Multivariable Predictive PID Control for Quadruple Tank

Qamar Saeed, Vali Uddin and Reza Katebi

Abstract—In this paper multivariable predictive PID controller has been implemented on a multi-inputs multi-outputs control problem i.e., quadruple tank system, in comparison with a simple multi-loop PI controller. One of the salient feature of this system is an adjustable transmission zero which can be adjust to operate in both minimum and non-minimum phase configuration, through the flow distribution to upper and lower tanks in quadruple tank system. Stability and performance analysis has also been carried out for this highly interactive two input two output system, both in minimum and non-minimum phases. Simulations of control system revealed that better performance are obtained in predictive PID design.

Keywords—Proportional-integral-derivative Control, Generalized Predictive Control, Predictive PID Control, Multivariable Systems

I. Introduction

The three term proportional, integral and derivative (PID) controllers ruled over the process industry for more than six decade, and still existing. The major selling point of PID controllers is due to its simplified structure, robustness and over a wide range of applicability and suitable performance, but limited to simple control problems. In 1939, the first commercial application of PID controller was introduced [1] and a great deal of research and development commenced. Since 1942, numerous PID tuning techniques have been developed and a summary of most popular tuning methods for PID controller are available in [2].

In last few decade, advancement and competition in process industry developed many complex control problems where classical PID were unable to cope and control community strive for better solution. Miller et. al., [3] have illustrated some of the main challenges faced by the control community. Considering the popularity and reliability of PID, many researcher tried to develop optimal PID. Rivera et. al., [4] introduced an Internal Model Control (IMC) based PID controller design using a first order process model, and this was later extended by Chien [5] for second order process model. Morari and Zafiriou [6], proposed IMC leads to PID controllers for virtually all models common in industrial practice. Wang et. al., [7] proposed a PID controller using a frequency response approach with least squares algorithm to equate with IMC. Rusnak [8] used linear quadratic regulator (LQR) theory to design PID controllers for a fifth order system. Grimble [9], derived H_∞ based PID structure. Katebi and Moradi [10] have introduced the predictive PID controller for SISO systems and Moradi et. al., [11] extended it for MIMO systems in polynomial form. The Generalized Predictive Control (GPC) method proposed by Clarke et. al., [12] is a reasonable representative of model based predictive control (MPC) methods and one of the most general way of posing the process control problem in time domain [13]. Tan et. al., [14] have presented a PID control design based on the GPC approach for a second order system with time delay but limited to single-input single output (SISO) systems. In this paper, we have develop a multi-input multi-output Predictive PID controller using the same approach as used by Tan et. al., [15] for his SISO systems.

In the recent past, multi-variable control system design have been in great demand and need much attention in the process industry and academia. In many processes, when some or all of the manipulated variable affects more than its corresponding controlled variable, mean there are some interaction between the controlled variable, which may result in poor performance or even in instability of control process. When the interaction are not negligible, the plant must be considered as multiple inputs and multiple outputs. In this paper, a highly interactive multi-variable process has been considered i.e., quadruple tank problem. This multi-variable systems contains a transmission zeros, which can vary from left half plane (minimum phase) to right half plane (non-minimum phase) depending on the ratio of the flow to upper and lower tanks [16].

The paper has been organized as follows: Section II briefly describe the model development of real processes. Manual multi-loop PI and predictive PID control design techniques has been discussed in section III and IV respectively. Stability analysis has been conducted in Section V. Performance analysis and simulation results are available in section VI. Finally, the conclusions are given in section VII.

