S_{V}}(U) \subseteq W_{B_p} \) and \( \Gamma_{F}^{S_{V}}(U) \subseteq W_{B_p} \) are the fuzzy coalition structures in \( S \cup U \), \( \Gamma_{F}^{S_{V}}(U) \subseteq W_{B_p} \) with respect to \( \Gamma_F \). \( \text{Supp}(L(S \cup U)) \subseteq W_{B_p}, \Gamma_{F}^{S_{V}}(U) \subseteq W_{B_p} \) denote the support of \( L(S \cup U) \subseteq W_{B_p}, \Gamma_{F}^{S_{V}}(U) \subseteq W_{B_p} \) and \( L(U) \subseteq W_{B_p}, \Gamma_{F}^{S_{V}}(U) \subseteq W_{B_p} \), respectively.

Example 3: (cf. Example 2). Let the player set \( N = \{1, 2, 3, 4\} \), and \( U = \{U(i)\}_{i} \in N \) be a fuzzy coalition in as defined in Example 3.1, i.e., \( U(i) > 0 \) for any \( i \in N \), and \( \Gamma_F = \{B_1, B_2\} \) is a fuzzy coalition structure in \( U \) as defined in Example 3.1, i.e., \( B_1 = \{U(1), U(2)\} \) and \( B_2 = \{U(3), U(4)\} \), then
\[
\text{Supp}(L(U, \Gamma_F)) = \{0, \{i\}_{i} \in N\}, \{1, 2\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{N\}.
\]
If \( S = \{U(1), U(2), U(3)\} \subseteq U \), then we get
\[
L(S, \Gamma_F^{S_{V}}) = \{0, \{i\}_{i} \in \{1, 2, 3\}, \{U(1), U(2), U(3)\}, \{1, 2, 3\} \}
\]
and
\[
\text{Supp}(L(S, \Gamma_F^{S_{V}})) = \{0, \{i\}_{i} \in \{1, 2, 3\}, \{1, 2, 3\} \}.
\]

Theorem 3: Let \( v_{C} \in G_{C}(U) \), the function \( \beta_{C} : (U, v_{C}, \Gamma_F) \rightarrow \mathbb{R} \) is defined by
\[
\beta_{C}(U, v_{C}, \Gamma_F) = \sum_{i=1}^{q(U)} \beta_{i}([U]_{h_i}, v_{0}, \Gamma_F^{[V]_{h_i}})(h_i - h_{i-1}) \\
\quad \forall i \in \text{Supp}(U) \quad (6)
\]
where \( \beta_{i}([U]_{h_i}, v_{0}, \Gamma_F^{[V]_{h_i}}) \) denotes the Banzhaf-Owen value in \([U]_{h_i}, \Gamma_F^{[V]_{h_i}}\) shown as Eq.(1). Then \( \beta_{C} \) is the unique Banzhaf-Owen value in \((U, v_{C}, \Gamma_F)\).

Example 4: Let the player set \( N = \{1, 2, 3, 4, 5\} \), \( v \in G(N) \) and \( U = \{U(i)\}_{i} \in N \) be a fuzzy coalition in \( L(N) \) such that \( U(i) > 0 \) for any \( i \in N \), where \( U(1) = 0.6, U(2) = 0.3, U(3) = 0.5, U(4) = 0.7 \) and \( U(5) = 0.8. \Gamma_F = \{B_1, B_2, B_3\} \) is a fuzzy coalition structure in \( U \), where \( B_1 = \{U(1), U(2)\}, B_2 = \{U(3), U(4)\} \) and \( B_3 = \{U(5)\} \). The crisp coalition values are given by Table 1.

When we restrict \( v \in G(N) \) in the setting of \( v_{O} \in G_{O}(U) \), from Eq.(5), we get the player Banzhaf-Owen values with respect to \( v_{O} \) are
\[
\beta_{O}(U, v_{O}, \Gamma_F) = 0.84, \beta_{O}(U, v_{O}, \Gamma_F) = 0.83,
\]
\[
\beta_{O}(U, v_{O}, \Gamma_F) = 0.78, \beta_{O}(U, v_{O}, \Gamma_F) = 1.63,
\]
TABLE I (VALUES FOR CRISP COALITIONS)

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$\beta_C(U, v_C, \Gamma_F) = 1.51$.

When we restrict $v \in G(N)$ in domain of $v_C \in G_C(U)$, from Eq.(6), we get the player Banzhaf-Owen values with respect to $v_C$ are

$\beta_C^1(U, v_C, \Gamma_F) = 1.56$, $\beta_C^2(U, v_C, \Gamma_F) = 1.69$,

$\beta_C^3(U, v_C, \Gamma_F) = 1.5$, $\beta_C^4(U, v_C, \Gamma_F) = 2.6$,

$\beta_C(U, v_C, \Gamma_F) = 2.2$.

V. CONCLUSION

We have researched the Banzhaf-Owen value for fuzzy games with coalition structures, which extends the using scope of the Banzhaf-Owen value. Like other payoff indices, there are other axiomatic systems to show the existence and uniqueness of the Banzhaf-Owen value, such as Owen [2], Alonso-Mejide et al. [3]. Moreover, we briefly study two special kinds of fuzzy games with coalition structures.

At present, the research about payoff indices for fuzzy games with coalition structures is not perfect. We shall continue to study in this area. Furthermore, we shall study other kinds of game theory, such as network formation games and playing online games.

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