II. Model Development

Johansson [16] described a laboratory quadruple-tank process which consists of four interconnected water tanks and two pumps as shown in fig. 1. The first principle mathematical model for this process using mass balances and Bernoulli's law is

$$\begin{aligned}
\frac{dh_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}v_1 \\
\frac{dh_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}v_2 \\
\frac{dh_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}v_2 \\
\frac{dh_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}v_1
\end{aligned} \tag{1}$$

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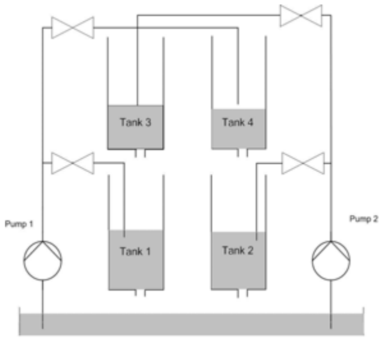

Fig. 1. Schematic diagram for Quadruple Tank process

where γ_i is the flow distribution to lower and diagonal upper tank, A_i is the cross-section area, a_i is the outlet hole cross-section and h_i is the water level, in tank i respectively.

There are two inputs (manipulators) and two outputs (controlled variable) in quadruple tank system, and control objective is to maintain water level in lower tanks around its setpoint with the manipulation of water flow with two pumps. The process inputs are v_1 and v_2 and the outputs are $y_1 = k_c h_1$ and $y_2 = k_c h_2$.

The voltage applied to Pump i is v_i and the corresponding flow is $k_i v_i$. The parameters $\gamma_1, \gamma_2 \in (0, 1)$ are valves setting for the distribution of flow to lower and upper diagonal tank respectively. The flow to Tank 1 is $\gamma_1 k_1 v_1$ and the flow to Tank 4 is $(1 - \gamma_1)k_1 v_1$ and similarly for Tank 2 and Tank 3 as shown in fig. 1. The g is denoted as acceleration of gravity. The parameter values of the laboratory process and operating parameter for minimum (P_-) and non-minimum phases (P_+) are given in [16], also shown in table 1 and table 2 respectively.

This typical system has two finite zeros for $\gamma_1, \gamma_2 \in (0, 1)$. One always lie in the left half-plane, but the other can be placed either in the left or the right half-plane depending on the valve setting of γ_1, γ_2.
If $1 < \gamma_1 + \gamma_2 \leq 2$ then system is minimum phase, means transmission zero is in left half plane.
If $0 \leq \gamma_1 + \gamma_2 < 1$ then system is non-minimum phase, means transmission zero is in right half plane.
If $\gamma_1 + \gamma_2 = 1$ then system has transmission zero at origin, a difficult case to handle using simple multi-loop PID control system design without de-coupler.

TABLE I
PARAMETER VALUE FOR QUADRUPLE TANK

Parameters	Units	Values
A_1,A_3	$[cm^2]$	28
A_2,A_4	$[cm^2]$	32
a_1,a_3	$[cm^2]$	0.071
a_2,a_4	$[cm^2]$	0.057
k_c	$[V/cm]$	0.5
g	$[cm/s^2]$	981

The linearized state-space equation at operating points $x_i = h_i - h_i^0$ and $u_i = v_i - v_i^0$ is given

TABLE II
OPERATING PARAMETER FOR MINIMUM (P_-) AND NON-MINIMUM (P_+) PHASES

Operating Values	Units	P_-	P_+
(h_1^0,h_2^0)	$[cm]$	$(12.4, 12.7)$	$(12.6, 13.0)$
(h_3^0,h_4^0)	$[cm]$	$(1.8.1.4)$	$(4.8, 4.9)$
(v_1^0,v_2^0)	$[V]$	$(3.00, 3.00)$	$(3.15, 3.15)$
(k_1,k_2)	$[cm^2/Vs]$	$(3.33, 3.35)$	$(3.14, 3.29)$
(γ_1,γ_2)	-	(0.70,0.60)	$(0.43, 0.34)$

in [16]

$$\begin{aligned} \frac{dx}{dt} &= \begin{pmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1 T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2 T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{pmatrix} x \\ &+ \begin{pmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{pmatrix} u \\ y &= \begin{pmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{pmatrix} x \end{aligned} \tag{2}$$

where the time constants are $T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}$.

Linearized transfer function matrix model for both minimum ($P-$) and non-minimum ($P+$) phases are given in [16] and [17].

III. MANUAL MULTI-LOOP PI CONTROL

The discrete position and velocity form of PID controller are described by equations (3) and (4) respectively [10].

$$u(k) = k_p e(k) + k_i \sum_{j=1}^{k} e(j) + k_d[e(k) - e(k-1)] \tag{3}$$

$$\begin{aligned} \Delta u(k) = u(k) - u(k-1) = k_p[e(k) - e(k-1)] + \\ + k_i e(k) + k_d[e(k) - 2e(k-1) + e(k-2)] \end{aligned} \tag{4}$$

where, k_p, k_i and k_d are the proportional, integral and derivative gains, respectively.

Johansson [16] applied multi-loop PI controller on quadruple tank manually for both minimum and non-minimum phase configuration in frequency domain. However, in this section same tuning parameters are implemented in control system design using state space formulation. All simulation results have been discussed in section VI in comparison with [16].

If valve setting of γ_1 and γ_2 for the distribution of water flow to upper and lower tanks are chosen as $\gamma_1 + \gamma_2 = 1.0$, then this typical control problem become extremely interactive between two controlled variables, with one transmission zero at origin, and it will not possible to obtain a suitable tuning parameters using multi-loop manual tuning without using de-coupler, as mentioned in section I.

IV. Predictive PID Control

Consider a two-input two-output square multi-variable system is given as

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{5}$$

In equation (5), each g_{ij} (where $i, j = 1, 2$) contains a subsystem which can be represented as [14] and [15]

$$g_{ij}(s) = \frac{ds + c}{(s+a)(s+b)} e^{-sL} \tag{6}$$

The equation (6) in discrete-time transfer function can be represented as

$$\acute{g}_{ij}(z) = \frac{\acute{b_1} z + \acute{b_2}}{z^2 + \acute{a_1} z + \acute{a_2}} z^{-t_d} \tag{7}$$

With some algebraic manipulation in equations (5 - 7), two multi-input single output (MISO) model can be obtained as,

$$\begin{aligned} &\begin{pmatrix} z^2 + \acute{a}_{11}^1 z + \acute{a}_{11}^2 & 0 \\ 0 & z^2 + \acute{a}_{22}^1 z + \acute{a}_{22}^2 \end{pmatrix} \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} \\ &= \begin{pmatrix} \acute{b}_{11}^0 z + \acute{b}_{11}^1 & \acute{b}_{12}^0 z + \acute{b}_{12}^1 \\ \acute{b}_{21}^0 z + \acute{b}_{21}^1 & \acute{b}_{22}^0 z + \acute{b}_{22}^1 \end{pmatrix} \begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} \end{aligned} \tag{8}$$

The error to the controller is represented as $e = y_d - y$, where y_d is the setpoint and y is the controlled output, then in terms of error, with $y_d = 0$, equivalent equation can be represented as,

$$\begin{aligned} e_1(k+1) &= \acute{a}_{11}^1 e_1(k) - \acute{a}_{11}^2 e(k-1) \\ &\quad - \acute{b}_{11}^0 \tilde{u}_1(k) - \acute{b}_{12}^0 \tilde{u}_2(k) \end{aligned} \tag{9}$$

$$\begin{aligned} e_2(k+1) &= \acute{a}_{22}^1 e_2(k) - \acute{a}_{22}^2 e(k-1) \\ &\quad - \acute{b}_{21}^0 \tilde{u}_1(k) - \acute{b}_{22}^0 \tilde{u}_2(k) \end{aligned} \tag{10}$$

where

$$\begin{aligned} \tilde{u}_1(k) &= u_1(k) + \frac{\acute{b}_{11}^1}{\acute{b}_{11}^0} u_1(k-1) \\ \tilde{u}_2(k) &= u_2(k) + \frac{\acute{b}_{12}^1}{\acute{b}_{12}^0} u_2(k-1) \end{aligned}$$

Equations (9 - 10) can be represented in state space form as

$$X(k+1) = AX(k) + B\tilde{u}(k) \tag{11}$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -\acute{a}_{11}^2 & -\acute{a}_{11}^1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ -\acute{a}_{22}^2 & -\acute{a}_{22}^1 & 0 \\ 0 & 1 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 0 \\ -\acute{b}_{11}^0 & -\acute{b}_{12}^0 \\ 0 & 0 \\ 0 & 0 \\ -\acute{b}_{21}^0 & -\acute{b}_{22}^0 \\ 0 & 0 \end{pmatrix}$$

$$X(k) = \begin{pmatrix} e_1(k-1) \\ e_1(k) \\ \theta_1(k) \\ e_2(k-1) \\ e_2(k) \\ \theta_2(k) \end{pmatrix}, \tilde{u}(k) = \begin{pmatrix} \tilde{u}_{11}(k) \\ \tilde{u}_{12}(k) \\ \tilde{u}_{21}(k) \\ \tilde{u}_{22}(k) \end{pmatrix}$$

and $\theta_i = \sum_{j=1}^{k} e_i(j)$ is the integral error.
Using p and m prediction and control horizons respectively, the predicted error in compact form can be represented as [14] and [15],

$$\mathbf{\bar{X}} = \mathbf{\bar{L}AX(k)} + \mathbf{B_M\tilde{U}} \tag{12}$$

where

$$\begin{aligned} \mathbf{\bar{X}} &= \begin{pmatrix} X^T(k+1) & X^T(k+2) & \cdots & X^T(k+p) \end{pmatrix}^T \\ \mathbf{\bar{L}} &= \begin{pmatrix} I \\ A \\ \vdots \\ A^{p-1} \end{pmatrix} \\ \mathbf{B_M} &= \begin{pmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ A^{p-1}B & A^{p-2}B & \cdots & A^{p-m}B \end{pmatrix} \\ \mathbf{\tilde{U}} &= \begin{pmatrix} \tilde{u}(k) & \tilde{u}(k+1) & \cdots & \tilde{u}(k+m-1) \end{pmatrix}^T \end{aligned}$$

Since GPC is an optimal control strategy, therefore a performance index or cost function must be minimized in order to obtain an optimal control signal. Considering the following cost function

$$J = \sum_{l=1}^{p} \|x(k+l)\|_{Q(l)}^2 + \sum_{j=1}^{m} \|u(k+j-1)\|_{R(j)}^2 \tag{13}$$

where Q and R are the error and control weighting matrices respectively. Substitution of prediction equation (12) in cost function (13) i.e., an optimization step, resulted an optimal control sequence, like [14] and [15]

$$\bar{U} = -[B_M^T Q B_M + R]^{-1} [B_M^T Q L A] X(k) \tag{14}$$

Under the receding horizon principle, only the first value of the optimal control sequence is applied at each sampling time while the rest are discarded. Therefore,

$$\begin{aligned} \tilde{u}(k - t_d) &= -H[B_M^T Q B_M + R]^{-1} [B_M^T Q L A] X(k), \\ &= -DX(k) \end{aligned} \tag{15}$$

where $D = H[B_M^T Q B_M + R]^{-1} [B_M^T Q L A]$
and $H = \begin{bmatrix} I & \mathbf{0} & \ldots & \mathbf{0} \end{bmatrix}$
From equation (15), it follows that $\tilde{u} = -DX(k + t_d)$, which means that current control value depends on the future predicted state. In case of significant time delay, this problem would be solved in two different range i.e., $0 \leq k < t_d$ and $k \geq t_d$, [14] and [15]. However, in absence of time delay (i.e., $t_d = 0$) then control law would simply be

$$\tilde{u} = -DX(k) \tag{16}$$

Similarly,

$$\begin{pmatrix} \tilde{u}_{11,k} \\ \tilde{u}_{12,k} \\ \tilde{u}_{21,k} \\ \tilde{u}_{22,k} \end{pmatrix} = \begin{pmatrix} D_{11} & 0 \\ D_{12} & 0 \\ 0 & D_{21} \\ 0 & D_{22} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \tag{17}$$

There is no significant time delay in quadruple tank system, so PID tuning parameters would only be consider for $k \geq t_d$

$$\left. \begin{cases} K_P = -(K_1(t_d) + K_2(t_d)) \\ K_I = -K_3(t_d) \\ K_D = K_1(t_d) \end{cases} \right\} \quad k \geq t_d$$

Earlier we have assumed that,

$$\tilde{u}_{11,k} = u_{1,k} + \frac{\acute{b}^1_{11}}{\acute{b}^0_{11}} u_{1,k-1} \quad (18)$$

$$\tilde{u}_{12,k} = u_{2,k} + \frac{\acute{b}^1_{12}}{\acute{b}^0_{12}} u_{2,k-1} \quad (19)$$

$$\tilde{u}_{21,k} = u_{1,k} + \frac{\acute{b}^1_{21}}{\acute{b}^0_{21}} u_{1,k-1} \quad (20)$$

$$\tilde{u}_{22(k} = u_{2,k} + \frac{\acute{b}^1_{22}}{\acute{b}^0_{22}} u_{2,k-1} \quad (21)$$

On combining equations (18 & 20) and (19 & 21), we obtain

$$\tilde{u}_{11,k} + \tilde{u}_{21,k} = 2u_{1,k} + (\frac{\acute{b}^1_{11}}{\acute{b}^0_{11}} + \frac{\acute{b}^1_{21}}{\acute{b}^0_{21}}) u_{1,k-1} \quad (22)$$

$$\tilde{u}_{12,k} + \tilde{u}_{22(k} = 2u_{2,k} + (\frac{\acute{b}^1_{12}}{\acute{b}^0_{12}} + \frac{\acute{b}^1_{22}}{\acute{b}^0_{22}}) u_{2,k-1} \quad (23)$$

Similarly,

$$u_{1,k} = \frac{1}{2}\left\{(\tilde{u}_{11,k} + \tilde{u}_{21,k}) - (\frac{\acute{b}^1_{11}}{\acute{b}^0_{11}} + \frac{\acute{b}^1_{21}}{\acute{b}^0_{21}}) u_{1,k-1}\right\} (24)$$

$$u_{2,k} = \frac{1}{2}\left\{(\tilde{u}_{12,k} + \tilde{u}_{22(k)}) - (\frac{\acute{b}^1_{12}}{\acute{b}^0_{12}} + \frac{\acute{b}^1_{22}}{\acute{b}^0_{22}}) u_{2,k-1}\right\} (25)$$

V. Stability Analysis

The stability is one of the major concern in all control system design. In case of linearized model, the stability of overall closed loop system is determined by characteristic equation.

In terms of transfer function two input two output (TITO) system can be described as,

$$y_1 = g_{11}u_1 + g_{12}u_2 \quad (26)$$

$$y_2 = g_{21}u_1 + g_{22}u_2 \quad (27)$$

In multi-loop PI control tuning for minimum phase problem, the control law is

$$u_1 = g_{c1}(y_{d,1} - y_1) \quad (28)$$

$$u_2 = g_{c2}(y_{d,2} - y_2) \quad (29)$$

As setpoint do not play any role in system stability, so let $y_{d,2} = 0$. Now substituting the equation (29) in equation (26) with algebraic manipulation, we get the characteristic equation for minimum phase ($C.E_{mp}$) as,

$$C.E_{mp} = 1 + g_{c1}g_{11} + g_{c2}g_{22} + g_{c1}g_{c2}(g_{11}g_{22} - g_{12}g_{21}) \quad (30)$$

Similarly, the control law for non-minimum phase multi-loop PID controllers are described as

$$u_1 = g_{c1}(y_{sp,2} - y_2) \quad (31)$$

$$u_2 = g_{c2}(y_{sp,1} - y_1) \quad (32)$$

On substitution of equations (31-32) in equations (26-27) yield

$$\begin{pmatrix} 1 + g_{12}g_{c1} & g_{11}g_{c2} \\ g_{22}g_{c1} & 1 + g_{21}g_{c2} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} g_{12}g_{c1} & g_{11}g_{c2} \\ g_{22}g_{c1} & g_{21}g_{c2} \end{pmatrix} \begin{pmatrix} y_{sp,1} \\ y_{sp,2} \end{pmatrix} \quad (33)$$

Eventually, characteristic equation for non-minimum phase ($C.E_{nmp}$) obtained as

$$C.E_{nmp} = (1 + g_{12}g_{c1})(1 + g_{21}g_{c2}) - g_{c1}g_{c2}g_{11}g_{22} \quad (34)$$

Johansson [16] describe that non-minimum phase configuration in quadruple tank system, is relatively a difficult control problem as one of the pole is very close to unit circle.

The stability of Predictive PID design based on GPC can also be carried out in a similar manner. From (24-25), we can obtained $u_{1,k}$ and $u_{2,k}$ as

$$u_{1,k} = \left\{2 + \left(\frac{\acute{b}^1_{11}}{\acute{b}^0_{11}} + \frac{\acute{b}^1_{21}}{\acute{b}^0_{21}}\right) z^{-1}\right\}^{-1} (\tilde{u}_{11,k} + \tilde{u}_{21,k}) (35)$$

$$u_{2,k} = \left\{2 + \left(\frac{\acute{b}^1_{12}}{\acute{b}^0_{12}} + \frac{\acute{b}^1_{22}}{\acute{b}^0_{22}}\right) z^{-1}\right\}^{-1} (\tilde{u}_{12,k} + \tilde{u}_{22,k}) (36)$$

From equation (17), we have,

$$\tilde{u}_{11,k} + \tilde{u}_{21,k} = \begin{pmatrix} D_{11} & D_{21} \end{pmatrix} X(k) \quad (37)$$

$$\tilde{u}_{12,k} + \tilde{u}_{22,k} = \begin{pmatrix} D_{12} & D_{22} \end{pmatrix} X(k) \quad (38)$$

where $X(k)$ is defined as,

$$X(k) = \begin{pmatrix} X_1(k) & X_2(k) \end{pmatrix}^T \quad (39)$$

in which

$$X_1(k) = \begin{pmatrix} e_1(k-1) & e_1(k-1) & \theta_1(k) \end{pmatrix}^T$$

$$X_2(k) = \begin{pmatrix} e_2(k-1) & e_2(k-1) & \theta_2(k) \end{pmatrix}^T$$

Using equations (26 - 27) and (35 - 38), a characteristic equation can be obtained and stability of the closed loop system are determined. In predictive PID design, we obtained the PID tuning parameters on the basis of GPC tuning parameters and equate to the PID tuning parameters in result as mentioned in section IV. Moreover, all simulation results revealed that systems are stable along the selected tuning parameters.

VI. Performance Analysis & Simulation Results

The multi-loop PI tuning parameters are obtained from [16], where $(K_1, T_{i1}) = (3.0, 30)$ and $(K_2, T_{i2}) = (2.7, 40)$ for minimum phase (P_{-}) while $(K_1, T_{i1}) = (1.5, 110)$ and $(K_2, T_{i2}) = (-0.12, 220)$ for non-minimum phase (P_{+}) system respectively. In this paper, simulation have been carried out in time domain using state space approach in comparison with earlier Johansson [16], frequency domain approach. Moreover, we have also plotted the upper tanks (i.e., tank 3 and tank 4) level along with the lower tanks (i.e., tank 1 and tank 2), for better illustration and understanding.

For minimum phase, we have observed a peak (i.e., overshoot) upto 7.45 with settling time is 60 sec as shown in fig. 2, while [16] indicated peak upto 7.3 with settling time 80 sec.

For non-minimum phase, a slight inverse response along with much higher overshoot upto 11 followed by a minor undershoot in output y_2 with settling time around 2450 sec (i.e., 35 times more than minimum phase) has been observed in our simulation by simple multi-loop PI control as shown in

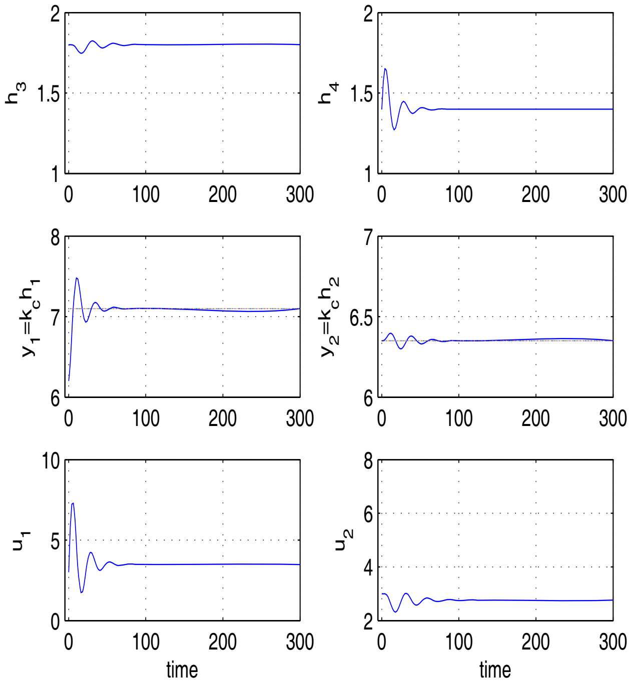

Fig. 2. Manual PID design for minimum phase

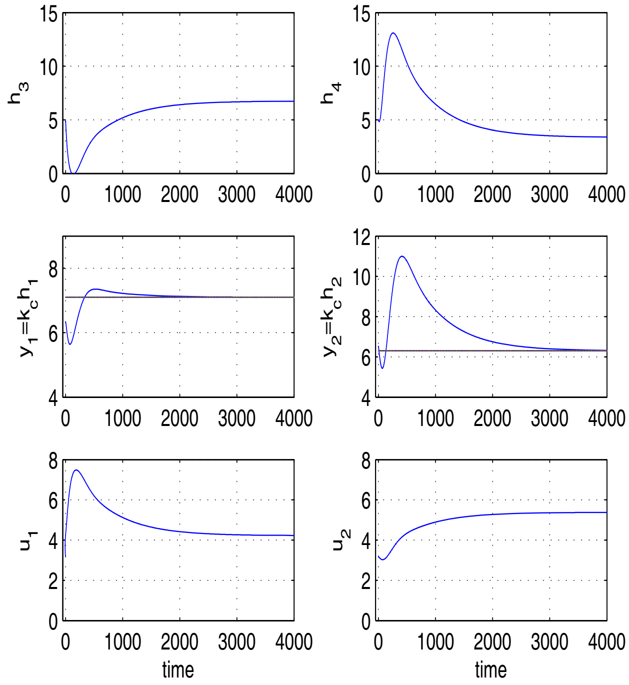

Fig. 4. Manual PID design for non-minimum phase

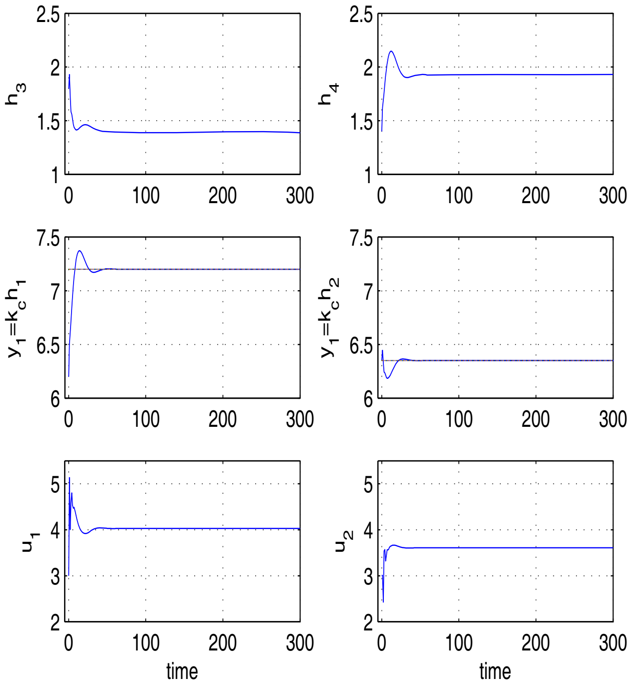

Fig. 3. Predictive PID design for minimum phase

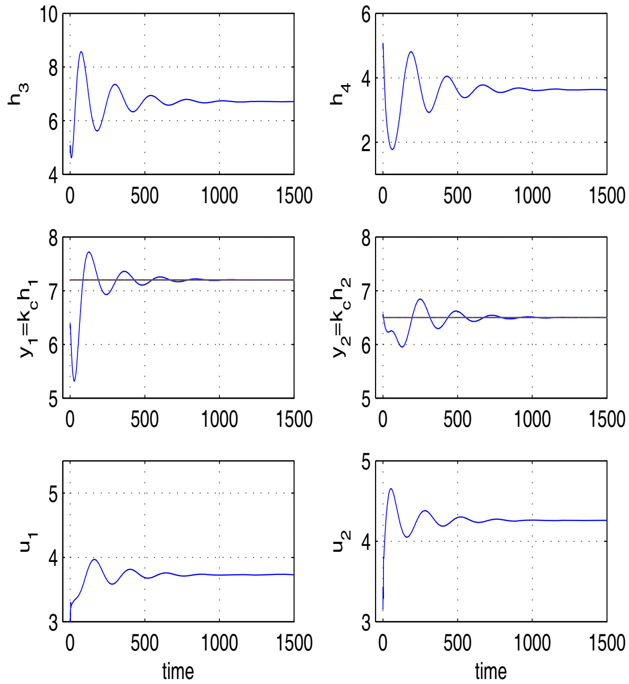

Fig. 5. Predictive PID design for non-minimum phase

fig. 4, while [16] indicated no inverse response and overshoot only upto 7.9 with settling time around 1200 sec (i.e., 15 times more than minimum phase).

Our simulation results are based on state space model in comparison with [16], which are based on transfer function model, the results we have obtained are slightly different than [16], but the trends are almost the same. Moreover, we are in concordance that non-minimum phase problem is much more difficult to tackle using manual multi-loop PID control design as concluded in [16].

In section IV, we have developed a MIMO predictive PID controller for quadruple tank problem using the same technique as given in [14] and [15] for SISO system. For minimum phase, we have used prediction horizon as $N_1 = 1$, $N_2 = 40$ and control horizon as $N_u = 40$ with weighting $R = \rho I_2 = 10I_2$ and $Q = I_3$ and observed the closed loop response of y_1 with peak as 7.35 for setpoint $y_{sp,1} = 7.2$ while y_2 regain its initial position within settling time i.e., 35 sec as shown in fig. 3.

Similarly for non-minimum phase, we have used the prediction horizon $N_1 = 1$, $N_2 = 100$ and control horizon $N_u = 2$ with weighting $Q = I_3$ and $R = \rho I_2 = 1000I_2$ an inverse response have been observed with settling time around 700 sec i.e., more than 20 times of the minimum phase problem as shown in fig 5. However, in real plant i.e., a nonlinear system result could be slightly different as the water drain from any respective tank depends on the square root of its level, not directly to its level.

VII. Conclusions

The structure of the P-PID is not much different from the conventional PID, therefore implementation does not make any difference. The effectiveness of all these methods have been well illustrated in simulations.

It has been observed that in each design technique, non-minimum phase is quite difficult to control. Multivariable system with unstable transmission zeros usually come across with internal instability problems i.e., a difficult aspect to control a process with RHPT zero canceled by a RHP pole. An important characteristic of RHPT zeros of multi-input multi-output (MIMO) systems is that it contains hidden dynamics.

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